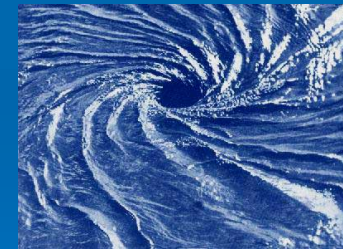
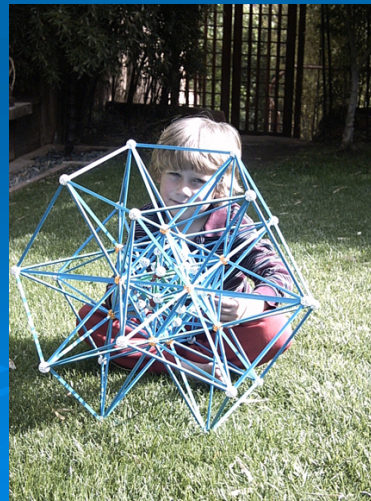
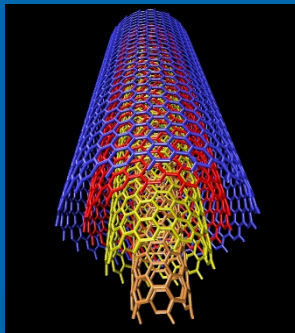
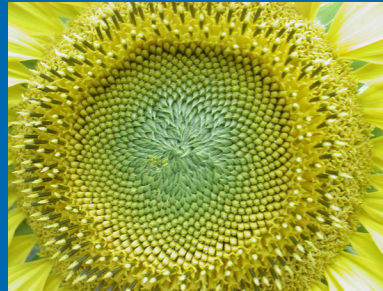
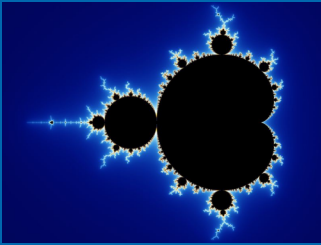


# SG203.Ia The Resurgence of Sacred Geometry in Science

## Online Module SG 203-A (Interm III-A)



Presented by AYA  
& the School of Sacred Geometry  
P. O. Box 3714  
Sedona, AZ 86340

[www.SchoolOfSacredGeometry.org](http://www.SchoolOfSacredGeometry.org)  
[phi@schoolofsacredgeometry.org](mailto:phi@schoolofsacredgeometry.org)  
[www.starwheels.com](http://www.starwheels.com)

# SG203.Ib Sacred Geometry in Science - Contents

## PART A (SG203-A)

### Introduction

Timeline

### 1. Fractals & Self-Similarity

- 1.1 Ancient Roots of Fractality (1-2)
- 1.2 Simple Iterative Fractals (1-2)
- 1.3 Proto-Fractals (1-3)
- 1.4 Sierpinski Triangle (1-3)
- 1.5 Fractals & B. Mandelbrot (1-3)
- 1.6 The Mandelbrot Set (1-2)
- 1.7 Faces of the M-Set (1-7)
- 1.8 M-Set and Phi (1-3)
- 1.9 Golden Fractals (1-6)
- 1.10 Phi: a Continued Fractal
- 1.11 Chaos Theory & Fractals (1-5)
- 1.12 Fractals in Human Body
- 1.13 Fractals in Nature
- 1.14 Fractal Models (1-2)
- 1.15 Fractals in technology (1-2)
- 1.16 Fractals in Culture
- 1.17 Fractals in Art
- 1.18 A Golden Fractal Universe

### 2. Penta-Symmetries

- 2.1 Penrose Tiling (1-2)
- 2.2 The two Penta-Modules
- 2.3 Penrose Tiling 2D (1-2)
- 2.4 Penrose Tiling 3D (1-4)
- 2.5 Quasicrystals - The Story (1-3)
- 2.6 Quasicrystals - Diffusion Patterns
- 2.7 Quasicrystals - R & D (1-2)

2.8 Quasicrystals - Design (1-2)

2.9 Water Micro Clusters (1-3)

2.10 Fullerenes - Buckyball (1-3)

2.11 Fullerenes - Variations (1-2)

2.12 Fullerenes - Applications (1-2)

### 3. Platonic Models in Science

- 3.1 The Nesting of the Cosmic Figures
- 3.2 The All Five Puzzle
- 3.3 The Dodeca-Icosa Doctrine
- 3.4 Kepler's Cosmic Cup
- 3.5 Felix Klein and the Icosahedron
- 3.6 New model Atom (1-4)
- 3.7 Platonic Wave Resonance
- 3.8 Oscillons & Solitons (1-2)
- 3.9 Rotating Platonics Atom Model
- 3.10 Atomic Nucleus Moon Model (1-3)
- 3.11 Tetrahedral Physics (1-2)
- 3.12 Platonic Chemistry (1-4)
- 3.13 Platonic Cosmology (1-2)

## PART B (SG203-B)

### 4. Fibonacci/Lucas & PHI

- 4.1 Fibon/Lucas Reminder (1-3)
- 4.2 Fibon in Maths (1-3)
- 4.3 Golden Maths (1-4)
- 4.4 Pascal Triangle (1-4)
- 4.5 Fibon in Technology (1-2)
- 4.6 Fibon in Nature (1-5)
- 4.7 Fibon Reflections

### 5. Vortex Science

- 5.1 The Insights of V. Schauburger (1-2)
- 5.2 What is Implosion? (1-2)
- 5.3 Theodor Schwenk (1-2)
- 5.4 What is a Vortex? (1-2)
- 5.5 Walter Russell (1-2)
- 5.6 Lawrence Edwards (1-2)
- 5.7 Golden Vortex (1-5)
- 5.8 Solitonic Vortices
- 5.9 Vortex Gallery (1-3)
- 5.10 Vortex Water (1-5)
- 5.11 Egg-Shape power (1-2)
- 5.12 Vortex Technology (1-2)
- 5.13 Vortex Sculptures

### 6. A Golden Matrix Universe

- 6.1 New Paradigm (1-4)
- 6.2 A Coherent Cosmos
- 6.3 A Non-Local Cosmos (1-2)
- 6.4 A Scale-Invariant Cosmos (1-4)
- 6.5 A Cosmos of In-Formation
- 6.6 The Field (1-3)
- 6.7 A Cosmos of Consciousness
- 6.8 Phi in Quantum (1-2)
- 6.9 E-Infinity / El Naschie (1-3)
- 6.10 Raji Heyrovska (1-4)
- 6.11 Golden Implosion / Dan Winter (1-5)
- 6.12 Spotlights on PHI
- 6.13 Mathematics of Harmony / Russia (1-4)
- 6.14 Cosmic Harmonics

### Conclusion

# SG203.Ic Sacred Geometry in Science

PART A (SG203A):

Chapter 1: Fractals & Self-Similarity

Chapter 2: Penta-Symmetries

Chapter 3: Platonic Models



All contents (texts, concepts, images, diagrams...)

© Aya, Aya Productions & StarWheel Foundation.

*We have acknowledged image and text credits (with hyperlinks) to the best of our ability.*

*Kindly contact us for omissions or corrections.*

[phi@schoolofsacredgeometry.org](mailto:phi@schoolofsacredgeometry.org)

# SG203.Id Sacred Geometry in Science - Intro 1

All great advances in knowledge & science have stemmed from the primordial belief that there are intelligible universal patterns behind manifested phenomena. This quest for order in apparent chaos has fueled human knowledge & fostering many cultures throughout history. In ancient times, these patterns of creation were highlighted by certain cultures and we have inherited hints of this knowledge passed on as “tradition”. In modern times, science has strived to uncover these patterns again but the process has been slow & fragmented as science engaged in the mechanical understanding of the separated parts rather than the whole.

Nowadays, perception has been turned around: the old paradigm is, one generation at a time, replaced by the much vaster approaches implied by quantum physics and a more encompassing “post quantum” physics & cosmology is made possible and is rapidly emerging in all fields of science & consciousness research. Research itself of course is no longer the naive scenario of an “objective observer” compiling data but is progressively experienced as an interactive conscious co-creation.

Contemporary cutting-edge science is rediscovering the geometric, harmonic & fractal “*self-similar*” patterns & symmetries of the universe, on all scales. A new loop is being completed on the evolutionary spiral as this is exactly what traditional Sacred Geometry has been speaking about all along. Rather than blindly exploiting convenient niches, this New Paradigm science aims at understanding the entire universe as a living whole, a unified field, a plenum of supra-intelligent energy/information. It also aims at acquiring the wisdom necessary to enter in conscious communication with that cosmic-level intelligence. The universe is glimpsed again by some as a gigantic orchestrated Song, with personal consciousness as the music instrument.

The keywords, at the core of this cosmic science, are: *Connectivity & Fractality*.

Quantum CONNECTIVITY is now described as *super-coherent*, *all-inclusive* and *non-local* i.e. beyond the boundaries of space-time. And universal connectivity is now glimpsed on a cosmological level and, on a human scale, in biology, both as intra-organic coherence and trans-organic coherence, including consciousness. The Zero Point Energy or Vacuum Fluctuation field is the medium of this hyper-dimensional connectivity of the Web of Life.

Within FRACTALITY, the optimal “self-similar” nesting or cascading has been found to be PHI-based. This has been glimpsed in a few domains already and is rapidly uncovered as an invariant universal constant of optimization for the transfer of energy and information, on all scales. PHI is the Harmonic Scaling Universal constant.

## SG203.1e Sacred Geometry in Science - Intro 2

Up to one generation ago, it took a kind of mystical or spiritual “faith” in life, nature and the universe to perceive & see HARMONIC ORDER as the source, manifestation & destination of the cosmos and of the human adventure. The researchers who proclaimed the extraordinary qualities of the Golden Ratio had little to back them up except the support of ancient traditions: in academia and the media, and therefore the public opinion, they were brushed aside as “golden-numberists”, an innocuous category of spiritualists. While not sent to the stake like the “algorists” were a few centuries ago [◆SG202.2], these pioneers were pictured as lacking credibility and hopelessly stuck in obsolete beliefs: at best, they were laughed at or relegated to “recreational mathematics”.

Since around 1975 however, a flurry of scientific (re)discoveries has emerged to corroborate the traditional teachings of Sacred Geometry. (See a partial list next page). These discoveries are in the process of completely turning around the public image of the Golden Ratio and its manifestations, as shown by the success of the Da Vinci Code (2003). As we are awakening to our inter-active & co-creative place in the Web of Life, on all scales, we are also understanding that there are universal codes & patterns of energy transfer underlying this exquisitely precise dance of nature, life & consciousness.

Now, at the onset of this 21st century and in time to provide a foundation to the New Paradigm taking shape in the global consciousness, we are in a position to realize that the Phi Ratio is indeed a key to the universe and we can start to possibly piece together the picture of a Golden Cosmos where Phi Ratio fractal cascades are the optimal pattern/code of relationship.

In this SG203 module, we will act as co-weavers of this collective loom of consciousness: we will survey how this New Paradigm science is encountering the ubiquitous presence of the Golden Ratio PHI & its vortexing spiral, as well as the related mathematical series of Fibonacci & Lucas and the volumetric building blocks of the universe represented by the 5 Platonic Solids and their derivatives the Archimedean Solids. PHI IS THE ULTIMATE FRACTAL!

New global technologies of nature-harmonious energy generation and use are upon us. Stay alive and creative!

Note that we will cover more extensively in coming up modules the following scientific domains: Human Biology & the DNA (SG204), the various “kingdoms” of Mother Nature (SG205) and the Celestial Cosmos (SG301).

The advertisement features a vibrant orange and yellow background with fractal patterns and a purple DNA double helix. The text is arranged in a white box at the bottom.

**Fractal Field  
Implosion Science Technologies**

[www.Breakthru-Technologies.com](http://www.Breakthru-Technologies.com)

Technology is evolving – are you?

# SG203.If Sacred Geometry in Science - Intro 3

This SG203 module is split into two parts of 3 chapters each:

• **PART 203-A:** We start, in chapter 1, with the most exciting & popular revolution in mathematics: the re-discovery by science of the ancient concept of gnomonic growth in nature under the new name of *self-similarity* and its mathematical formalization under the name of *fractal geometry*. The Mandelbrot Set and other types of fractal objects are now pervading the scientific language and the popular culture. We will focus on a special category of fractals and their unique & universal properties: the Golden Fractals.

Chapter 2 is a brief exposition of a fast expanding area: Penta-Symmetry. From Penrose Tiling to Quasi-Crystals to the "Buckyball"  $C_{60}$  molecule and the family of Fullerenes and much more uncovered daily...

Chapter 3 is a survey of the coming back of the Platonic Models in Science, from the atomic nucleus to chemistry to cosmology...

• **PART 203-B:** Chapter 4 is a look into science from the specific perspective of the Fibonacci Series and the Golden Ratio. We review the appearance of the Fibonacci Series in various fields of science, specially mathematics and we introduce a variety of applications of the Fibonacci Series in technology and nature...

Chapter 5 presents new perspectives about spirals structures and vortex dynamics. We introduce the work of pioneers such as Viktor Schauberger, Walter Russell, Lawrence Edwards... And we play with the wonderful power of vortices...

Finally, chapter 6 introduces the subject of a "Universal Golden Matrix" and the new paradigm understanding of a non-local, scale-invariant, all inclusive, self-similar Field of In-formation underlying what we experience as the 3D reality.

Note: as an artist, historian and practitioner/teacher of Sacred Geometry, Aya is not in a *scientific* position to test or prove some of the sometimes complex mathematical theories or experimental data popping up in all fields of cutting-edge science.

However, the recognized "New Paradigm" in science, born out of the quantized nature of the universe, is our departure point. Based on this new understanding and with the benefit of the higher "mandala overview" (acquired after an extensive study of Sacred Geometry, both artistic and experimental) as well as the *intuitive* wisdom gleaned from many years of meditation & yoga, Aya is confidently sharing his insights about the overall trend of contemporary science: re-discovering the primordial Patterns of Harmony on all scales of the universe.

Our intentions in this module are:

- To point to new scientific models or data that are based on a Golden-related geometry structure. And there are many...
- To provide lines of new enquiry and research to scientifically-minded students of Sacred Geometry & cosmology.
- To suggest the re-appearance of a cosmic-level knowledge capable of weaving together the harmonic strands of the universal fabric or, for music lovers, the harmonic chords of the universal orchestra.

Happy Journey upon the Golden Road Freeway!

# SG203.Ig Sacred Geometry in Science - Timeline (1)

## List of accelerating scientific discoveries uncovering PHI, since around 1975. Western Science.

- 1974. Roger Penrose proposed a set of two tiles providing a non-periodic tiling in 2D. Based on the Phi-based Penta-Modules.
- 1975 & 1979. *Synergetics I* and *Synergetics II*, two seminal works by Buckminster Fuller on the geometry of design / Platonic Solids.
- 1976. Robert Ammann discovered the 3D Golden Rhombohedra, formed by the fat & thin rhombi (a Penta-Modules combination).
- 1978. First computer-generated top down view of DNA by Robert Langridge. DNA viewed as a pentagonal/dodecahedral helix.
- 1983. Publication of *The Fractal geometry of Nature* by Benoit Mandelbrot who coined the term “fractal” in 1975.
- 1984. The Becker-Hagens planetary grid, following up on works by B. Fuller & Russian researchers. Icosa-dodeca geometry.
- 1984. Quasi-Crystals: Dan Shechtman & others published their discovery of penta-symmetry in quasi-crystals (Al-Mn alloys).
- 1985. Discovery of the C<sub>60</sub> carbon molecule (“Buckminsterfullerene”) and the family of “Fullerenes”. Curl, Kroto and Smalley.
- 1986. Nuclear model based on nested Platonic Solids proposed by Dr. Robert Moon (“Moon Model”).
- 1986. Global Scaling Theory: matter resonates harmonically at logarithmic/fractal scales, like a melody. Hartmut Müller.
- 1990. Tetrahedral Physics in solar system. Planetary anomalies at Star-Tetrahedron points. Richard Hoagland.
- 1990. Discovery of DNA’s nucleotides showing super-resonance to Phi-based Fibonacci/Lucas series. Jean-Claude Perez.
- 1990. Golden Ratio models of optimum coherence in brain and heart waves. Dan Winter & HeartMath Institute.
- 1991. The structure of the C60 molecule is proven to be a truncated icosahedron (Archimedean Solid) as predicted by B. Fuller.
- 1992. Physics of the Growth Angle in Phyllotaxis explained by Douady & Couder. Growth (Golden) Angle =  $137.5^\circ = 360 / \Phi^2$
- 1994. “E-Infinity” theory by El Naschie showing that the mass of quarks & subatomic particles is a function of Phi and 1/Phi.
- 1997. Octahedral geometry structure of super-clusters of galaxies. Battaner & Florido.
- 1998. Experimental verification by Steinhardt & Jeong of quasi-periodic tiling with overlapping decagons. STM images show that the ratios of height between terraces are based on Phi.
- 2002. “Spira Solaris” data proving that Golden Ratio is the key geometry factor in the ordering of the solar system. John N. Harris.
- 2003. Dodecahedral space topology of the universe proposed by J. P. Luminet & al.
- 2003. Octopole & quadrupole concepts of the Cosmic Microwave Background (CMB) radiation, based on WMAP data. Max Tegmark
- 2006. Implosion Physics. Golden Ratio fractal compression of charge as electric cause of gravity. Dan Winter.
- 2008. Fractal holographic model of Unified Field & Scaling Law of Organized Matter. N. Hamein & E. Rausher.
- 2009. Mineralogical evidence of naturally-formed quasi-crystals in samples from Russia. Bindi, Steinhardt, Yao and Lu.
- 2009. Golden Ratio measurements in radii of hydrogen atom. R. Heyrovska.
- 2009. The 3 known radii of Hydrogen as perfect multiples of Golden Ratio x Planck’s length. Dan Winter.
- 2010. Phi-based Unified Theories: Golden Matrix Cosmos / Winter. Golden Quantum Field / El Naschie. E8 Theory / G. Lisi.
- 2010. New Fractal Field technologies using optimized Phi compression. Breakthru-technologies and al.
- 2010. Golden Ratio discovered at the quantum scale. Coldea, Tennant et al. Science, Jan. 8, 2010.

## SG203.Ih Sacred Geometry in Science - Timeline (2)

List of accelerating scientific discoveries uncovering PHI, since around 1975. Russian/Slavic Science.

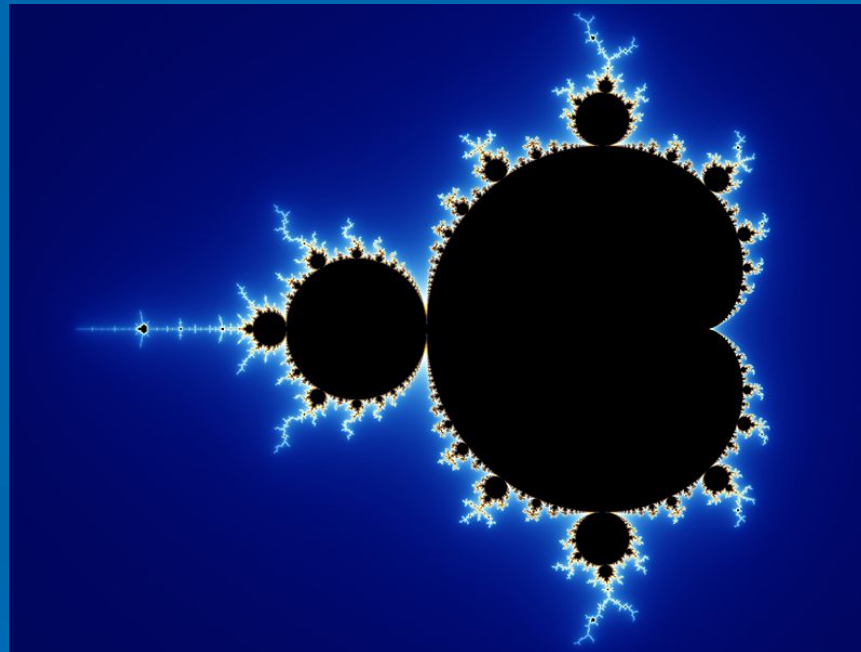
Note: the list below is presented here for parallel timeline comparison with western science.

This list will be presented again in Chapter 6: Golden Matrix Science

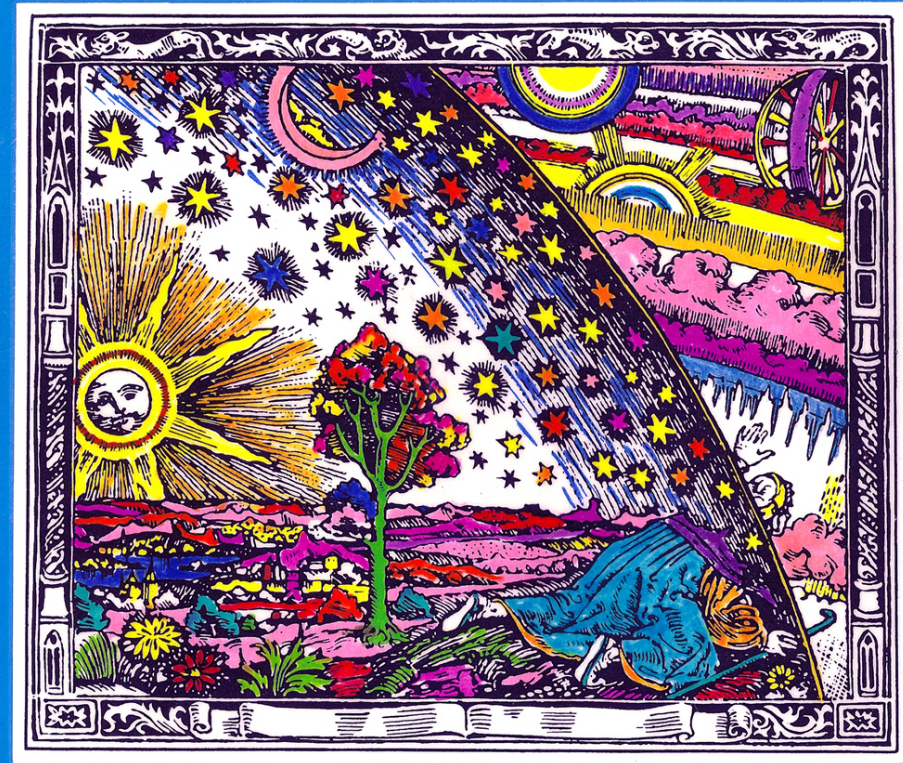
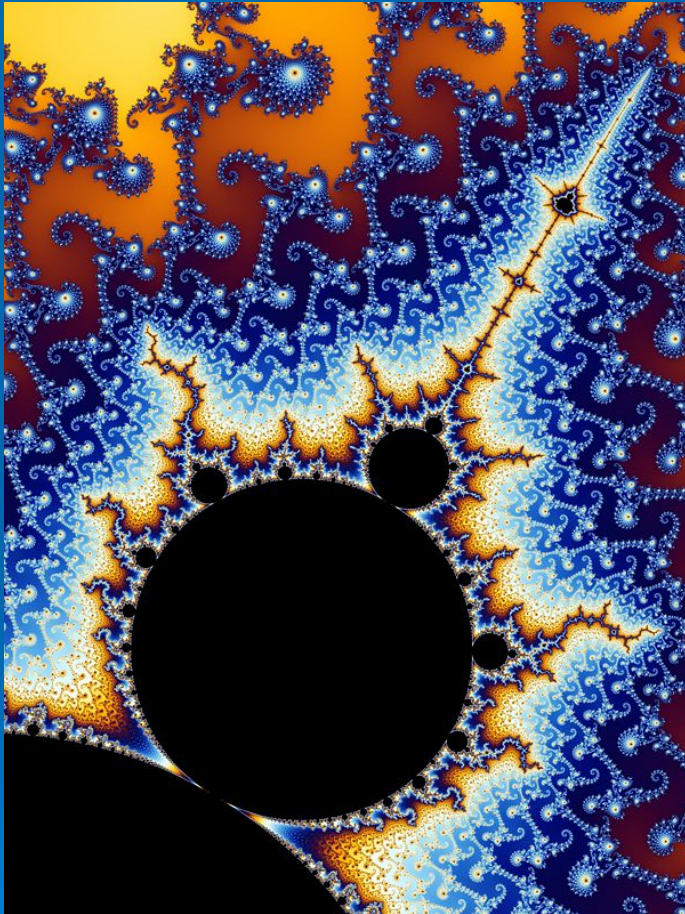
- 1961. Nikolay Vorobyov. *Fibonacci Numbers*.
- 1977. A. Stakhov. *Algorithmic Measurements Theory*. (Measurements based on Fibo Numbers).
- 1978. Butusov, K. P. *The golden Ratio in the Solar System*.
- 1979-95. Stakhov et al. *Soviet Fibonacci Computer Project*. 65 foreign patents.
- 1984. Eduard Soroko. *Structural Harmony of Systems*. (Reviews the Pythagorean idea of the numerical harmony of the universe).
- 1984. A. Stakhov. *Codes of Golden Proportion*. (New number systems).
- 1985. Natalia Pomerant. *Aesthetic Foundations of Ancient Egypt*. (Role of Golden Section in Egypt).
- 1986. Jan Grzedzielski. *Energy-Geometry Code of Nature*. (The Golden Mean as the proportion of thermo-dynamic equilibrium in self-organizing systems.)
- 1986. Kovalev, F. V. (Russian art teacher). *The Golden Section in Painting*. (Manual for artists).
- 1990. Nikolay Vasyutinsky (Ukrainian scientist). *The Golden Section: Three Approaches to the Nature of Harmony*.
- 1992. *First International Seminar on the Golden Section and Problems of System Harmony*. Kiev.
- 1993. *Second International Seminar*. Kiev. 1994, 1995, 1996: Follow-up Seminars. Stavropol.
- 1994. Oleg Bodnar. *The Golden Section and Non-Euclidian Geometry in Nature & Art*. (New geometric theory of phyllotaxis).
- 1996. A. Stakhov. Seminal Lecture. *The Golden Section and Modern Harmony Mathematics*.
- 1997. Tsvetkov (Russian biologist). *Heart, Golden Section & Symmetry*. (Role of Golden Ratio in optimizing heart activity).
- 1998. Professor Korobko (Russia). *The Golden Proportion and Problems of System Harmony*. (Comprehensive manual for universities).
- 1999. A. Stakhov et al. *Introduction to Fibonacci Coding & Cryptography*. (Fibonacci matrices).
- 2000. Shevelev (Russian architect). *The Meta-Language of Living Nature*.
- 2001. A. Stakhov and A. Sluchenkova. Website: *Museum of Harmony and the Golden Section*. [www.goldenmuseum.com](http://www.goldenmuseum.com)
- 2003. International Conference. *Problems of Harmony, Symmetry and the Golden Section*. Vinnitsa (Ukraine).
- 2003. A. Stakhov. *Hyperbolic Fibonacci & Lucas functions*. (A new mathematics for the living nature).
- 2005. Petrunenko, V. V. *The Golden Section in Quantum States and its Astronomical & Physical Manifestations*.
- 2006. Soroko, E. M. *The Golden Section, Processes of self-organization and Evolution of System*. (Introduction to a general theory of System Harmony).
- 2006. Vladimirov, Y. S. *Metaphysics of the 21st Century*. (Golden Mean applications in modern science).
- 2006. Petoukhov, S. V. *Metaphysical Aspects of the Matrix Analysis of the Genetic Code and the Golden Section*. (Discovery of the "golden geno-matrices").
- 2005. Creation of the *Institute of the Golden Section*.
- 2006. Stakhov, Sluchenkova & Scherbakov. *The Da Vinci Code & Fibonacci Series*.
- 2009. A. Stakhov. *The Mathematics of Harmony*. (A new inter-disciplinary science).



# SG203.1 Chapter 1. Fractals & Self-Similarity



## SG203.1.1.1 Ancient Roots (1)



For some reasons, these two images, a few centuries apart, have called on me to sit together. They seem to point to a common experience of peering at... infinity. They both offer a feeling of sacred awe... They both are cosmology and sacred geometry... Gaze at them for a moment and let them take you on a journey...

This is how they speak to me:

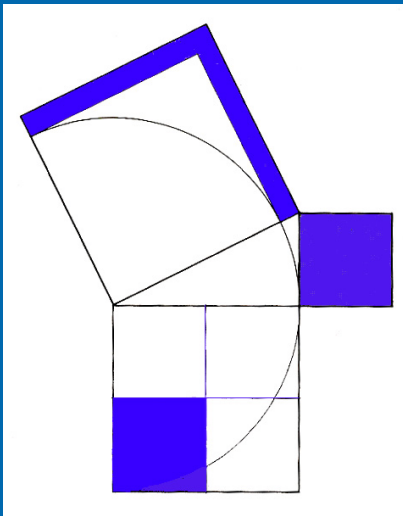
- They suggest breaking through to another dimension and discovering therein new and endless worlds to explore. Overall Oneness and yet each part completely unique. There is always one more level nested in Self. It never ends...
- The star in the sky is just like a flower on earth... or a newborn fractal seed. The tree's branching is a computer's model of fractal filaments... Each word we sing is a delicate cymatic mandala...
- All elements, small & big, are proportionally connected. Unity in infinity. A tremendous sense of the majestic harmonic order of the cosmos. A benevolent, all-embracing, loving home...
- And it is all looping back onto itself. The shepherd's head is a star-like flower blossoming in another dimension & experiencing a new sun-like illumination... Self-reproducing seed Yes, life continues... by evolutionary progeny and consciousness Phi nesting...

## SG203.1.1.2 Ancient Roots of Fractality (2)

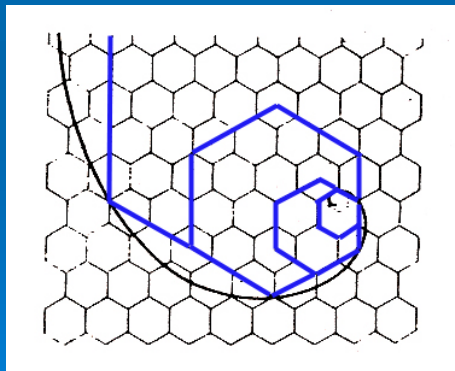
"There are certain things which suffer no alteration save in magnitude when they grow..." stated Aristotle (384-322 BC). He was referring to what the Greek mathematicians called the geometry of the *Gnomon* and the type of growth based on it, known as *Gnomonic Expansion*.

A *Gnomon*, as later defined by Hero of Alexandria (c. 10-70 BC), is "any figure which, when added to an original figure, leaves the resultant figure similar to the original".

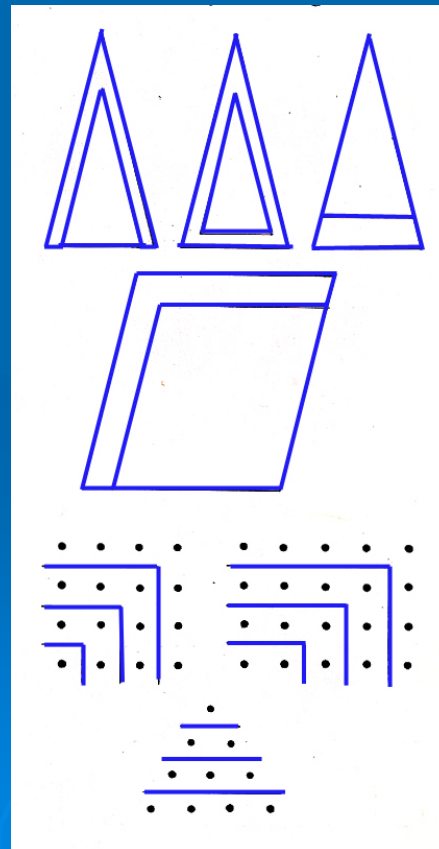
The dynamics of Gnomonic Expansion represents one of nature's most common forms of growth: growth by accretion or accumulative increase, in which the old form is proportionally contained in the new form. This type of growth is found in the more permanent aspects of the animal body (bones, shells, teeth, horns...) as well as in the architecture of sacred buildings such as the Hindu temple. Nowadays, this is described by fractal mathematics.



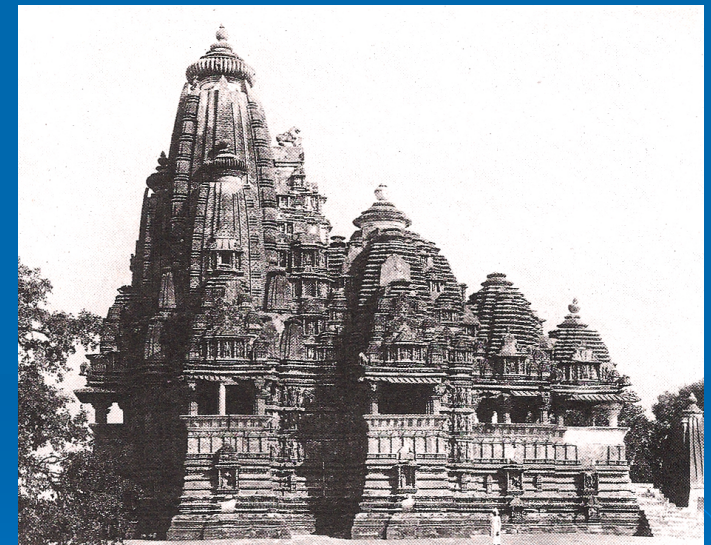
↑ Shown above is the gnomonic increase from the square surface area of 4 to the square surface area of 9, where the gnomon of the larger square 9 is equal to the 1/4 of the initial square of 4 (in blue). This method of figuring the gnomon shows its relationship to the Pythagorean theorem:  $a^2 + b^2 = c^2$



↑ Spirals can be traced from the gnomonic growth of hexagons (or triangles). [◆SG105]



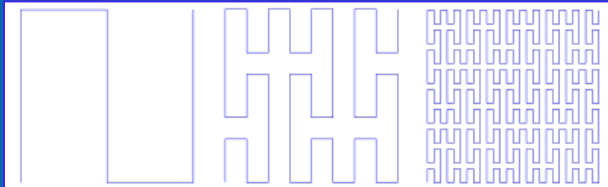
← Gnomonic expansion shown in geometric figures, and in the dotted shapes of square, oblong and triangular numbers.



↑ The typical Hindu temple is an extension of the initial cosmic square (the *Vastu Purusha Mandala*) [◆SG207]. The previous stage of growth remains as part of the *gnomonic* design of the subsequent stages (just like in the Nautilus shell geometry).

# SG203.1.2.1 Simple Iterative Fractals (1)

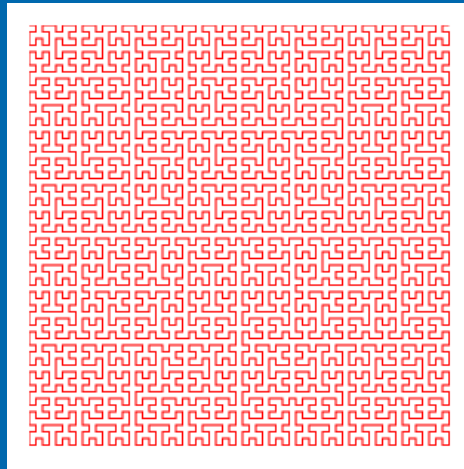
Before Fractals received their famous name, simple *iterative shapes* had already been studied. In 1890, Guiseppe Peano discovered a “*space-filling curve*”. This was a bombshell in the world of mathematicians who were dealing either with the 2D plane or the 1D lines but could not conceive of a curve with no slope and an ambiguity of dimensions. Even the mathematical giant Poincaré called these curves “a gallery of monsters”. Well, 80 years later, Mandelbrot proved that these *monsters* contained the secret of fractals and were the mathematics able to describe the “chaotic” regularities of nature.



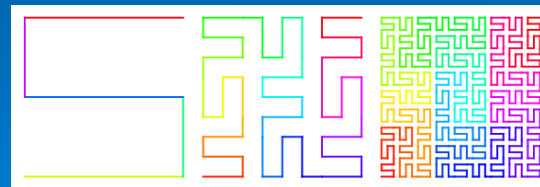
Peano Curve: the 3 first iterations



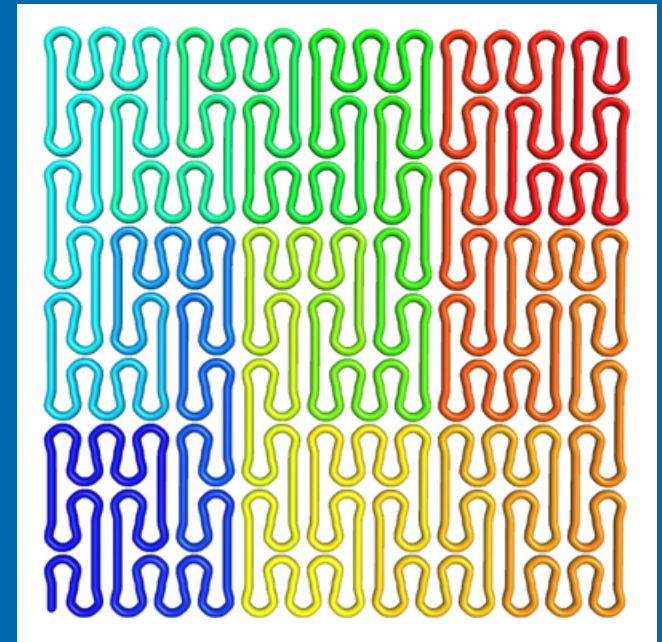
↑ The monument dedicated to Giuseppe Peano, the famous mathematician from Cuneo, Italy.



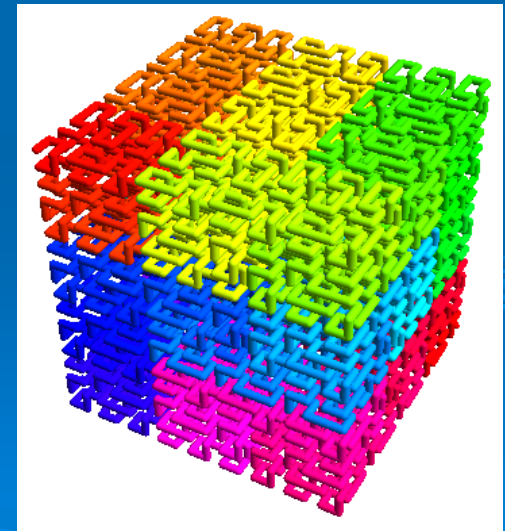
↑ Hilbert Iteration



↑ The Swastika Peano

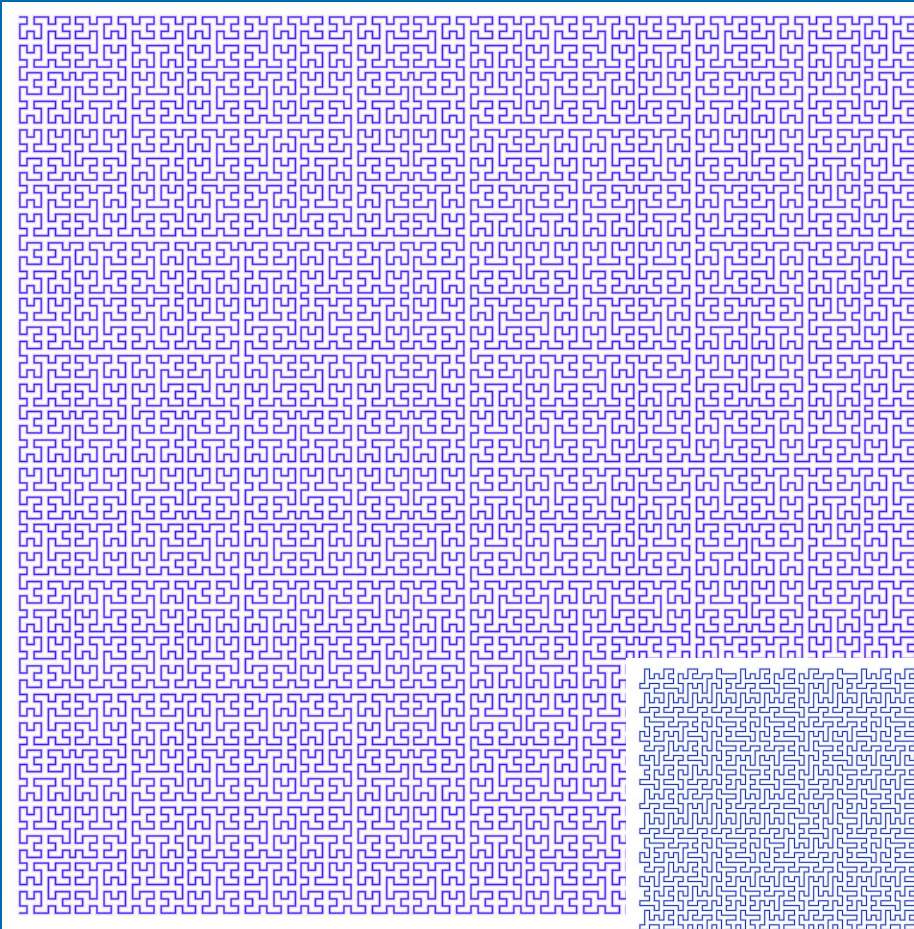


Peano Curves. ([www.seas.harvard.edu](http://www.seas.harvard.edu))



↑ The Peano Curve in 3D

## SG203.1.2.2 Simple Iterative Fractals (2)

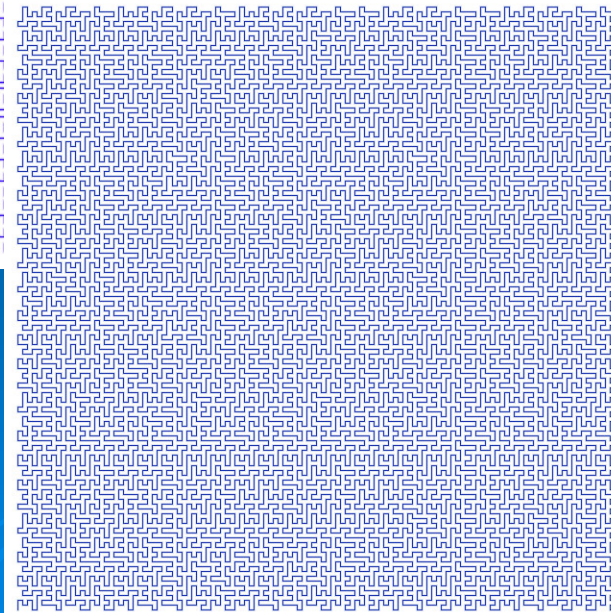


↑ Hilbert Iteration

Gosper Iteration →

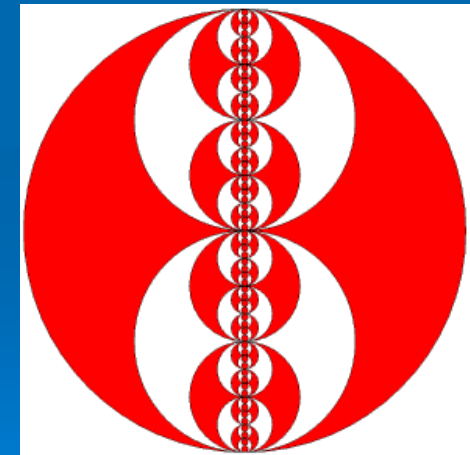
Imagine quite a few more iterations and you won't be able to distinguish any detail but a filled plane: the line and the plane merge in a "fractal" in-between dimension.

At infinity, the Peano's fractal dimension is two.



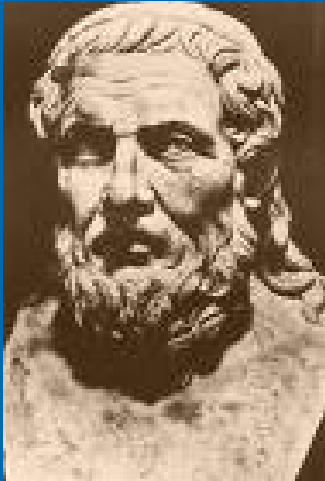
Note: this is the fractal structure used in the classic Yi Ching or *Book of Changes*.

← The Cantor set, introduced by German mathematician Georg Cantor, involves only the real numbers between 0 and 1, and is defined by repeatedly removing the middle thirds of the lines. The Cantor set is the prototype of a fractal. It is self-similar: it is equal to two copies of itself, if each copy is shrunk by a factor of  $1/3$ . (Wikipedia)



↑ Another oriental wisdom fractal: The "Tao Fractal"

# SG203.1.3.1 Proto-Fractals - Apollonian Gasket

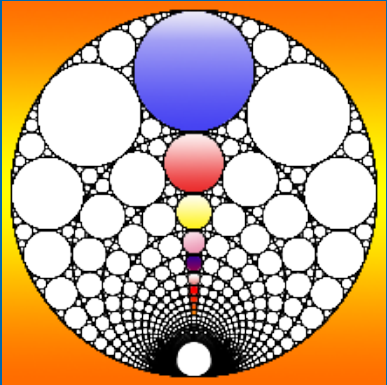
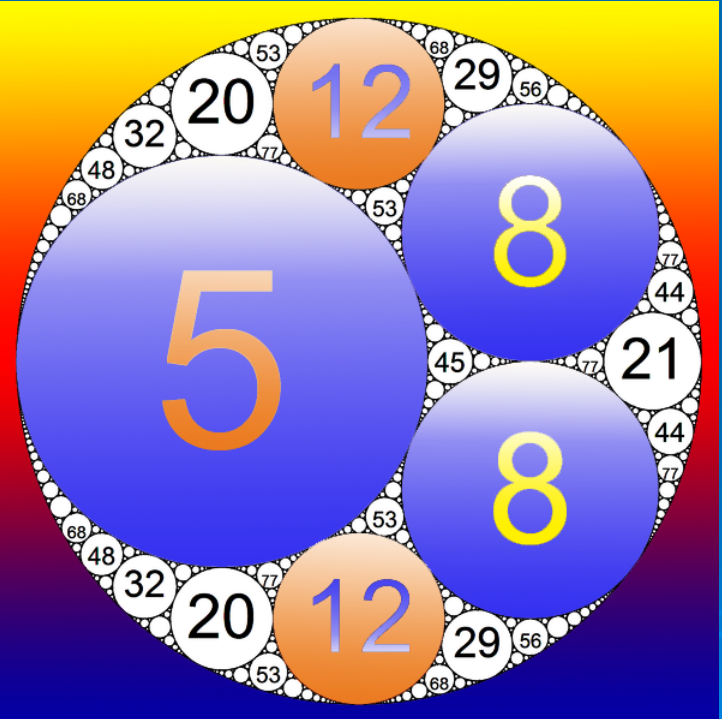


Apollonius of Perga

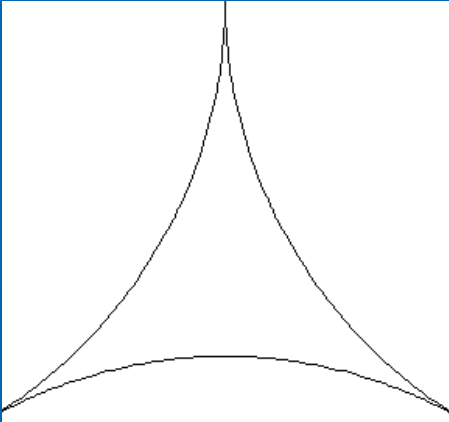
Apollonius of Perga (ca. 262 BC - ca. 190 BC) was a Greek geometer and astronomer noted for his writings on conic sections. It was Apollonius who gave the ellipse, the parabola, and the hyperbola the names by which we know them. Apollonius in the *Conics* further developed a method that is so similar to analytic geometry that his work is sometimes thought to have anticipated the work of Descartes by some 1800 years.

In mathematics, an *Apollonian gasket* or *Apollonian net* - one of the first fractals ever described - is a set of mutually tangent circles, formed by solving Apollonius' problem iteratively. It is a fractal generated from triples of circles, where any circle is tangent to two others. Construction: start with three circles  $C_1$ ,  $C_2$  and  $C_3$ , each one of which is tangent to the other two (in the general construction, these three circles can be any size, as long as they have common tangents). Apollonius discovered that there are two other non-intersecting circles,  $C_4$  and  $C_5$ , which have the property that they are tangent to all three of the original circles - these are called *Apollonian circles*. Adding the two Apollonian circles to the original three, we now have five circles. Continuing the construction stage by stage in this way, we can add  $2 \cdot 3^n$  new circles at stage  $n$ , giving a total of  $3n+1 + 2$  circles after  $n$  stages.

In the limit, this set of circles is an *Apollonian gasket*. The Apollonian gasket has a fractal "Hausdorff" dimension of about 1.3057.

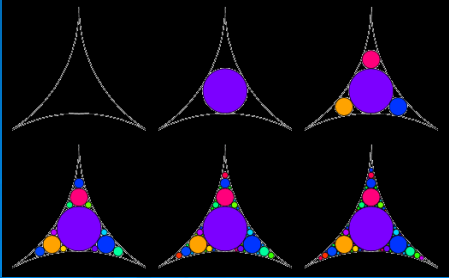


↑ A nested Apollonian gasket.

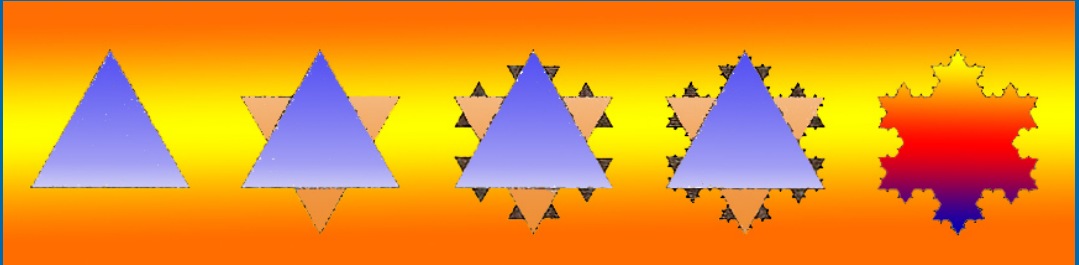


← Mathworld animation of the Apollonian gasket. [mathworld.wolfram.com](http://mathworld.wolfram.com)

← Integral Apollonian circle packing defined by circle curvatures of  $(-3,5,8,8)$ . A negative curvature indicates that all other circles are tangent to the interior of that circle (the bounding circle). Wikipedia.

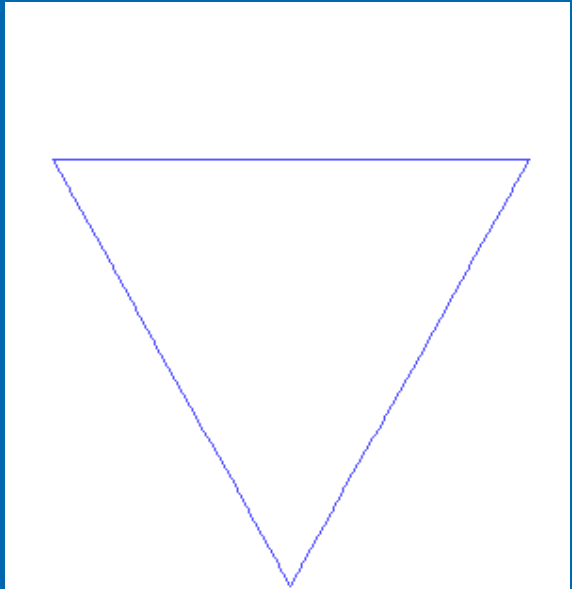


# SG203.1.3.2 Proto-Fractals - The Koch Snowflake

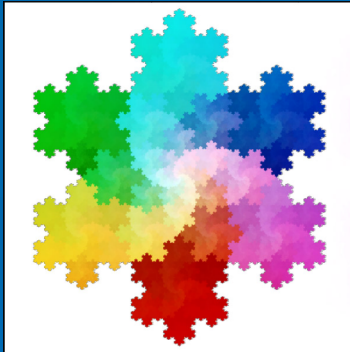


The *Koch Snowflake* was first described by the Swedish mathematician Helge von Koch (1870-1924). The ultimate result is a “fractal curve” that is infinitely “wiggly”: there are no straight lines in it whatsoever. The numbers grow very fast: at stage 8, there are 196,608 little sides. This fractal has an amazing geometric property: its area is finite but its perimeter is infinite. This means that you can color the inside of the Koch snowflake, but you can never wrap a string around its boundary. For each iteration, the length of the outline increases by a factor of  $4/3$  to infinity and the area converges to  $8/5$  (Fibonacci numbers) that of the original triangle.

The Koch Snowflake is *self-similar* and with its many bays, promontories and inlets, it resembles coastlines. In fact, it has a fractal dimension (the measure of its “wrinkleness”) of 1.2619 - which turns out to be about the same fractal dimension as the coastline of Britain.

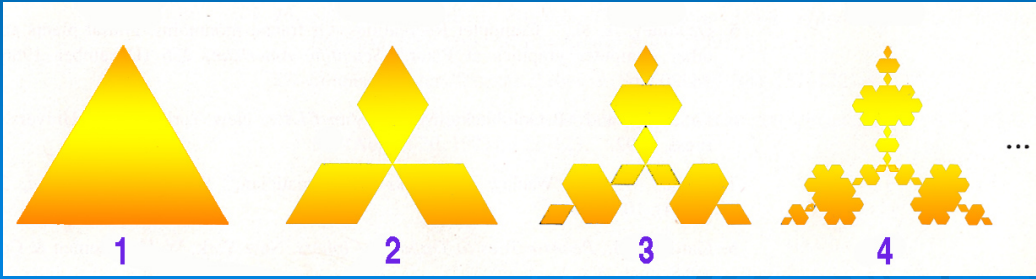
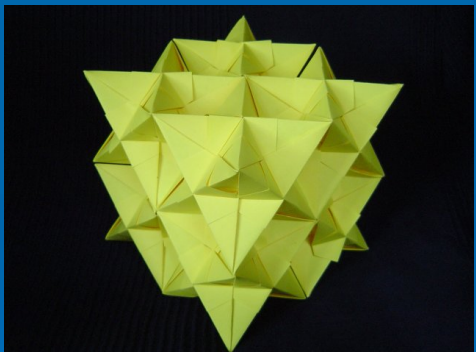


Koch fractal animation. Wikipedia.

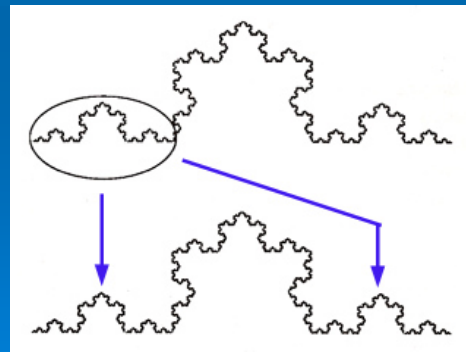


This fractal is a 3D → analogue of the Koch snowflake. In each iteration, a tetrahedron with half the edge length is placed in the middle of each triangular face. [Michal Kosmulski](#)

← [commons.wikimedia.org](https://commons.wikimedia.org)

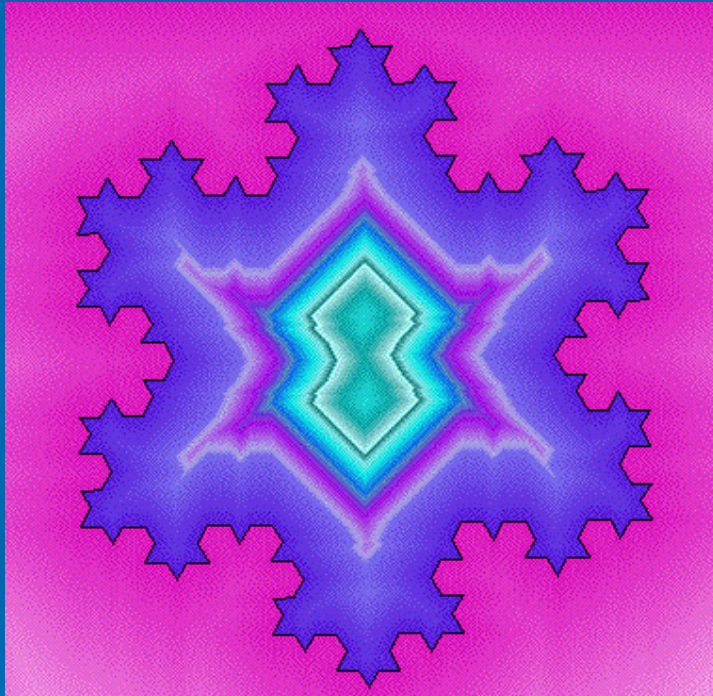


← The Koch Anti-Snowflake



↑ Self-similarity.

### SG203.1.3.3 Proto-Fractals - The Koch Snowflake



3rd iteration

It's a portal! Enter it  
or walk its crop circle...



Photo ©Lucy Pringle

↑ The Koch Snowflake (2nd iteration) at Silbury Hill.



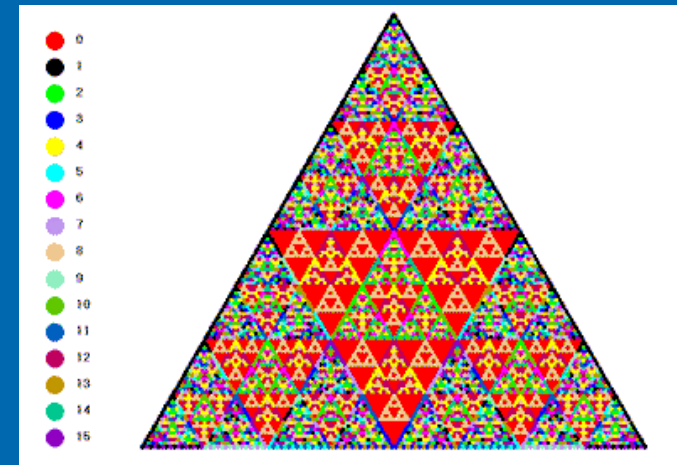
# SG203.1.4.1 The Sierpinski Triangle in 2D

The *Sierpinski Triangle*, also called the *Sierpinski Gasket* or *Sieve*, is a fractal named after Waclaw Sierpinski who described it in 1915, way before the term “*fractal*” was coined by Benoit Mandelbrot (1975).

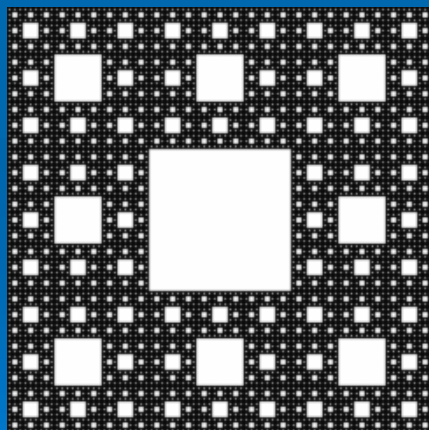
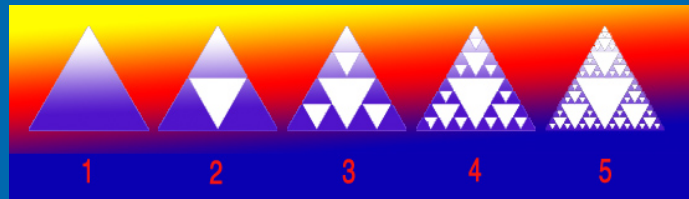
Originally constructed as a curve, this is one of the basic examples of self-similar sets or fractals: a mathematically generated pattern that can be reproduced at any magnification or reduction. The original equilateral triangle (1) gets divided into four identical triangles (2) by inserting one triangle pointing down at the mid-points. The same iteration rule repeats at smaller magnifications (3, 4, 5...)



↑ Waclaw Sierpinski



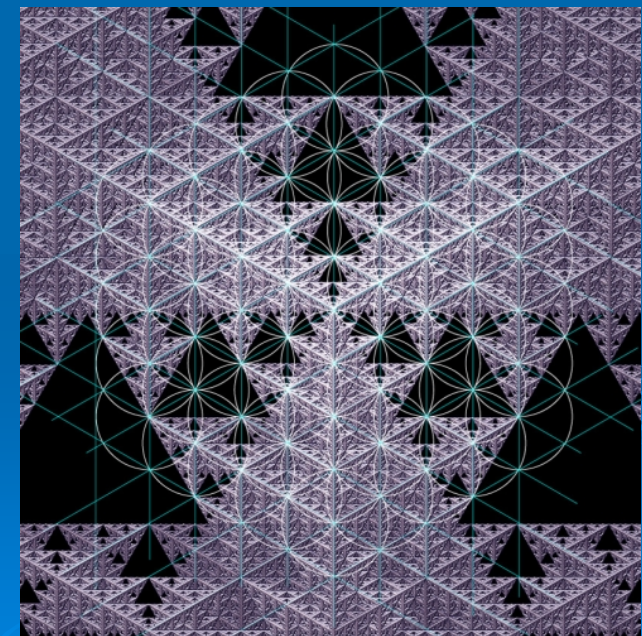
[curvebank.calstatela.edu](http://curvebank.calstatela.edu)



↑ The Sierpinski carpet is a plane fractal first described by Sierpinski in 1916. The carpet is a generalization of the Cantor set to two dimensions.

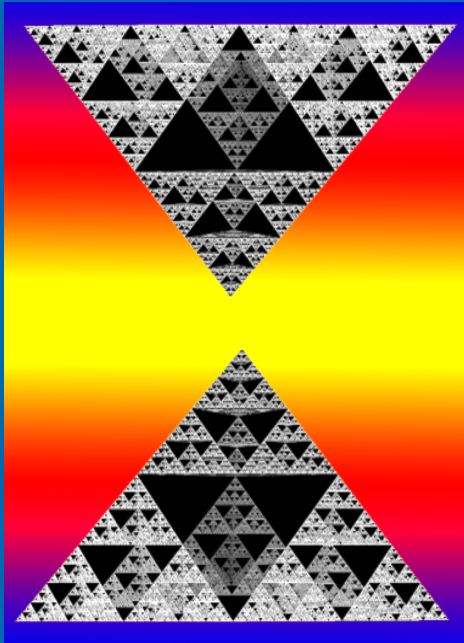


[math-art.net/](http://math-art.net/)

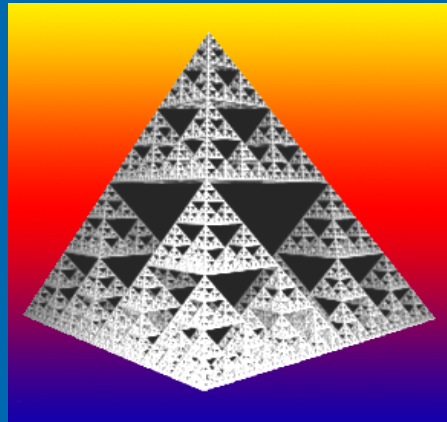


↑ The Flower of Life and the isometric triangular grid fit perfectly on top of the Sierpinski Triangle.

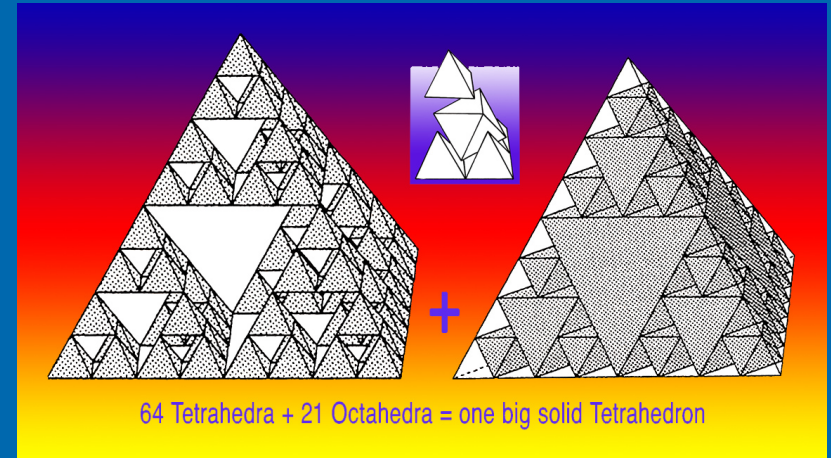
# SG203.1.4.2 The Sierpinski Triangle in 3D



↑ The Double Triangle as 3D Sierpinski Fractal (face view)

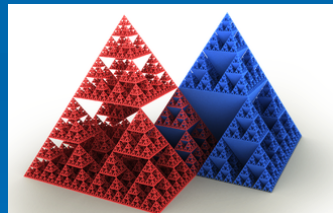
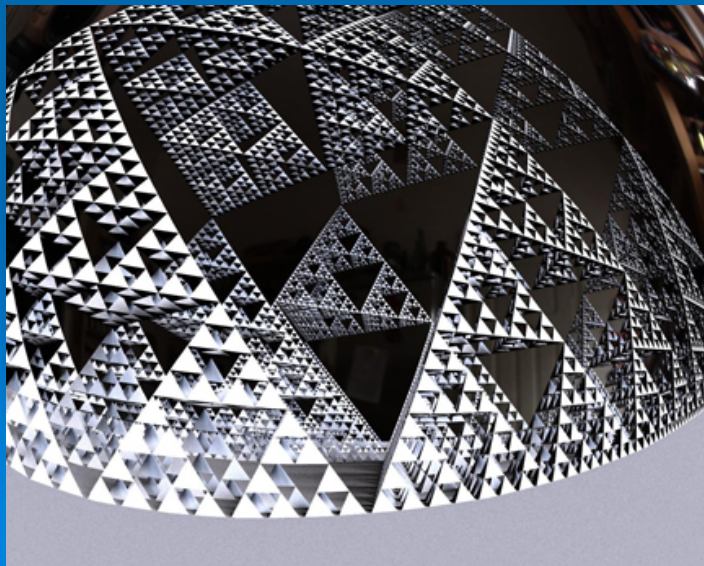


↑ 3D Sierpinski (edge view)

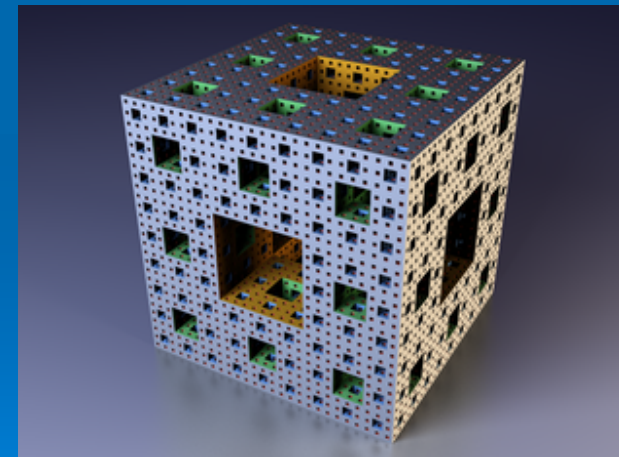


The nesting of the Tetrahedron (4 equilateral triangles) and the Octahedron (8 equilateral triangles). (B. Rawles [www.geometrycode.com](http://www.geometrycode.com))

←↑ Original images:  
[www.phidelity.com](http://www.phidelity.com)



← [www.3d-gfx.com](http://www.3d-gfx.com)

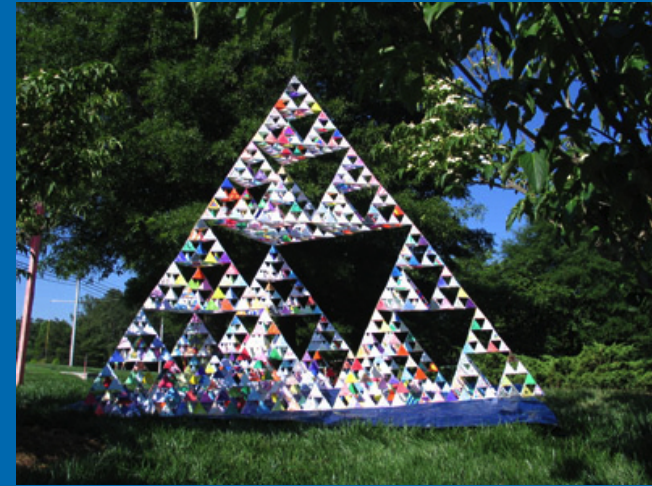


↑ The Menger Sponge is a 3D extension of the Cantor set and Sierpinski carpet. It was first described by Karl Menger (1926) while exploring the concept of topological dimension.



“Stage-3 Sierpinski and a bird’s nest”.  
2003 Gayla Chandler

## SG203.1.4.3 The Sierpinski Triangle in 3D



↑ This giant sculpture is a 3D version of the Sierpinski Triangle. This large tetrahedron consists of 1024 smaller tetrahedrons. It was created by students of [Alan A. Lewis School](#). The order of fractal is 6.

## SG203.1.5.1 1975-1983: Fractals Are Born

The term “fractal” (Latin *fractus*, from *frangere* = to break, to fragment) was coined in 1975 by mathematician Benoit Mandelbrot. Mandelbrot’s aim was to describe under a single heading new mathematical objects (like the Koch Snowflake, the Sierpinski Triangle or the Mandelbrot Set) as well as many shapes in nature such as clouds, mountains, coastlines, lightning paths, branching networks like the lungs, rivers or the vascular system...

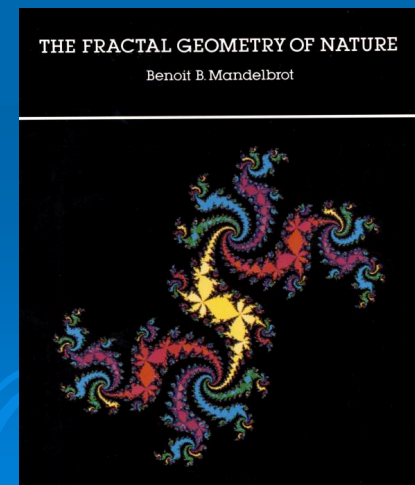


By studying a wide variety of irregular mathematical objects and natural forms, Mandelbrot realized that all these geometries had some essential features in common. He therefore invented a new type of mathematics to describe & analyze these features and, in 1983, publishes his landmark book “*The Fractal Geometry of Nature*” which had a great influence on a whole new generation of mathematicians studying chaotic and dynamical systems. In Mandelbrot’s terms:

“*Why is (standard) geometry often described as ‘cold’ and ‘dry’? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. CLOUDS ARE NOT SPHERES, MOUNTAINS ARE NOT CONES, COASTLINES ARE NOT CIRCLES, AND BARK IS NOT SMOOTH, NOR DOES LIGHTNING TRAVEL IN A STRAIGHT LINE...* Nature exhibits not only a higher degree but an altogether different level of complexity.” (The part bolded by Aya is the most famous sentence of Mandelbrot, oftentimes quoted.)

And so Mandelbrot created Fractal Geometry as a “*language for the clouds*”.

The most important property of fractal shapes is *self-similarity* or symmetry across size scale: their characteristic patterns are found repeatedly at ascending or descending scales in such a way that their parts, at any magnitude, are similar in shape to the whole. Break a piece of cauliflower: it looks just like a whole cauliflower. In other words, the shape of the whole is similar to itself at all levels of manifestation. Does that sound familiar? Remember the “small is to the large as the large is to the whole” = PHI?



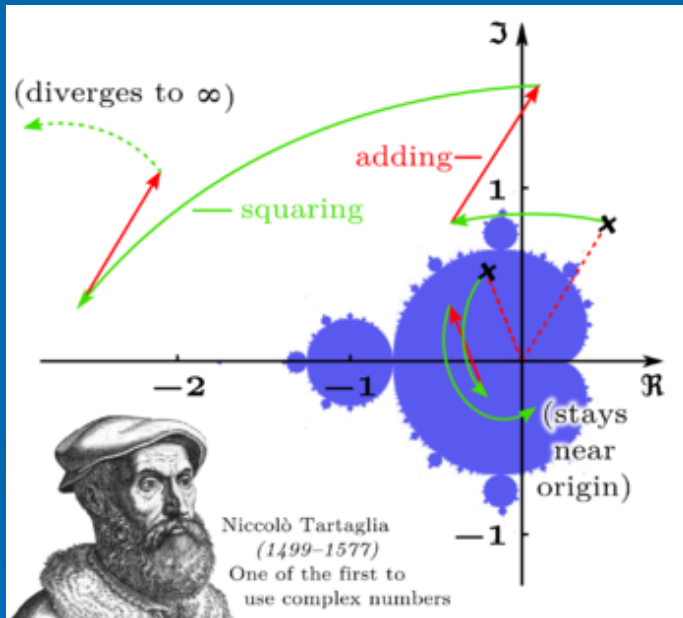
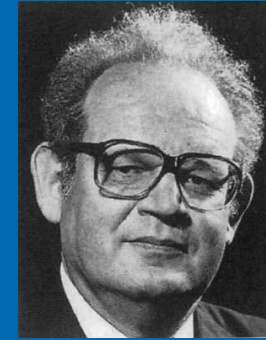
↑ The seminal book by B. Mandelbrot. 1983. 20

# SG203.1.5.2 Meet Benoit Mandelbrot (1)

Benoit B. Mandelbrot is a French /American mathematician, best known as the “father of fractal geometry”.

Mandelbrot was born in Warsaw, Poland, in a Jewish family from Lithuania. He was raised within a strong academic tradition: his mother was a medical doctor and he was introduced to mathematics by two uncles. Anticipating the threat posed by Nazi Germany, the family fled from Poland to France in 1936 when he was 11. At the *Ecole Polytechnique*, in Paris, Mandelbrot studied under Gaston Julia (who devised the formula for the Julia set and first investigated the field of complex dynamics).

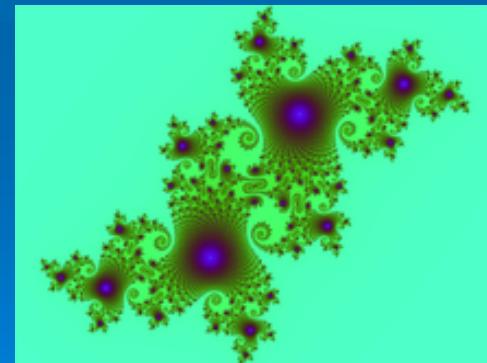
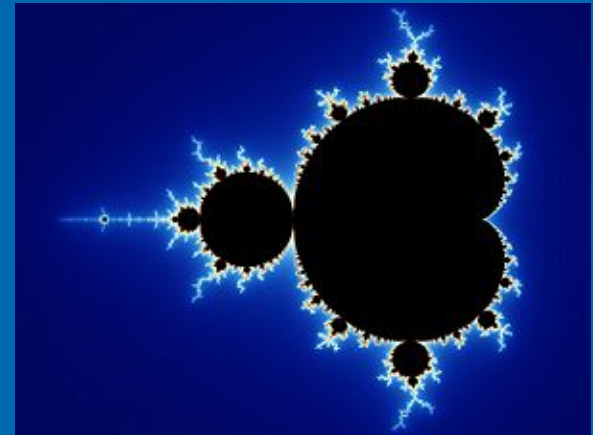
Later, Mandelbrot studied & taught at various scientific institutions to eventually become a fellow of IBM.



It was on March 1st, 1980, at IBM's Watson Research Center in upstate New York, that Benoit Mandelbrot first saw the shape that was later to be called the Mandelbrot Set or M-set. →

← The M-set is a subset of the c-plane (the complex number plane) i.e. the boundary set for all the values, in the c-plane, that do not diverge to infinity. The complex numbers contain the real numbers but extend them by adding imaginary parts such as  $i^2 = -1$ . This is in order to form an algebraically closed field, where any polynomial equation has a root, including examples such as  $x^2 = -1$ , according to the Fundamental Theorem.

Certain fractals are plotted in the complex plane, e.g. the Mandelbrot set and Julia sets.



↑ A Julia set, a fractal related to the M-Set.

The discovery and study of new fractal shapes has become arguably one of the hottest mathematical topics of the last 20 years. As a field of study, fractal geometry is a mathematician's dream come true: it combines complex and pioneering mathematics, stunning graphics, a poetic & mystical contemplation of harmonic infinity... and extreme relevance to the world of nature... which is increasingly understood as a nested fractal mini-cosmos. Using fractals, Mandelbrot was able, using no astronomical data, to mathematically visualize a distribution of galaxies that was confirmed by astrophysicists.

## SG203.1.5.3 Meet Benoit Mandelbrot (2)

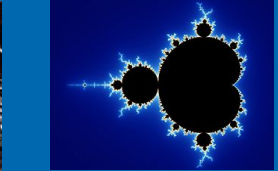
Although Mandelbrot coined the term fractal, some of the mathematical objects he presented in *The Fractal Geometry of Nature* had been described by other mathematicians. However, before Mandelbrot, they had been regarded as isolated curiosities with unnatural and non-intuitive properties. Mandelbrot brought these objects together for the first time and turned them into essential tools for the long-awaited effort to extend the scope of science to non-smooth objects in the real world. He highlighted their common properties, such as self-similarity (linear, non-linear, or statistical) and scale invariance.

He emphasized the use of fractals as realistic and useful models to accurately describe the development & resulting shape of many "rough" phenomena in the real world, as well as many growth processes evident in nature, both organic and inorganic. Natural fractals include the shapes of mountains, coastlines and river basins, the structures of plants, the clustering of galaxies, and Brownian motion. Fractals are found in the human body, such as the "bronchial tree" and blood vessels, and in human pursuits, such as music, painting, architecture, and stock market fluctuations. Mandelbrot believed that fractals, far from being unnatural & abstract, were in many ways more intuitive and natural than the artificially smooth objects of traditional Euclidean geometry. Mandelbrot's work has revolutionized the way researchers in many fields both perceive and characterize the phenomena of natural growth.

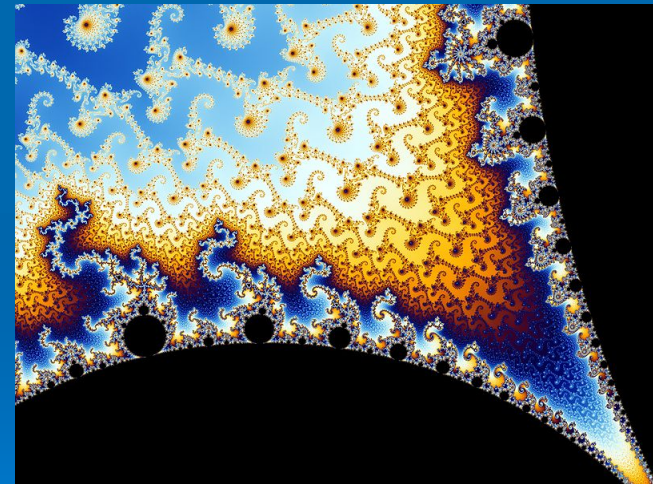
*"Mandelbrot has been called a visionary and a maverick. His informal and passionate style of writing and his emphasis on visual and geometric intuition made *The Fractal Geometry of Nature* accessible to non-specialists."* (Wikipedia).

Like Da Vinci, Tesla, Fuller and Coxeter, Mandelbrot belongs to the rare breed of thinkers who can VISUALIZE mathematics & scientific data. At France's prestigious *Ecole Polytechnique*, Mandelbrot could not do the algebra very well but he got top grades by translating the questions into mental images, the way traditional Sacred Geometry & Vedic Mathematics work. Mandelbrot claims, tongue in cheek, that using a telephone book is an ordeal, but he can SEE things that other people can't.

Mandelbrot's seminal work sparked widespread popular interest in fractals, chaos theory and other unifying fields of science and mathematics. It also launched a new art Renaissance and became a flagship for the New Paradigm in global consciousness whereby the universe is perceived again as a harmonic, resonant orchestration of musical scales & chords with ascending / descending overtones modulating the perfect octaval unisons.



← Natural water frost crystal growth on cold glass, showing fractal branching growth in a purely physical system.

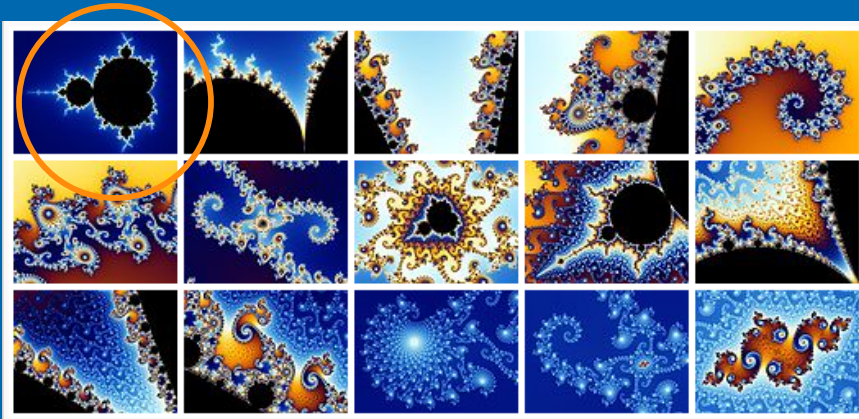


↑ Wikipedia

*"Every so often I was seized by the sudden urge to drop a field right in the middle of writing a paper, and to grab a new research interest in a field about which I knew nothing. I followed my instincts, but could not account for them until much much later".*

(Mandelbrot)

# SG203.1.6.1 The Mandelbrot Set (1)

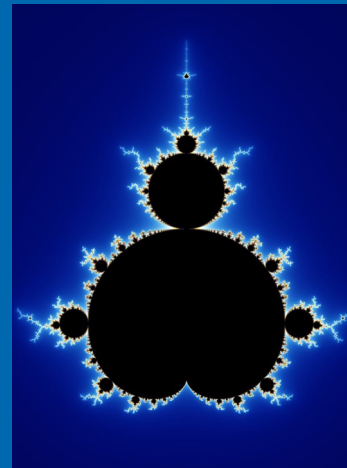


↑ Some of the myriad patterns & “worlds” implicate in the M-set. (Wikipedia).

## Some characteristics of the M-set:

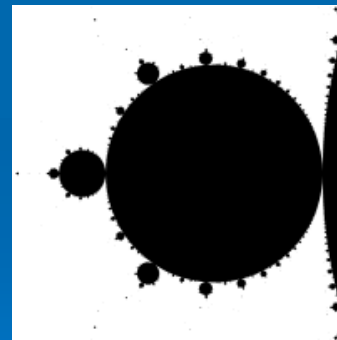
- The M-set can be viewed at infinite resolution without loss of detail. You can never zoom far enough to stop seeing the patterns.
- The M-set is SELF-SIMILAR. In mathematics, a self-similar object is exactly or approximately similar to a part of itself.
- Note that the M-set is not scale-invariant like an exact holographic replica: within the self-similarity, striking, unique differences show up (as pictured in some of the following pages).
- This blending of infinite repetition and infinite variety is a hallmark of the balance achieved by the M-set, just like we observe in nature.
- The M-set is mapped on an X-Y coordinate plane using the technique of computer iteration = the endless repetition of the same geometric operation.
- For every point on the X-Y grid there is an infinitely repeating fractal shape called a Julia set. The M-set is the collection of all the possible Julia Sets in the complex plane. Starting at the needle tip and moving to the bottom cusp, the M-set a map of every possible curve or spiral.

With the advent of computer graphics, the M-set could not only be investigated by mathematicians as a collective map of all the possible Julia sets for quadratic functions, but, for the first time in the modern era, the geometric visualization of mathematics was possible and exciting again. The movie-star popularity of the M-set has bridged over to the ancient traditions of visualizing the sacred patterns and geometries of the cosmos.



The M-set is a black rounded bud shape looking like a fat buddha (the “cardioid”) with an “antenna” (the tail or needle) on top, many branching arms (the mandelbuds) and a cusp at the bottom. Each of these buds is surrounded by hundreds of smaller buds and the process repeats itself to infinity.

The M-set contains many copies of itself buried deep in the nooks & crannies, each one unique and containing equally many *sub-mandelbuds*. The whole pattern (with all the curls, crimps, turns & trillions of spiral modulations) is one single membrane outlining & enclosing the boundary in black.



↑ Animation of the self-similarity in the M-set. (Wikipedia).

*The M-set is a magical cosmic toy partaking of the Great Mystery as an inter-dimensional interface.*

- The M-set was proven to be the absolute maximum space filling curve possible in 2 + dimensions. If the boundary region was one 'quanta' more curved inward on itself, it would have to overlap, popping into 3 dimensions.

Also Mandelbrot curves have been discovered in cross-sections of MAGNETIC FIELD borders, implying there is a 3-D Mandelbrot equivalent that is closely tied to electromagnetism and therefore a deep structural and fundamental aspect of life, and physical space/time.

Think about that for a moment, visualize it: take a slice of the magnetic field of the earth, sun, a plant, the data on audio or video tape, and there's our old familiar Mandelbulb. All these data are stored as the M-set! [From www.miqel.com](http://www.miqel.com)

## SG203.1.6.2 The Mandelbrot Set (2) $Z^2 + C$

Fractals in general are simultaneously highly complex and extremely simple.  
This is specially true for the M-set:

*“This set is an astonishing combination of utter simplicity and mind-boggling complication. At first sight, it is a ‘molecule’ made of bonded ‘atoms’, one shape like a cardioid and the other nearly circular. But a closer look discloses an infinity of smaller molecules shaped like the big one, and linked by what I proposed to call a ‘devil’s polymer’. Don’t let me go on raving about this set’s beauty!”*

Benoit Mandelbrot.

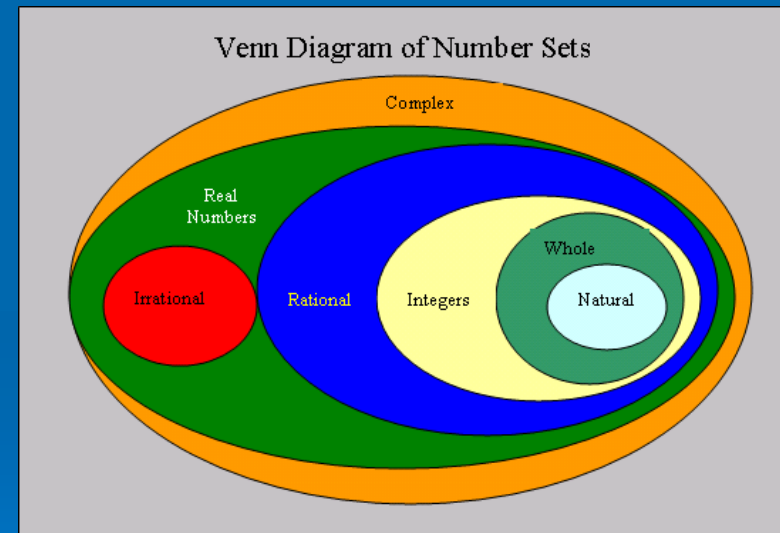
How does the “most complex object in mathematics” can be represented? Quite simply by following the following equation:  $Z^2 + C$ . It does look quite accessible, except this equation refers to the settings of coordinates on the “complex plane”. As explained by John Briggs and F. David Peat in their wonderful book *Turbulent Mirror* (1989):

*“Setting the figures on the equation is like setting dials on a spaceship and propel it toward a coordinate formed by the intersection of two parts which are called ‘real’ and ‘imaginary’. Any complex number is made up of these two parts. And any complex numbers can be represented by a point in the complex plane.*

*It’s pretty much like locating Phoenix, Arizona, in an atlas map by finding the intersection of the letter J and the number 10. The main difference is that on the complex plane the number of possible intersections is infinite and the real and imaginary parts of the coordinates can be positive, negative, whole numbers, or decimal expansions.*

*In the  $Z^2 + C$  equation, Z is a complex number allowed to vary and C is a fixed complex number. The adventurer sets his or her complex numbers into the equation and tells the computer to take the result of the addition of  $Z^2 + C$  and substitute it the next time around for Z... and again and again...*

*The computer searches for all the complex numbers in the area that are not so large as to exceed calculation capacity. The screen will display the fixed numbers in black, as they represent the M-set itself. The numbers that the iteration stretches toward infinity are represented in shades of color... The very boundary of the M-set has infinite depth because there is always an infinity of numbers between any two numbers on the complex plane.”*

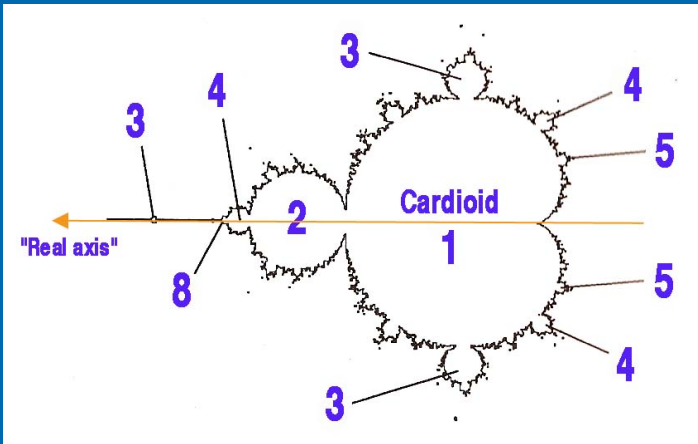


↑ The complex number plane C is the number field encompassing the other types of numbers, all the way to the “natural numbers” of daily life.  
[intermath.coe.uga.edu](http://intermath.coe.uga.edu)

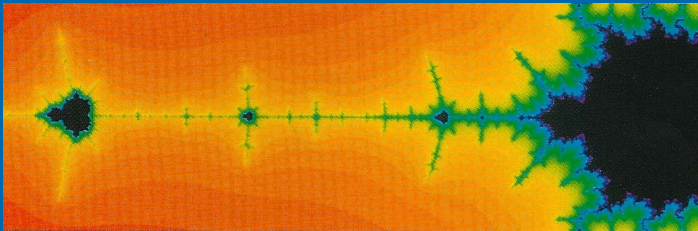


# SG203.1.7.1 Faces of the M-Set (1)

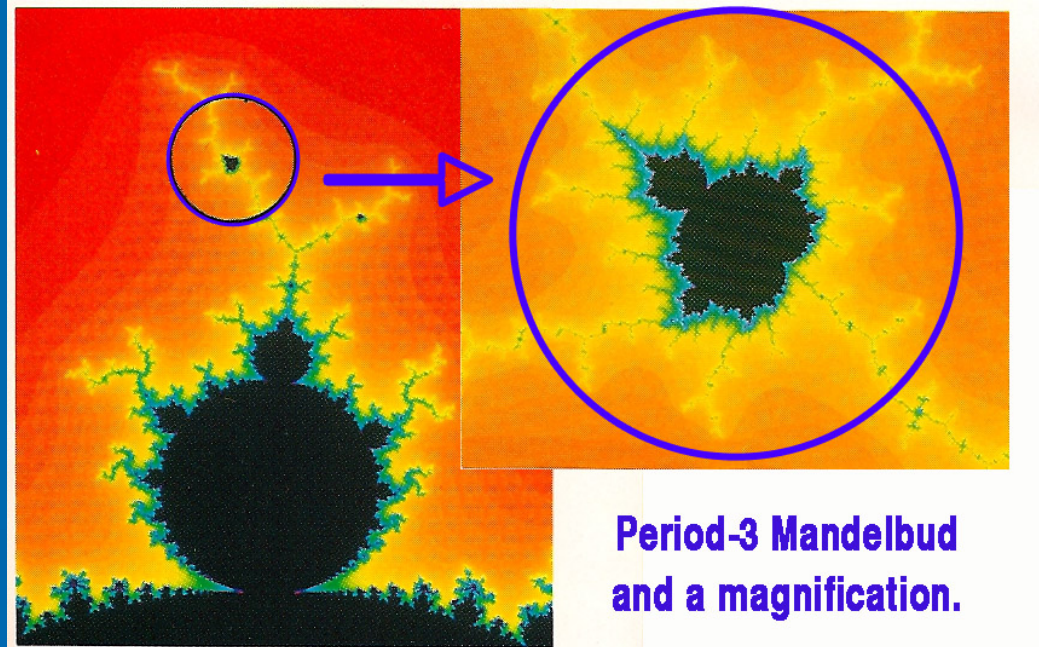
## Bulb Periods (1)



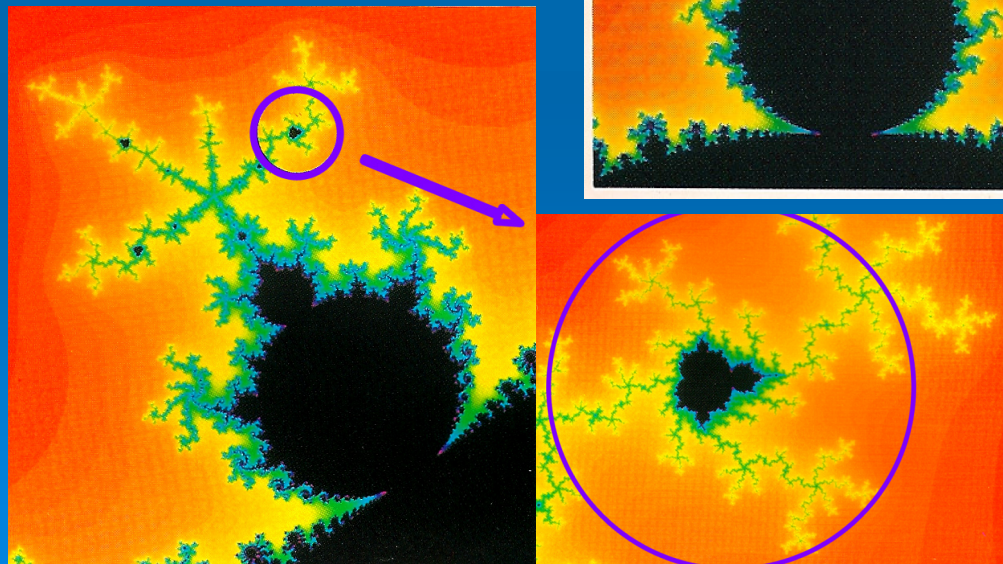
The M-set is symmetric about the real axis.  
 Moreover, the periods of the symmetrically located bulbs (buds) are the same.  
 The periods are the branching patterns.  
 The magnifications repeat the period branching of the original bulb.  
*(Below and next page are some examples.)*



↑ The M-set "antenna" showing symmetry about the axis.



**Period-3 Mandelbud and a magnification.**



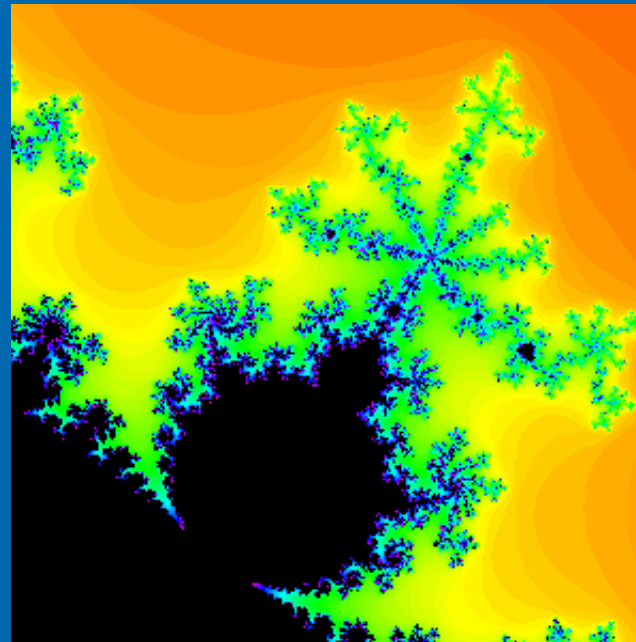
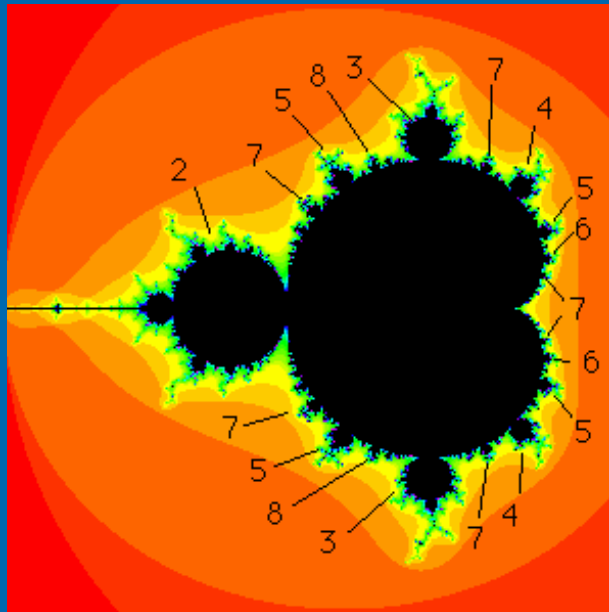
→ **Period-5 Mandelbud and a magnification.**



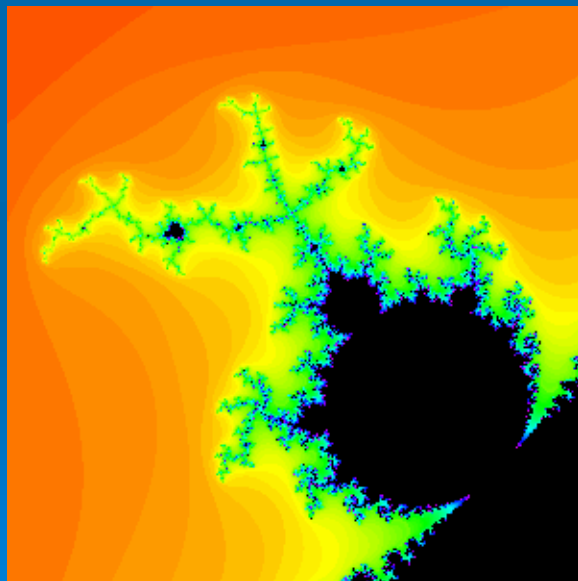
# SG203.1.7.2 The M-Set (2) Bulb Periods (2)

## More Bulb Periods

Images  
[www.bergen.edu](http://www.bergen.edu)



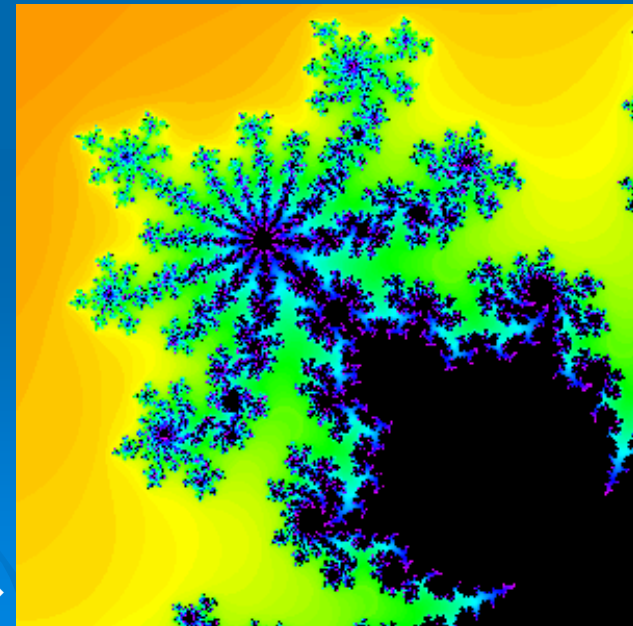
← 7-Period



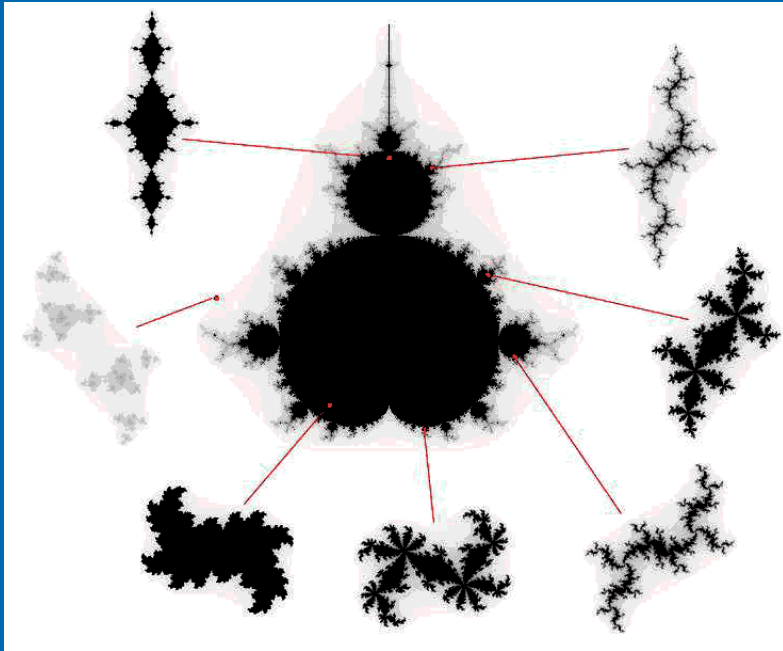
↑ 4-Period



12-Period →



## SG203.1.7.3 The M-Set (3) Julia-sets

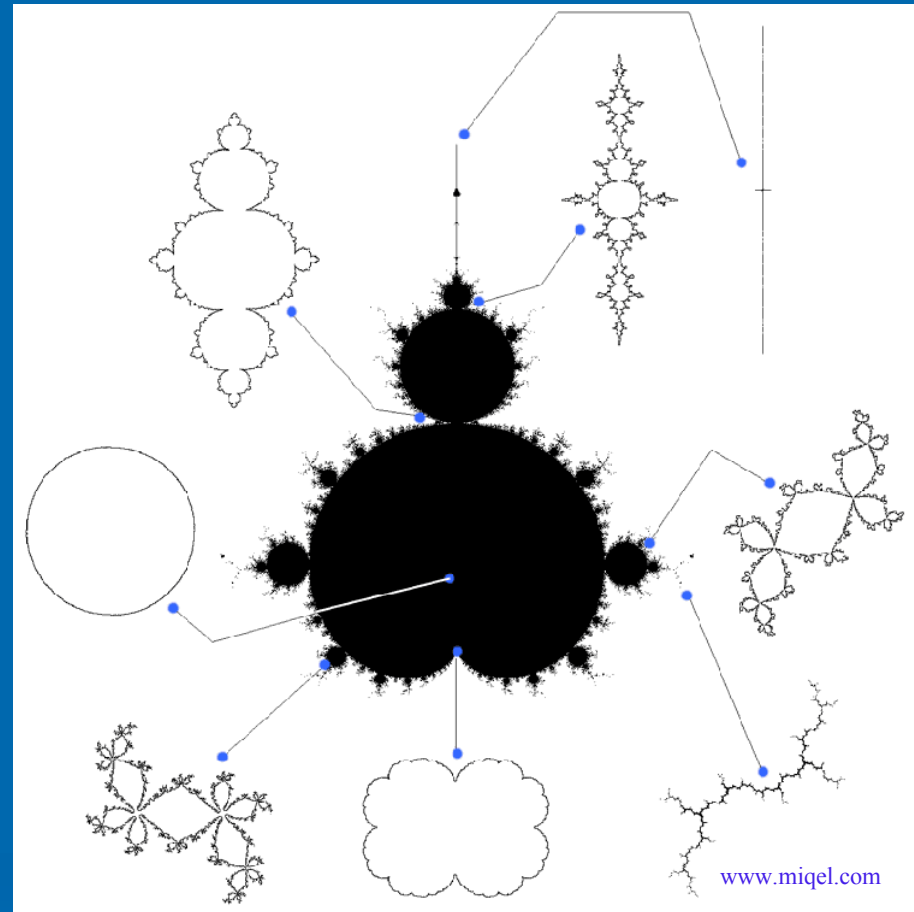


↑ Never-changing Julia-sets showing where they are located upon the self-similar yet ever changing M-set... Image: [Paul Burke](#).

*“For each point in the M-Set there is a corresponding Julia-set, the difference is J-sets repeat themselves perfectly over and over as you ‘zoom in’ by Iterating the equation into finer and finer points on the grid.*

*The M-set however changes constantly as you zoom in, and is a single continuous line that maps the transition between every possible Julia set (from a straight line to a million-coil spiral to lightning like fragments).”*

[www.miqel.com](http://www.miqel.com)



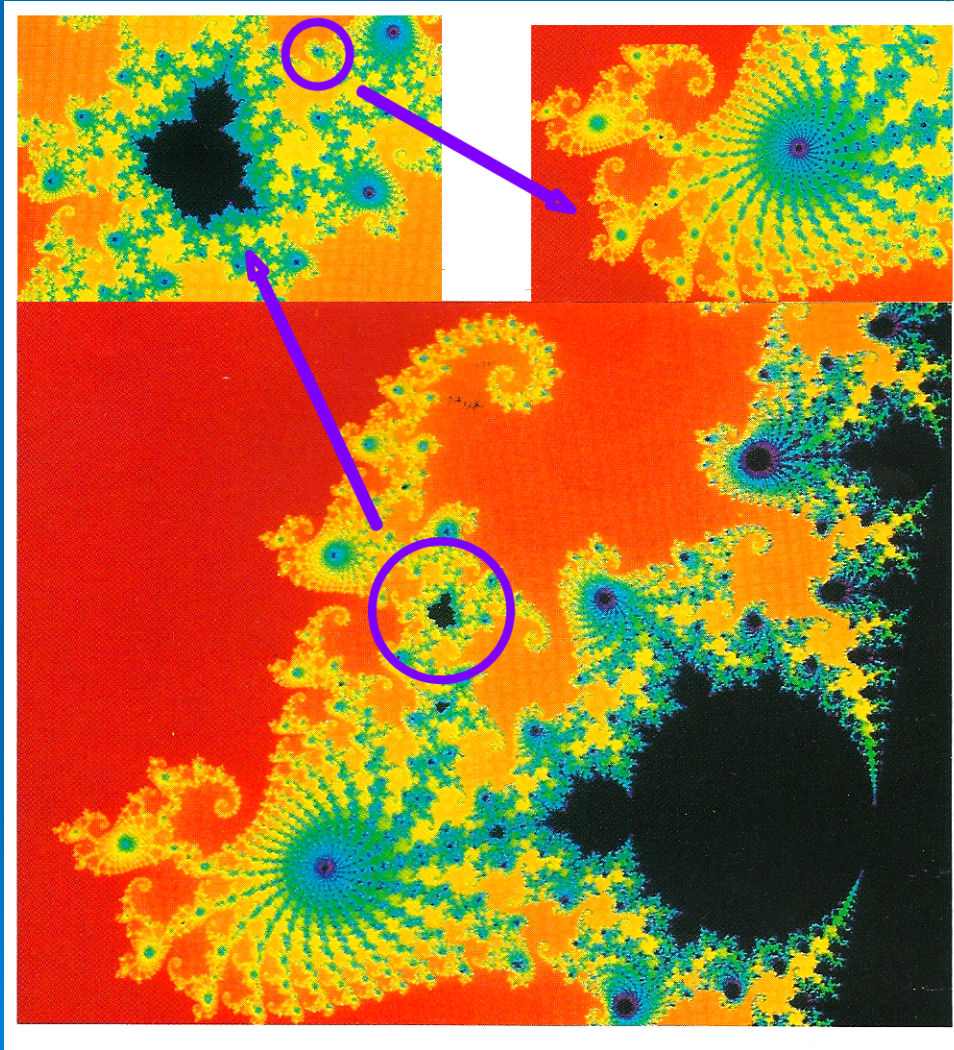
↑ The M-set is the *meta-pattern of all curves in 2D*: it contains every possible combination of curvature within its infinitely thin boundary.

Notice that one J-set (top right) is nearly a straight line (masculine/Yang) while on the left (middle) there is a nearly perfect circle (feminine/Yin). [◆SG101.5]

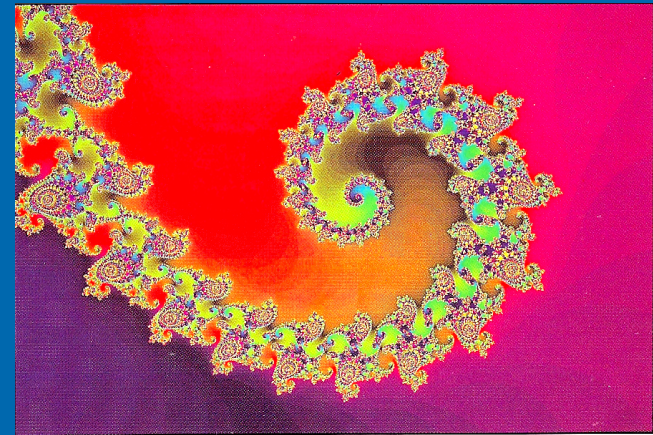
**The Mandelbrot set IS the two cosmic principles and everything in-between.**  
**The M-set is the dance between the line and the curve, the perfect geometer unifying the straight edge and compass.**

## SG203.1.7.4 The M-Set (4) Sea Horses

Many shapes are showing up in the exploration of the M-set, and like clouds, constellations or red rocks in Sedona, they evoke real or imaginary objects and beings: reefs, urchins, cliffs, starfish, snails, dragons....  
Here we show the popular images of "seahorses"...



Period-25 Mandelbud with 2 magnifications.

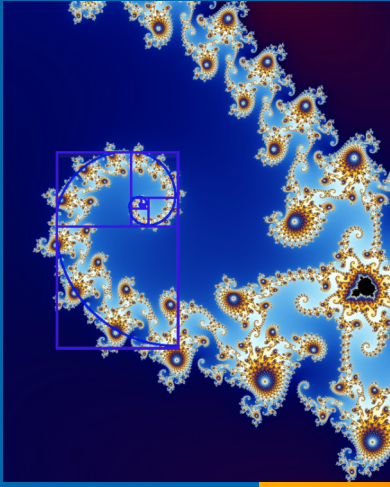


↑ Closeup on a "seahorse tail", a logarithmic spiral

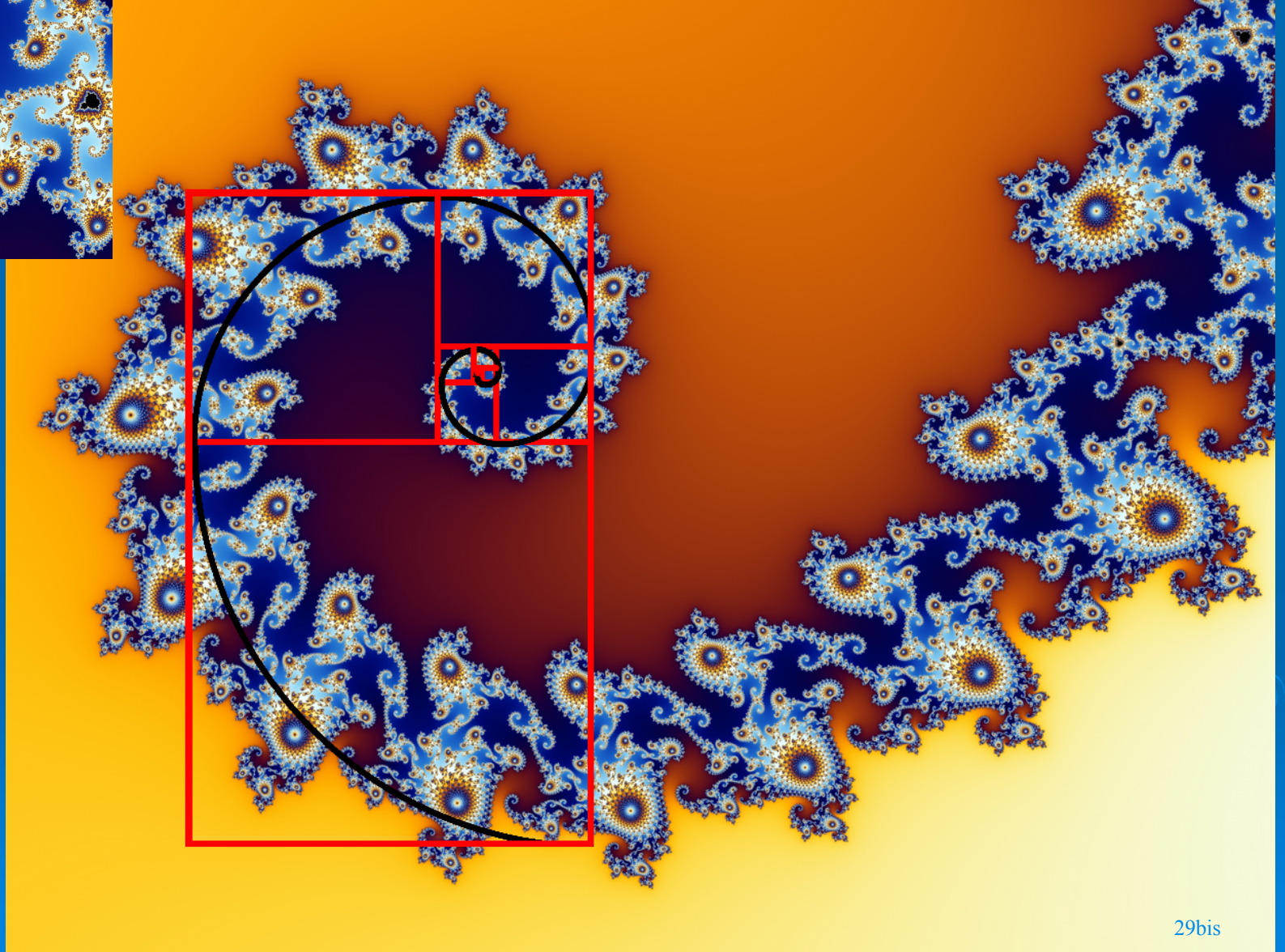


↑ Detail of above "seahorse tail".

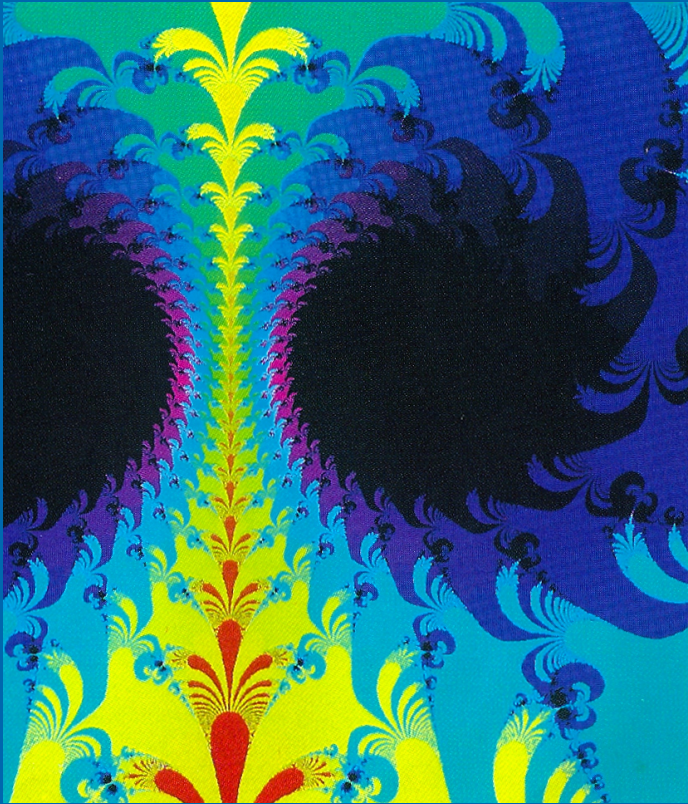
## SG203.1.7.5 The M-set (5) Seahorse Phi Spirals



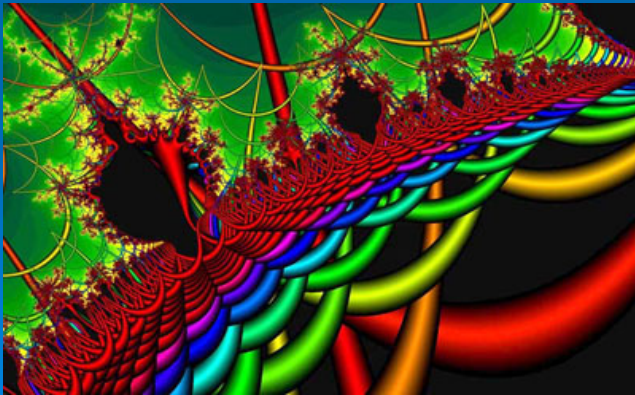
Like the Nautilus shell and many other spirals in nature, the M-set spirals of the "seahorses" approximate the Golden Spiral proportions - but are not exact class-book "whirling squares" Fibonacci spirals (90°): they have more specific expansion rates and variables waiting for analysis by harmonic mathematicians.



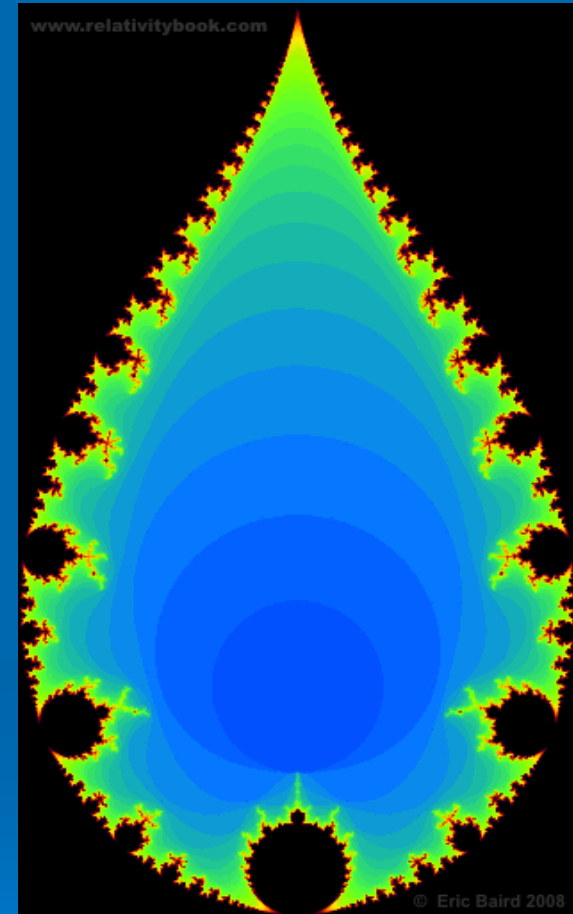
## SG203.1.7.6 The M-Set (6) Inverse & Exponential



↑ Figuration of a complex exponential.

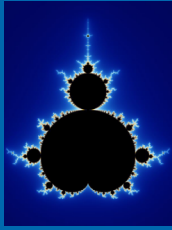


↑ Deep inside the fabric of the M-set. [www.miqel.com](http://www.miqel.com)

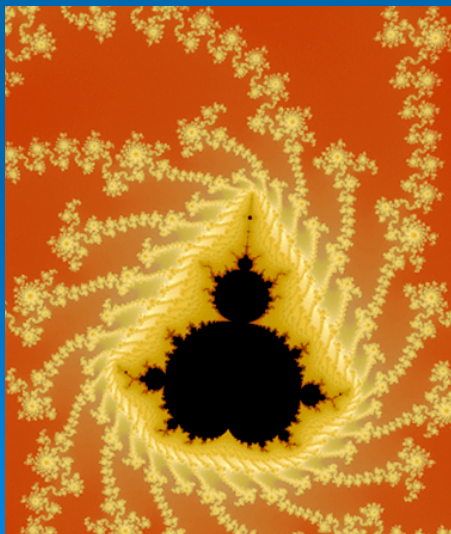


↑ Inverse M-sets.

Since the M-set doesn't exist outside a circle of radius 2 on the complex plane, it is possible to turn it inside out.



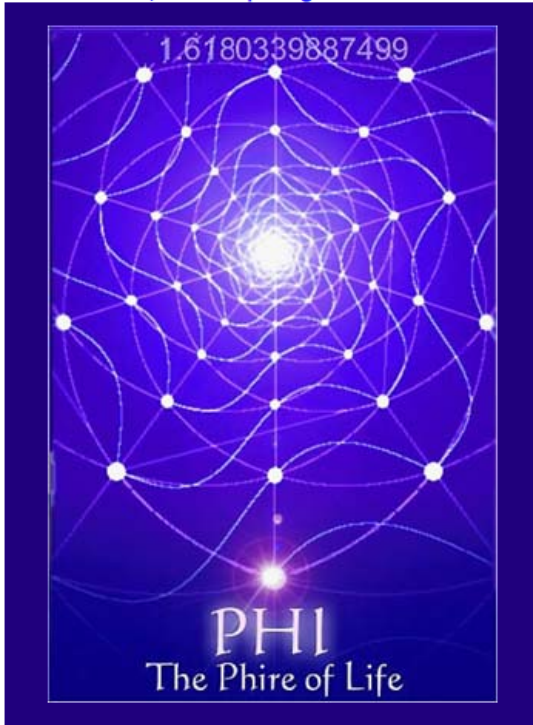
# SG203.1.7.7 The M-Set (7) Buddhas, Auras & Bulbs



Let your imagination create  
free associations...

## PHI and FRACTALS

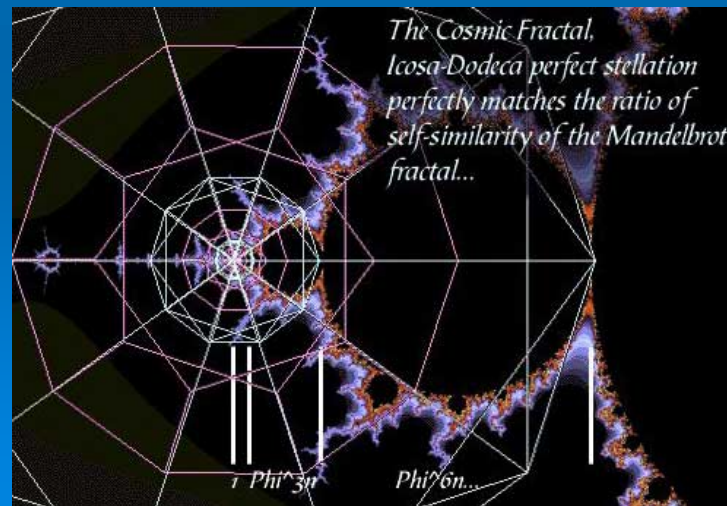
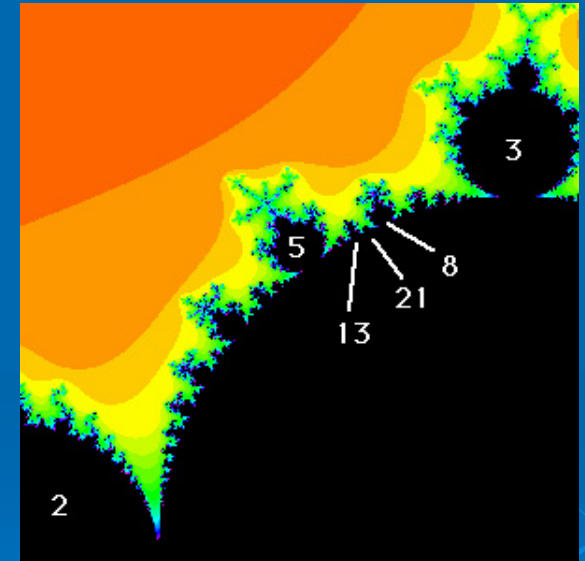
Phi Web, a nested pentagonal wave vortex



Phi and the MANDELBROT SET



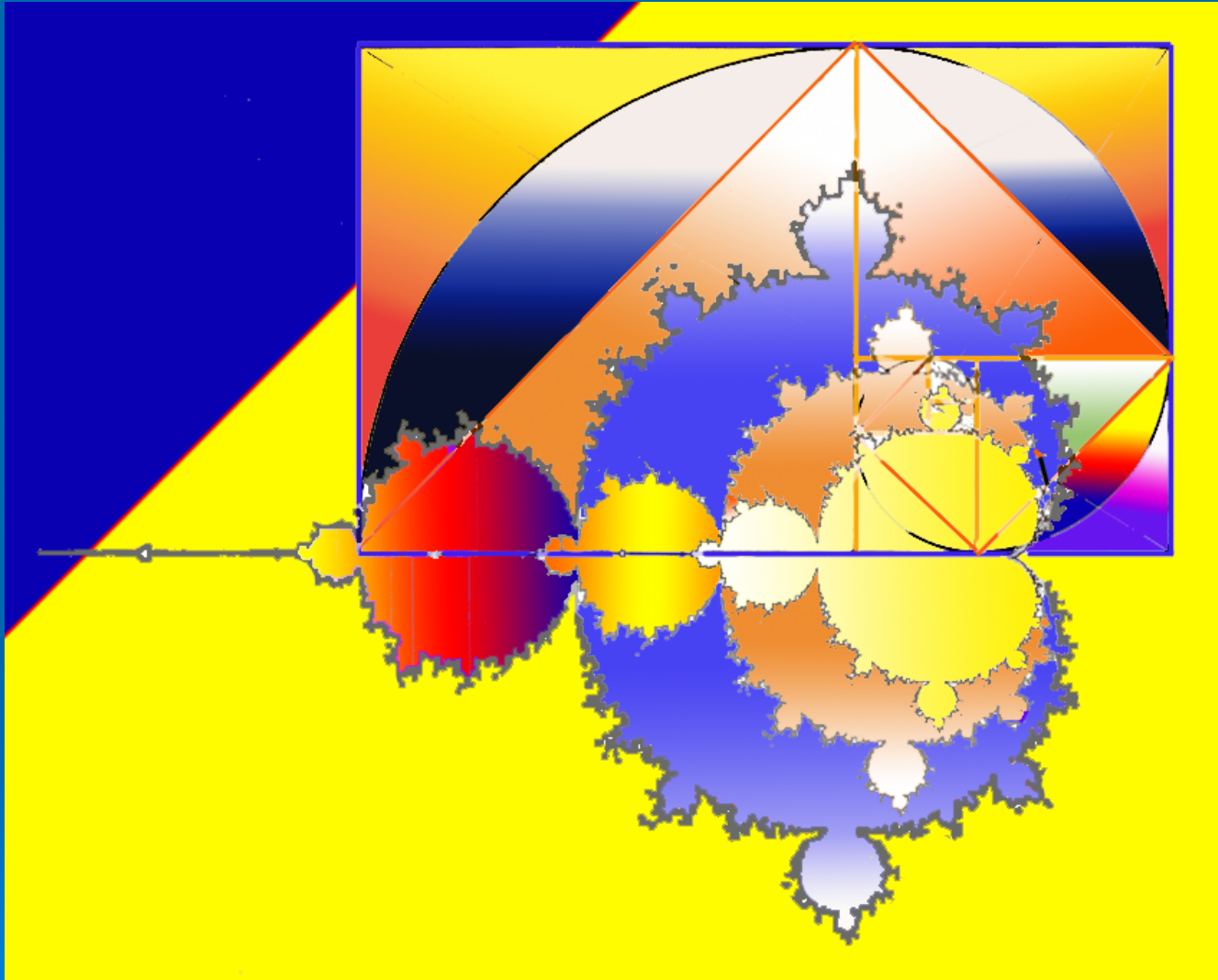
## SG203.1.8.1 Mandelbrot and Phi (1)



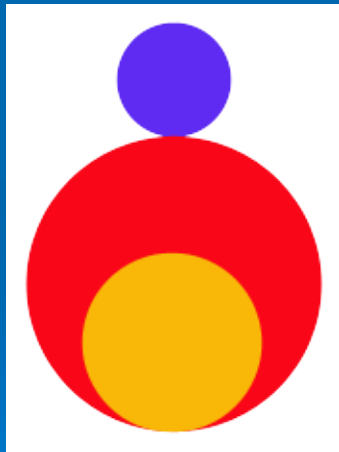
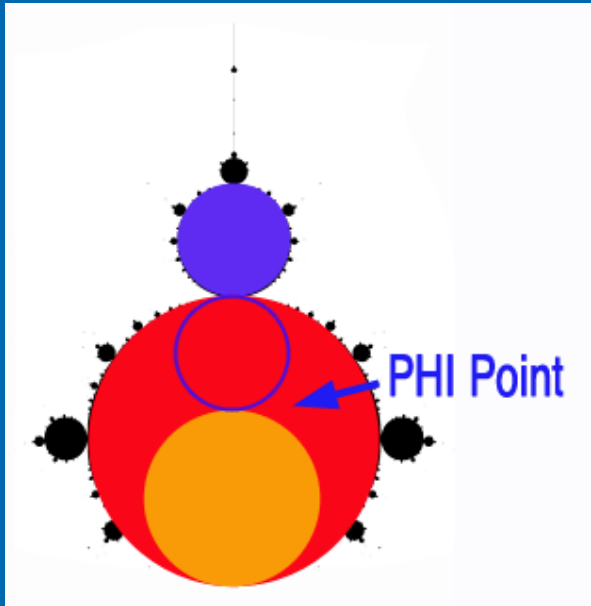
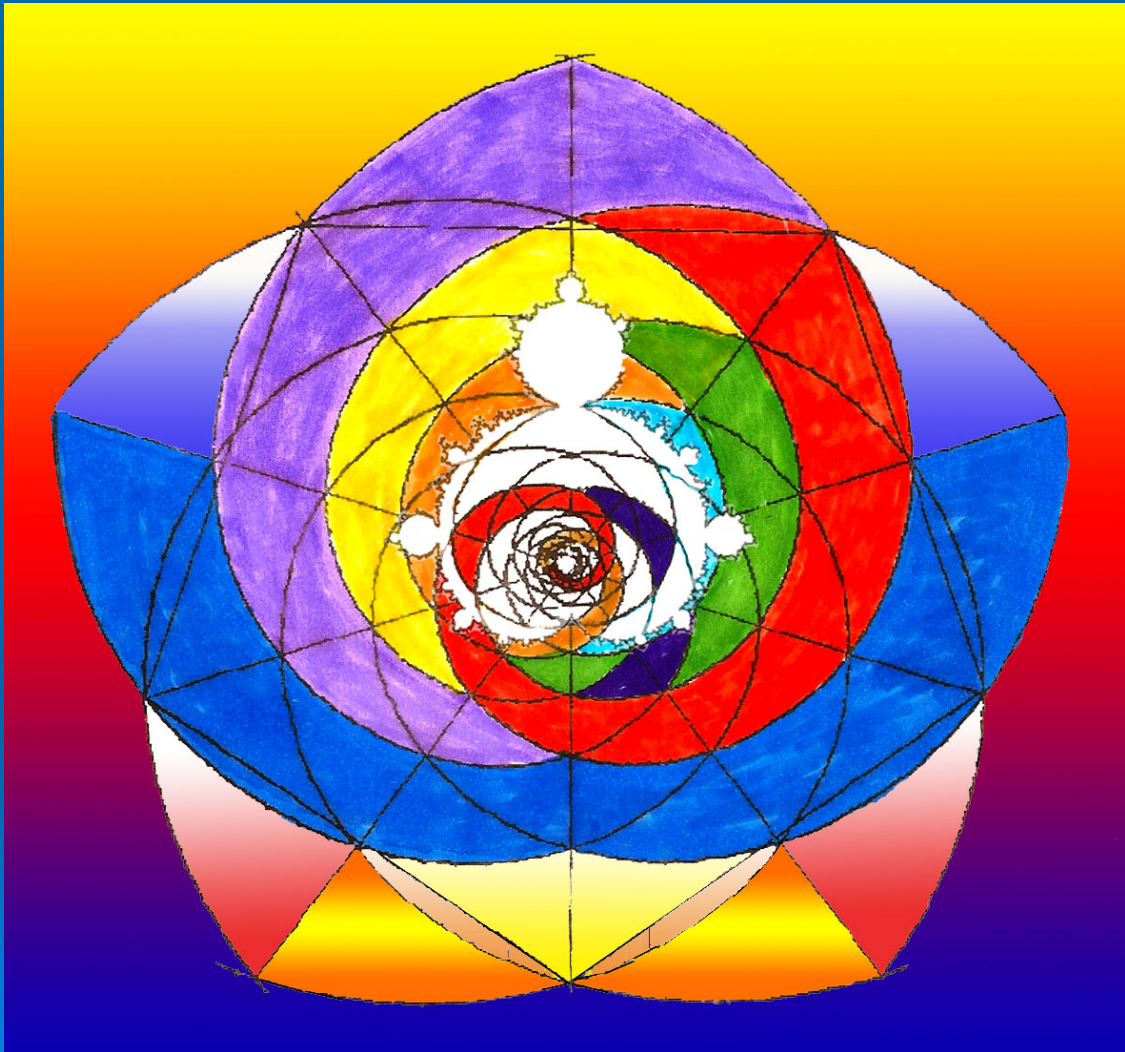
↑ → Renditions by Dan Winter showing the Phi correlations of the M-set.  
[www.goldenmean.info](http://www.goldenmean.info)



## SG203.1.8.2 Mandelbrot and Phi (2) Play 1



# SG203.1.8.3 Mandelbrot and Phi (3)



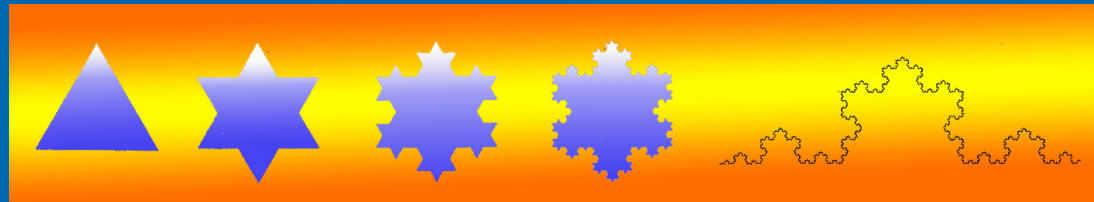
(Note the "self-similarity" with Sacred Geometry's Arbelos.)

[◆SG104.4]

# SG203.1.9.1 Golden Fractals (1)

## Reduction Factor

We have already met the famous Koch Snowflake, the result of a simple iterative function. The Koch fractal has a “length reduction factor” ( $f$ ) of  $1/3$ : the original side of  $1$  is replaced by  $3 \times 1/3$ .

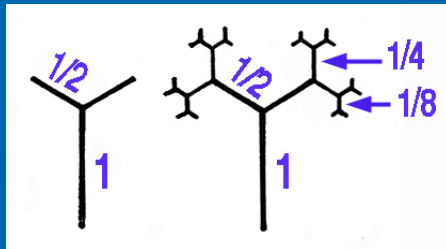
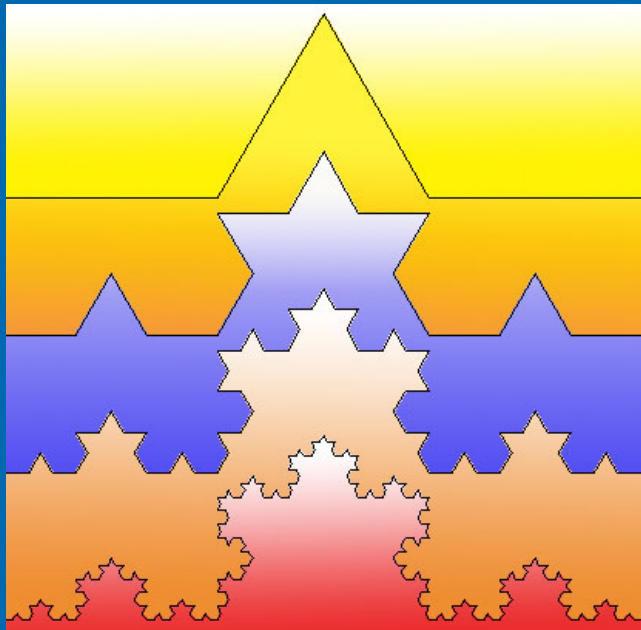


↑ Koch Snowflake Fractal with Reduction Factor  $1/3$

What interests us here is the question: what is happening when we vary the factor “ $f$ ”? If we use a reduction factor of  $1/2$ , we obtain elements that are still far apart.

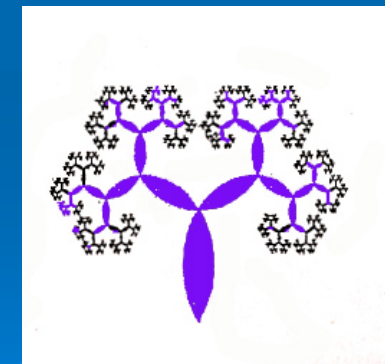
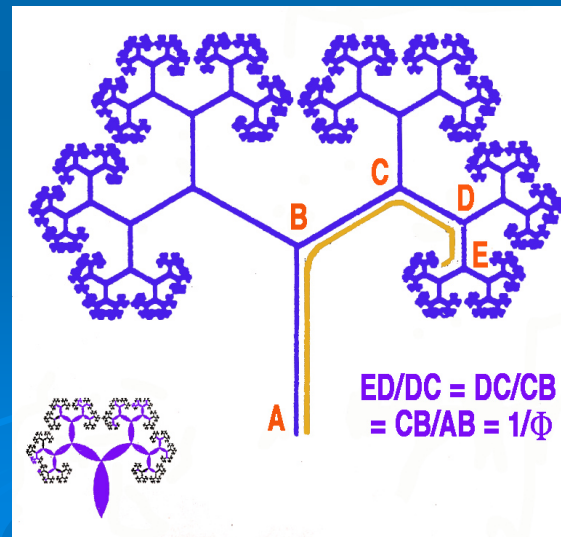
And the question arises: at what reduction factor  $f$  do the elements (branches) perfectly TOUCH but DO NOT OVERLAP?

This happens to be when  $f$  is precisely equal to the reciprocal of Phi or Minor Phi =  $1/\Phi = .618$



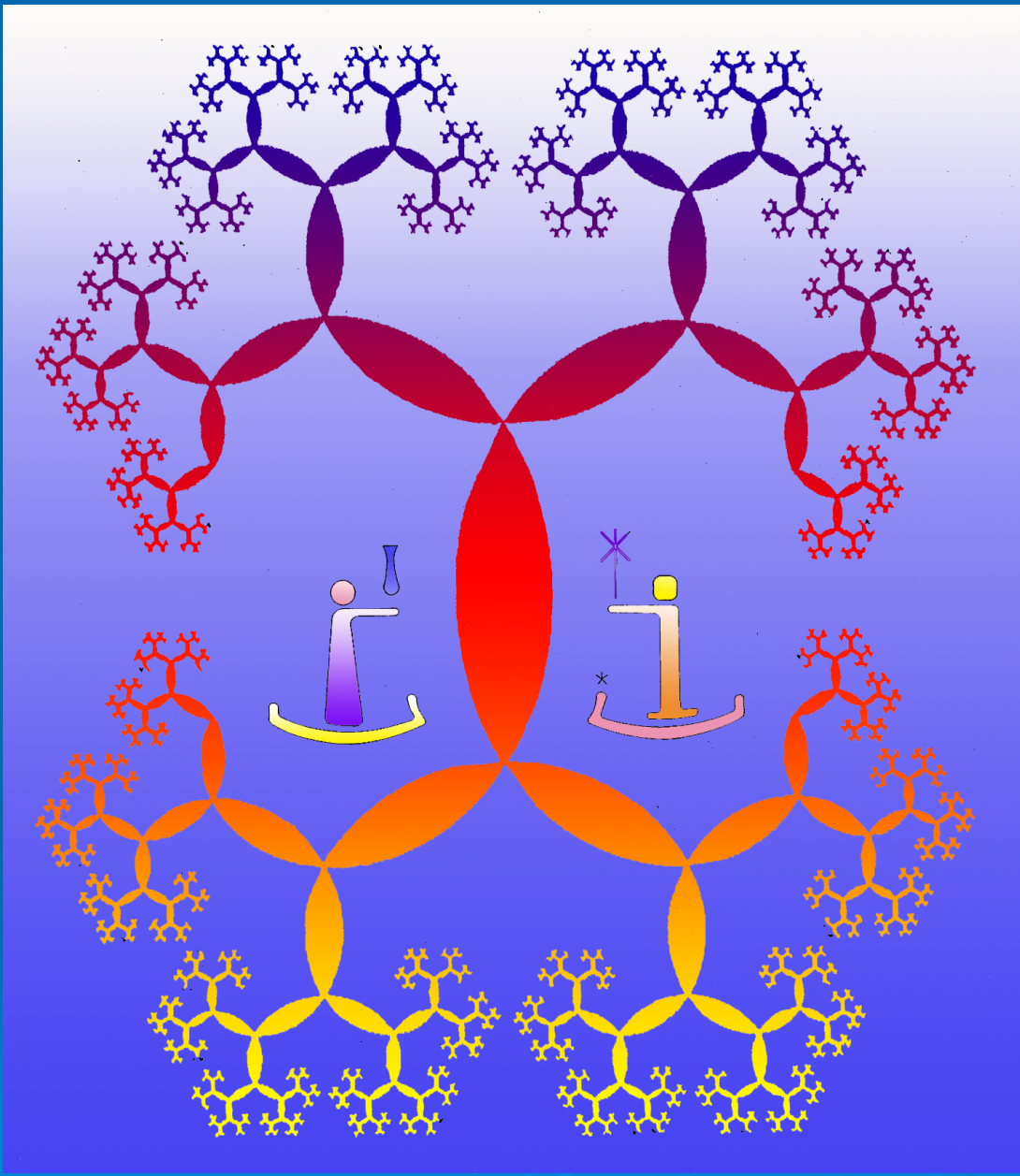
↑ If we use a reduction factor of  $1/2$  (instead of  $1/3$ ) and start with a line, we obtain the “ $f = 1/2$ ” tree. The elements are still far apart.

Note the appearance of the Golden Spiral →  
 $AB : BC :: BC : CD :: CD : DE$

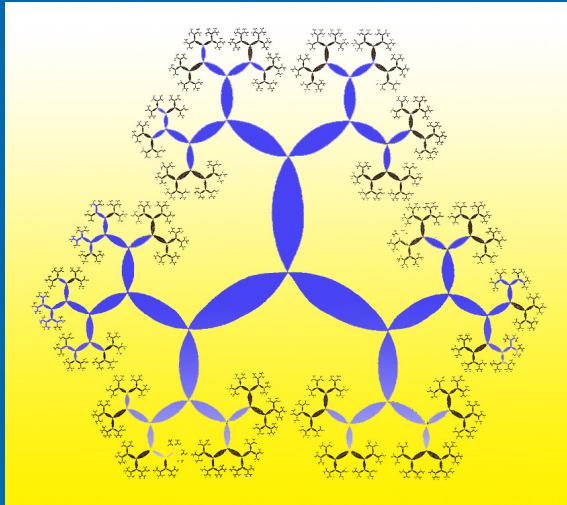


← ↑ Golden Fractal Tree with  $f = 1/\Phi = 0.618$  and fractal dimension =  $1.44$

**SG203.1.9.2**  
**Golden**  
**Fractals (2)**  
**Golden Trees**



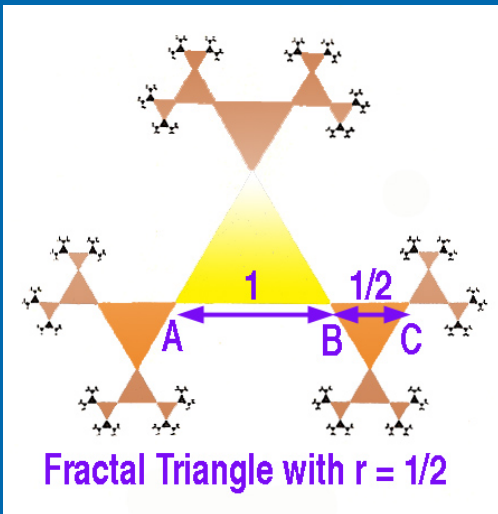
Four Golden Trees



Three Golden Trees

# SG203.1.9.3 Golden Fractals (3) Triangles

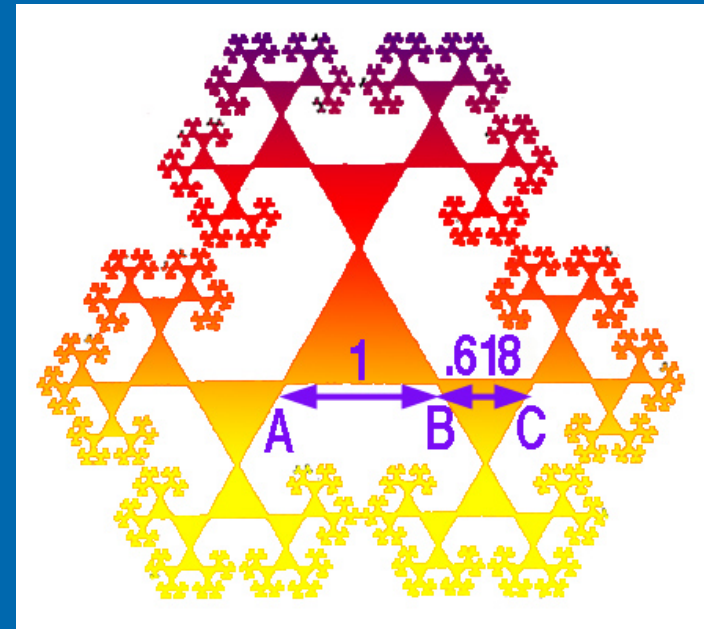
Fractals can not only be built from lines but also from geometric figures such as triangles & squares.  
 Interestingly enough, such fractal geometry brings up the Golden Ratio.



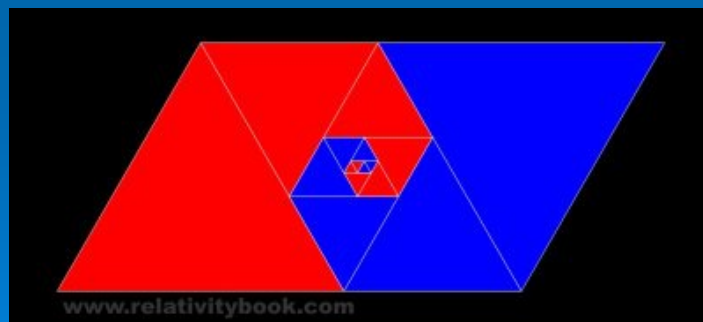
Let us ask the same question again: at what reduction factor would the boughs of this “tree” start to touch?

Yes, the answer again is:  
 $1/\Phi = .618$

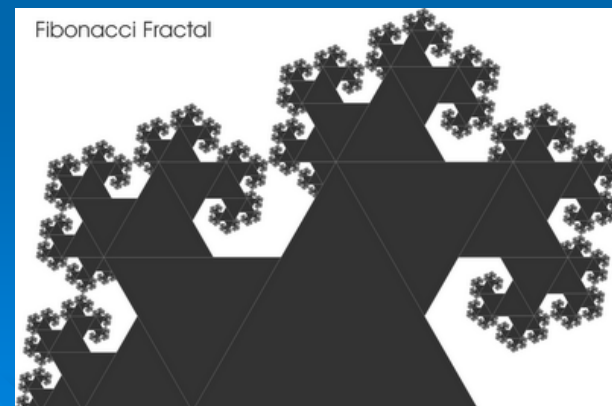
Golden Triangle Tree →  
 with  $f = 1/\Phi$



↑ Fractal Triangle Tree with a reduction factor  $f = .5$  (less than Phi): the branches do not touch.



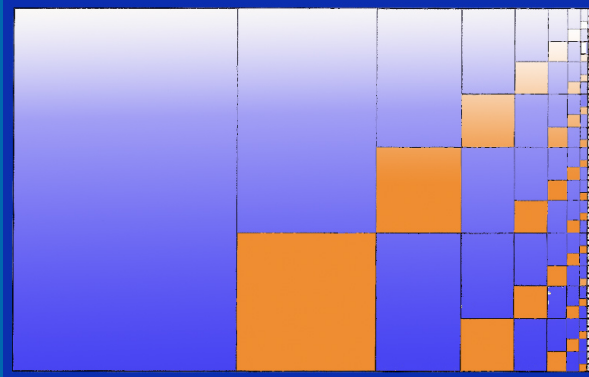
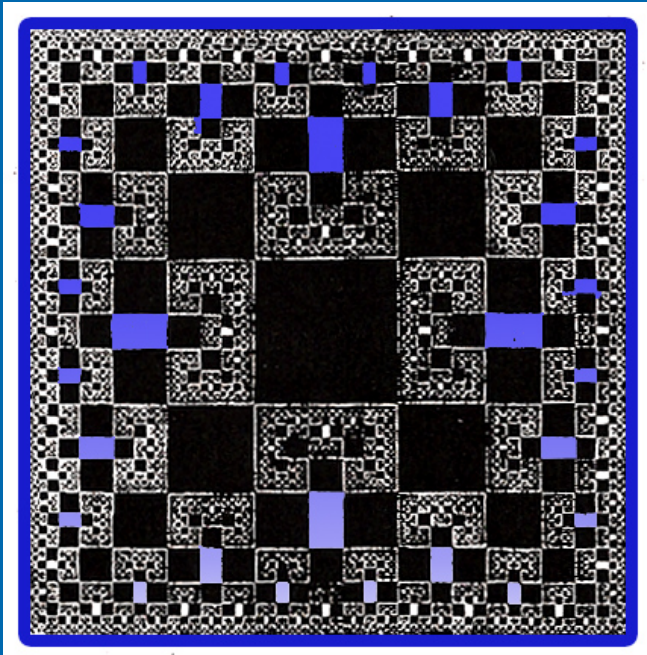
↑ Golden Spiral fractal triangles



↑ Another rendition of a Golden Triangle Tree

## SG203.1.9.4 Golden Fractals (4) Squares

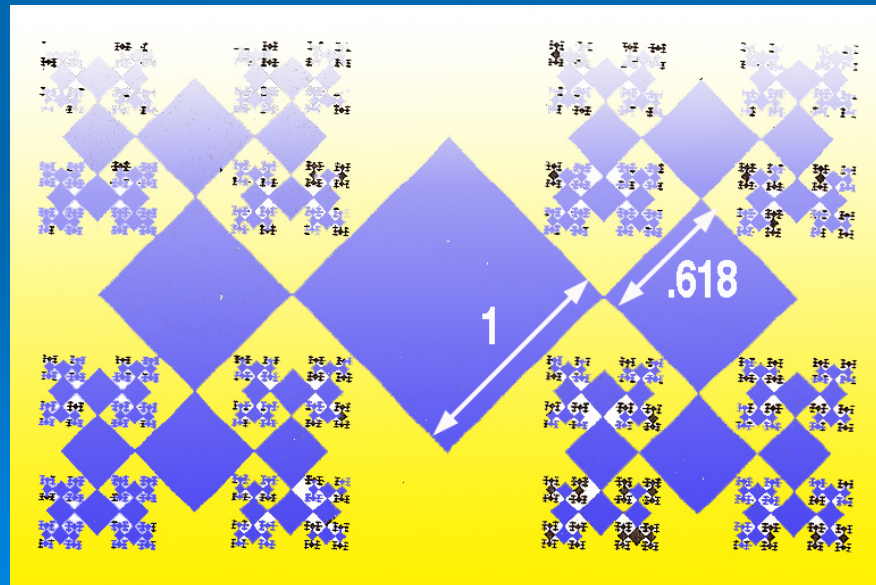
← The same answer appears again if we use a square: only at the reduction factor of  $1/\Phi$  will the squares touch but not overlap.  
 No wonder the Growth Angle in plants is based on  $\Phi$ : maximum exposure to elements but no overlap. Note that all rectangles (in blue) are Golden Rectangles!



← Fractal subdivision of the Golden Rectangle showing the Phi-based cascade of smaller squares and golden rectangles.  
 [♦SG105]

Mario Livio in his *Golden Ratio* (2002) comments:

*“We therefore find that while, in Euclidian geometry, the Golden Ratio originated from the Pentagon, in fractal geometry, it is associated even with simpler figures like squares and equilateral triangles.”*

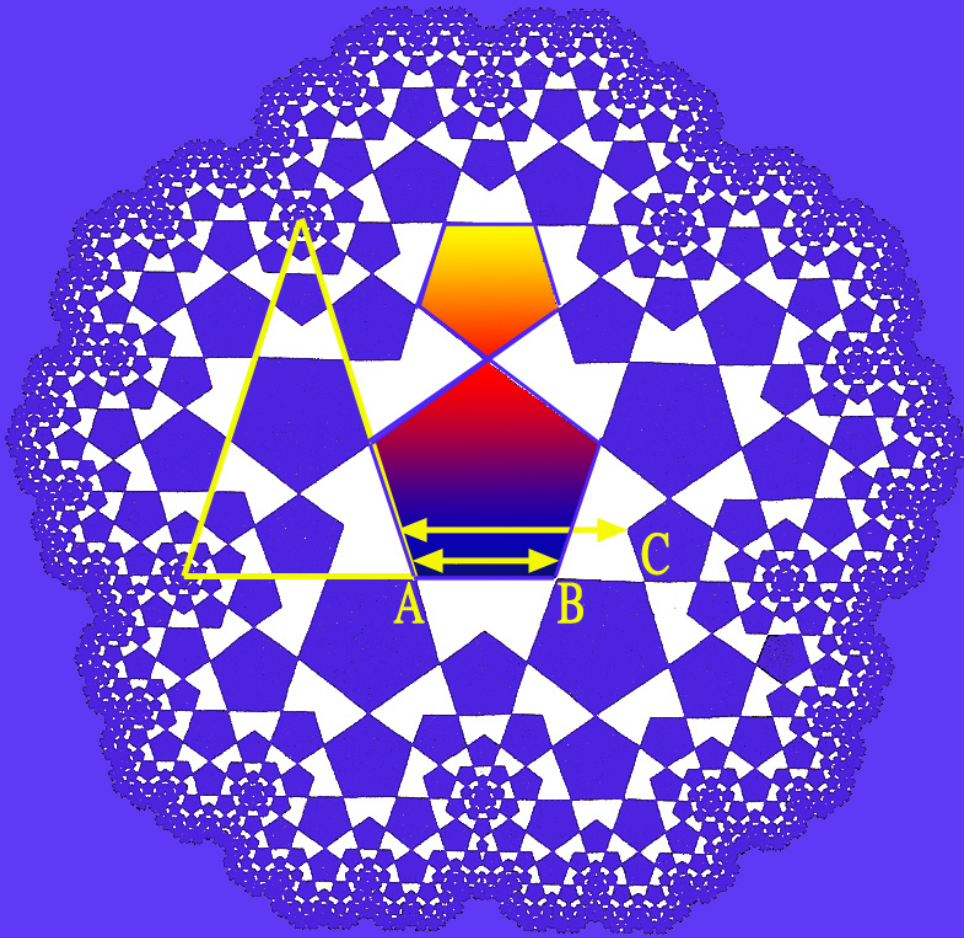


The Golden Ratio is the Fine Balance, Harmonic Scaling Constant of the universe.

← This square fractal is a Golden fractal with a T-shaped branching structure.

The sides of the squares are in a Phi Ratio relationship.

# SG203.1.9.5 Golden Fractals (5) Pentagons

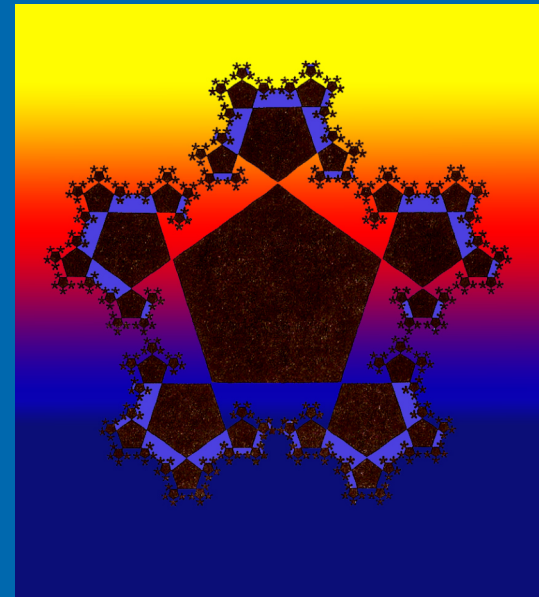


Golden Fractal based on pentagons.

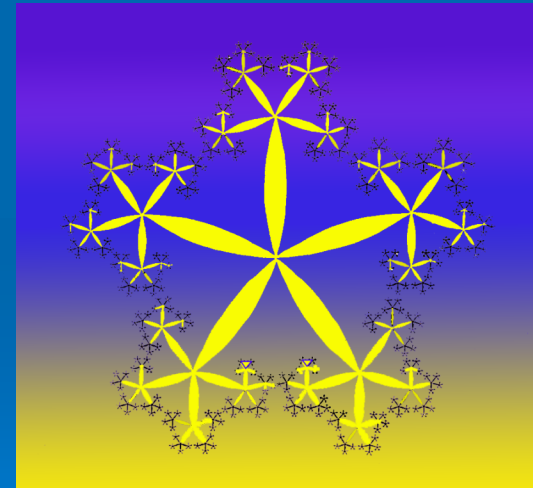
Reduction factor =  $1/\Phi = .618$

$AB = 1 \quad BC = 1/\Phi \quad AC = \Phi$

Note the Golden Triangle (in yellow)  
with base = unit 1 and sides = Phi.



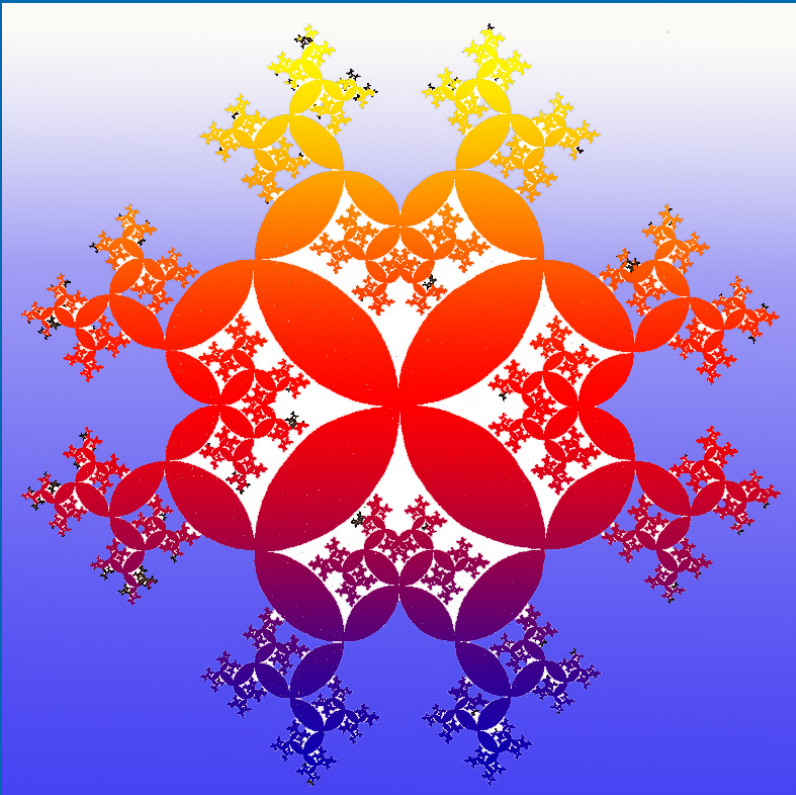
Reduction factor  $f = (1/\Phi)^2 = .382$



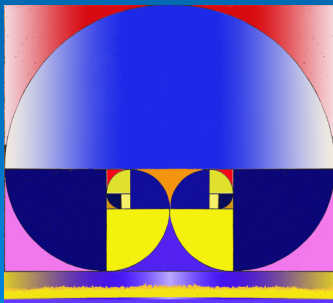
Reduction factor  $f = (1/\Phi)^2 = .382$

(After B&W images from:  
Hans Walser. *The Golden Section*. 2001)

## SG203.1.9.6 Golden Fractals (6) Golden Spirals

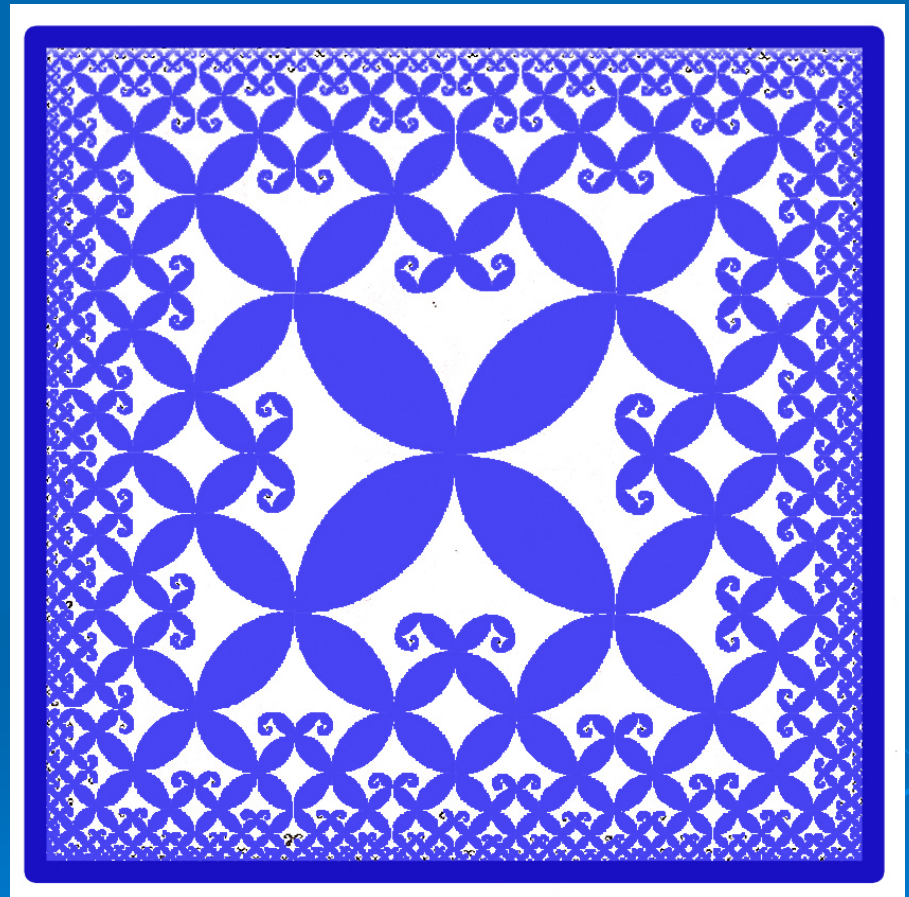


Golden Fractal based on Golden Spiral. (Hans Walser)



← Two Golden Spirals  
meeting face to face.

[◆SG105]



↑ Variation of the Golden Spiral fractal. (Hans Walser)





# SG203.1.11.1 Chaos Theory (1) Underlying Order

Ancient cosmologies and mythologies described the universe as a harmonic, although precarious, balance between the forces of chaos and the forces of order. [◆SG101.2] They conceived of chaos or "nothingness" (the current Zero Point Field) as a primordial, creative state, from which the material universe manifested (the formless abyss called Nut in Egypt or the goddess Tiamat in Babylonia). But chaos was always presented in a reciprocal and paradoxical relationship with order: they are seemingly opposite and yet they are an integral part of each other, like the Chinese Yin and Yang. Order is hiding in chaos and chaos contains order. Order and chaos are like mirror-images. This is what the new paradigm science of Chaos Theory describes as the dynamics of living systems. Remember the dynamics of cymatics: sound frequencies creating areas of upheaval (chaos) and order.

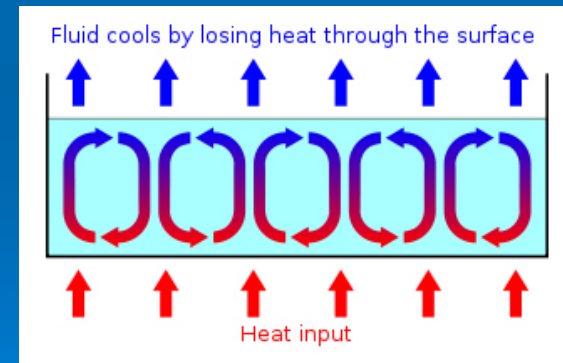
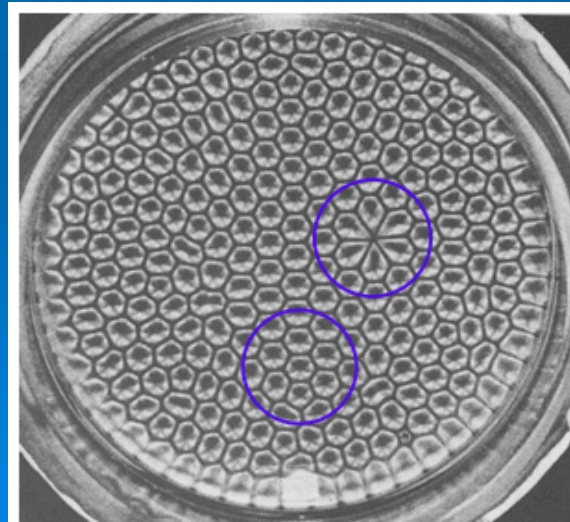
There is a Sufi saying: "You think because you understand *one*, you must understand *two*, because *one and one makes two*. But you must also understand *AND*." This is exactly what Chaos Theory does: it looks into the *inter-connectivity of systems* and the relationships between discrete parts. It looks into unseen, invisible forces & information signals existing between what were previously thought as separate entities only to turn out to be complementary phases of a larger process. We now realize that order and "chaotic" change are not the irreducible opposites we were taught. In fact, change and constant creation are nature's ways to maintain order and structure. We are daring to look into systems which, by their inherent design, fall apart and are seemingly destroyed only to be able to renew themselves. This understanding, applied to the human life experience, has large implications.

Russian-born Belgian-American chemist Ilya Prigogine (Nobel Prize 1977) is well-known for his work on "*dissipative structures*" which have the capacity to regenerate to higher levels of self-organization. In older mechanistic models, fluctuations & disturbances were viewed as signs of trouble. But dissipative structures show the ability of living systems to respond to disorder with new levels of life. Dissipative structures operate according to *non-equilibrium thermodynamics*: they are not in stationary states, are continuously subject to flux of matter and energy to and from other systems, and exhibit spontaneous breaking or creation of symmetry. Dissipative structures are patterns or *process structures* i.e. they maintain form over time yet have no rigidity of structure. Prigogine's work has helped explain a puzzle: if entropy is the rule, why does life flourish? Simple examples include convection, cyclones and hurricanes. More complex examples include lasers, Bénard cells, the Belousov-Zhabotinsky reaction and at the most sophisticated level, life itself.

*Bénard cells* → are convection cells that appear spontaneously in a liquid layer when heat is applied from below. They can be obtained using a simple experiment first conducted by Henri Bénard, a French physicist, in 1900.

The appearance of Bénard cells is an example of *order out of chaos* - which is the title of Prigogine's book on *dissipative structures*.

The local heating causes entropy to increase, but the density inversion induces complex & non-linear behavior: convection cells that arrange into a regular *hexagonal lattice*, which turns out to be a classic *geometry of emergent order*.

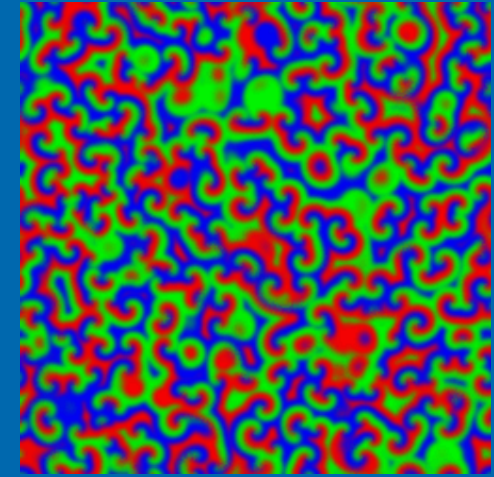


↑ Convection process. (Wikipedia)

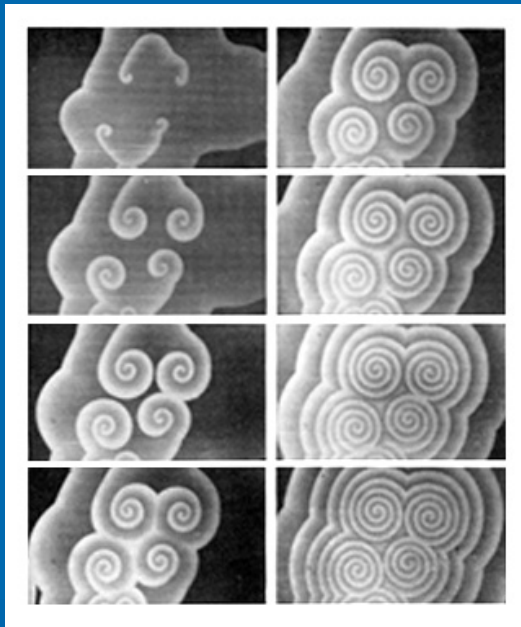
← Typical Bénard cells. Note the hexagonal geometry.

# SG203.1.11.2 Chaos Theory (2) B Z Reaction

A *Belousov-Zhabotinsky reaction* (BZ reaction) is a classical example of *non-equilibrium thermodynamics*, resulting in the establishment of a nonlinear chemical oscillator. These reactions are *far from equilibrium* and remain so for a significant length of time. During the BZ reaction, transition-metal ions catalyze oxidation of various, usually organic, reductants by bromic acid in acidic water solution and create beautiful patterns of chaotic order.



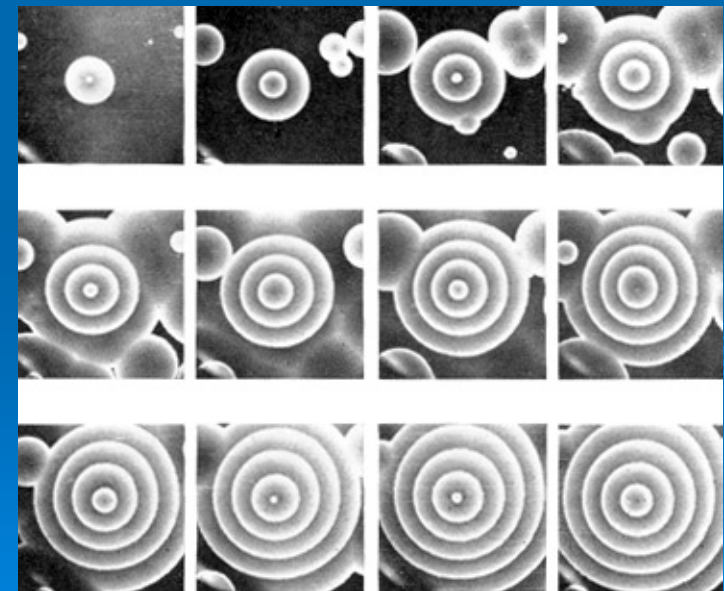
BZ reaction. Animation.  
Wikipedia.



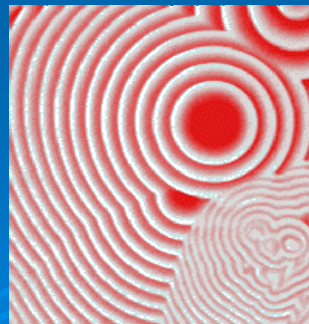
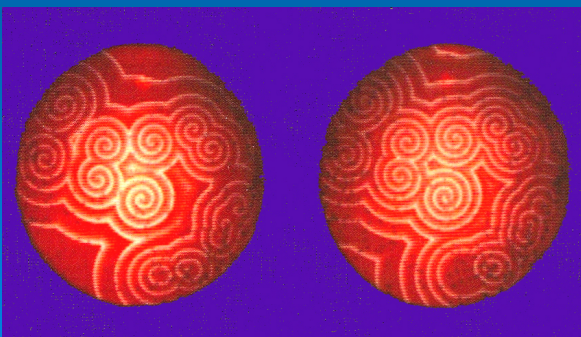
↑↓ Examples of BZ reaction



↑ Spiral coral  
similar to BZ patterns.  
[credit](#)



[www.scholarpedia.org](http://www.scholarpedia.org)



[www.faidherbe.org](http://www.faidherbe.org)

# SG203.1.11.3 Chaos Theory (3) Fractals

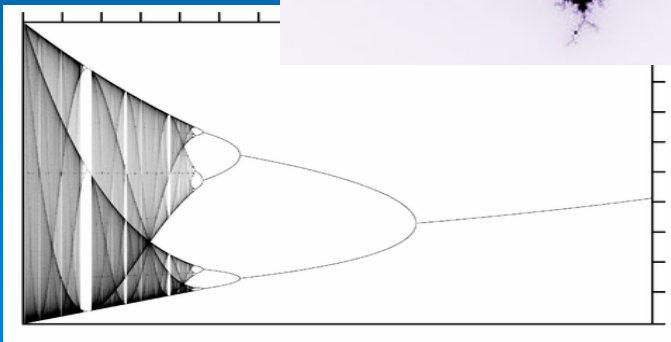
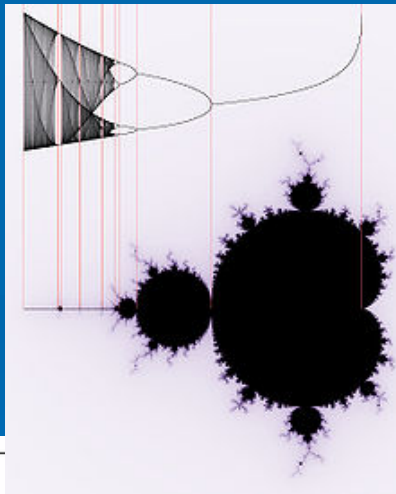
There is a deep and ubiquitous connection between chaos and fractals. Mathematicians like to state: *fractals are the complex systems of chaos made visible*. In recent years, it has become apparent that most chaotic regions for dynamical systems are fractals. The Julia set, for instance, is the place where all the chaotic behavior of a complex function occurs.

Chaotic systems are *complex*. Complexity can be called the "edge of chaos". Complex dynamical systems may be very large or very small. A complex system is neither completely deterministic nor completely random: it exhibits both characteristics. The different parts of complex systems are linked and affect one another in a synergistic manner. In a complex system, there are many interactive loops of feedback, resonance, self-similarity, fractality, ZPE field-level communication etc...

We no longer live in the old science linear world: we live in the dynamic chaos of a fractal cosmos. Now that computers can carry out extremely complex calculations in minimal time, chaotic fractal systems are known to be widespread and, in fact, to be the norm in the universe, both in natural and man-made systems.

Chaos theory describes complex motion and the dynamics of sensitive systems. Chaotic systems are mathematically deterministic but impossible to precisely predict: behavior in chaotic systems is *a-periodic* (no variable undergoes a regular repetition of values). However, a chaotic system can actually evolve in a way that appears to be smooth and ordered. The presence of chaos produces ordered structures and fractal patterns on a larger scale. It has been found that the presence of chaos is necessary for larger scale patterns (such as mountains and galaxies) to arise.

Correspondence →  
between the  
Mandelbrot set  
and the "bifurcation  
diagram"  
(top left & below)



## The Butterfly Effect

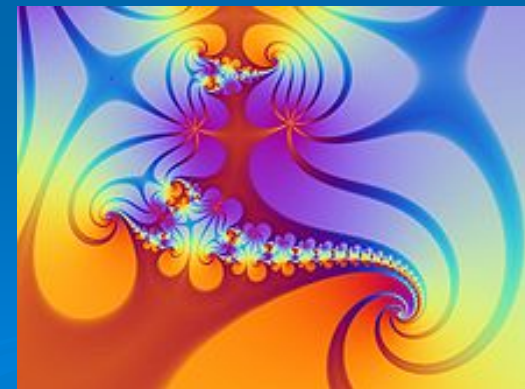
Chaotic systems are non-linear and non-deterministic: they are extremely sensitive to *initial perturbations*. This also means that they are sensitive to INTENTION.

The weather is an example of a chaotic system. In the 1960s, Edward Lorenz was a meteorologist at MIT working on a project to simulate weather patterns on a computer. He accidentally stumbled upon the "Butterfly Effect" which reflects how changes on the small scale affect things on the large scale: a butterfly flapping its wings in Hong Kong could change tornado patterns in Texas, by a form of *fractal resonance*.

→ Coloring of the field lines in the *Fatou* domain for an iteration of the form:  
 $(1 - z^3 / 6) / (z - z^2 / 2)^2 + c$ .

In the context of complex dynamics, the *Julia set* and the *Fatou set* are two complementary sets: regular and chaotic.

In a *Fatou set*, all nearby values behave similarly under repeated iteration of the function. In a *Julia set* the values are such that an arbitrarily small perturbation can cause drastic changes in the sequence of iterated function values.

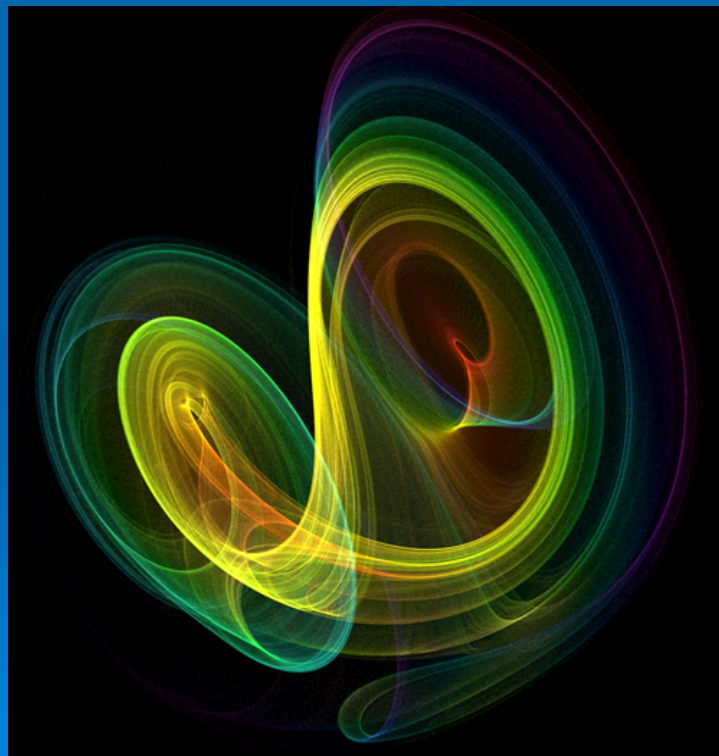


# SG203.1.11.4 Chaos Theory (4)

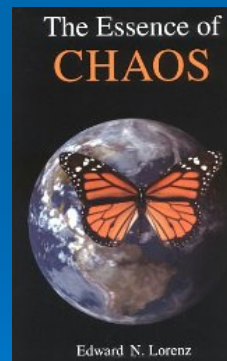
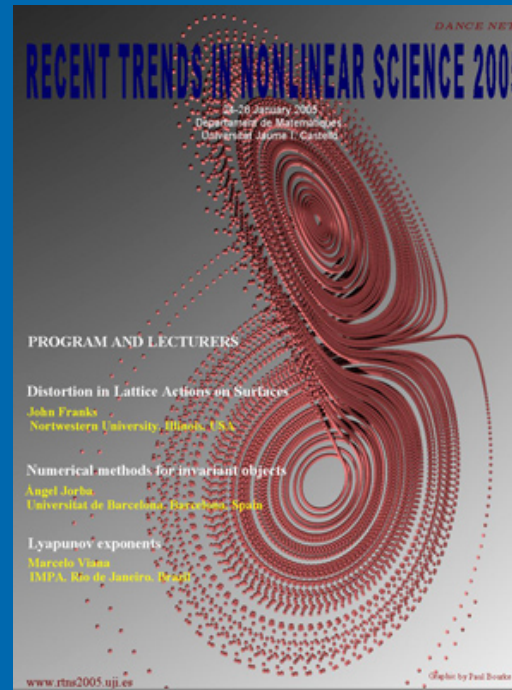
## Lorenz Attractor

From the simplified equations of convection rolls arising in the equations of the atmosphere, meteorologist Edward Lorenz derived the *Lorenz Attractor*, a fractal structure that pulls points towards itself and corresponds to the long-term behavior of the *Lorenz oscillator*. The Lorenz oscillator is a 3-dimensional dynamical system that exhibits chaotic flow, is noted for its lemniscate (infinity sign) shape, and is a *strange attractor*.

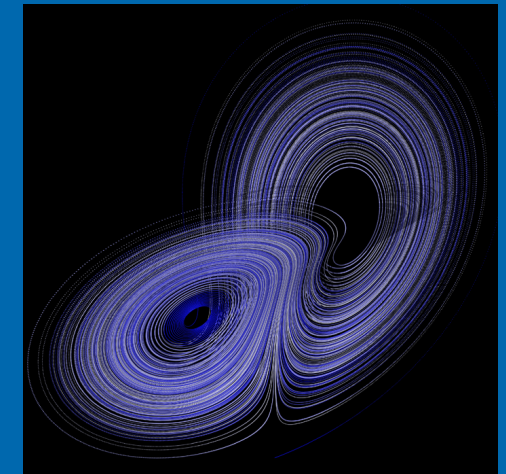
A *strange attractor* is a fractal structure set towards which a dynamical system evolves over time: points that get close enough to the attractor remain close even if slightly disturbed. Describing the attractors of chaotic dynamical systems has been one of the achievements of chaos theory.



[www.edc.ncl.ac.uk](http://www.edc.ncl.ac.uk)

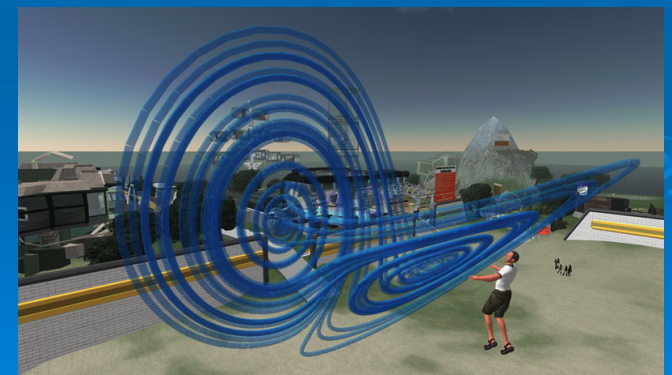


↑ The seminal book by Edward Lorenz



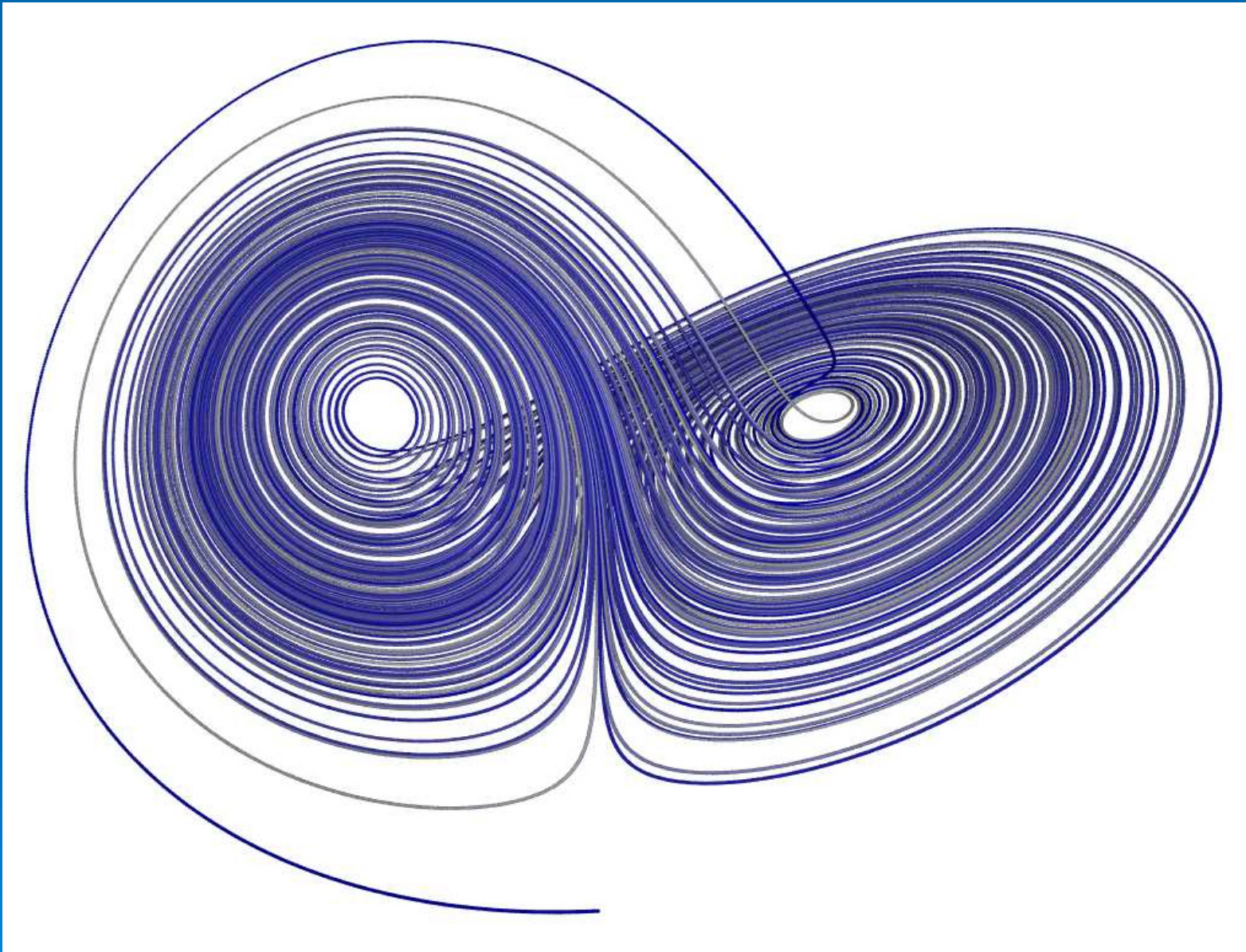
↑ Pov-Ray by Marcus Fritzsche

Images from the website of [Paul Bourke](#)



↑ The L-Attractor in Second Life.

## SG203.1.11.5 Chaos Theory (5) Lorenz Attractor



### Practice:

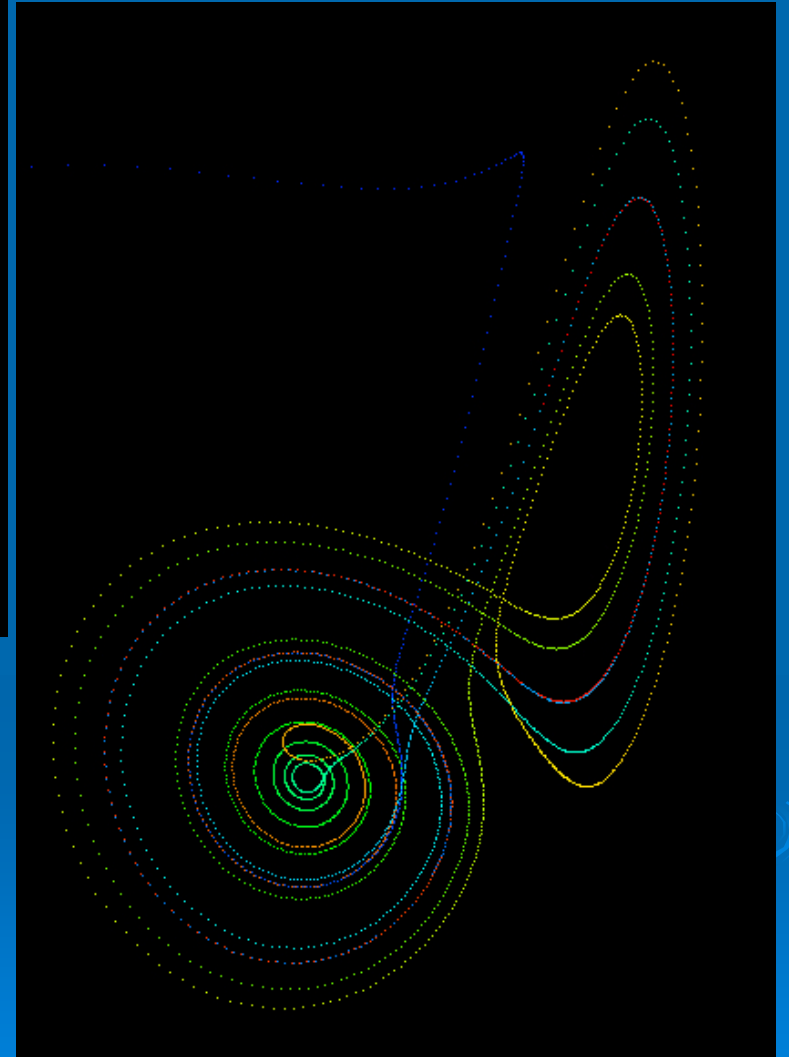
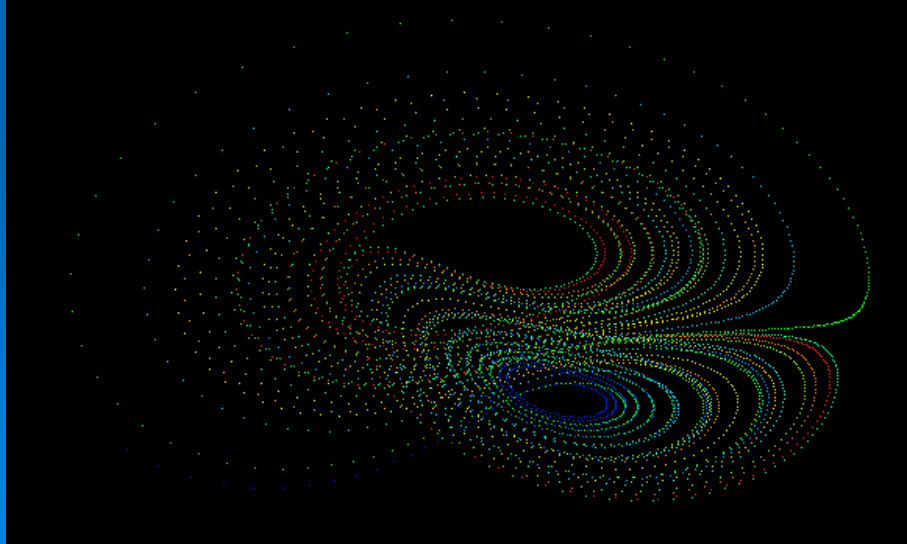
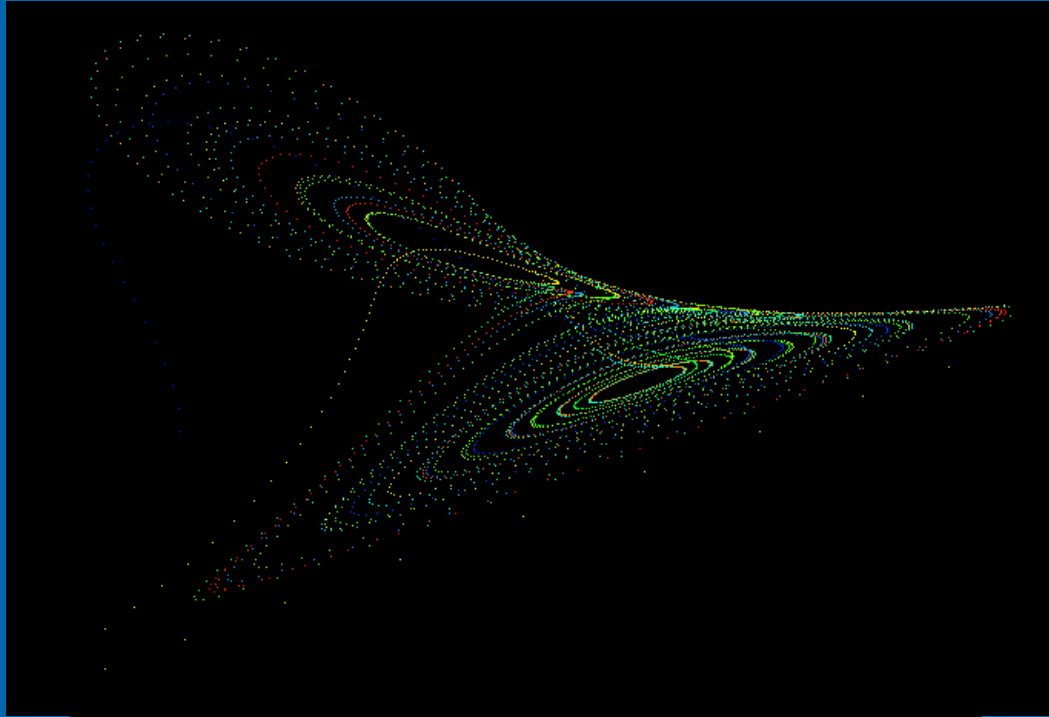
Allow your contemplative gaze to meander through this design...

See the dynamic flow, the alternation of polarities, the vortex formation, the clockwise and counter-clockwise motions, the symbolic archetypes, the infinity sign...

Surf a wave system of fractal creation...

## SG203.1.11.6 Chaos Theory (6) Lorenz Attractor

Some trajectory renderings of the Lorenz Strange Attractor are images of sparkling Beauty...  
(Graphics on Paul Bourke website:  
[local.wasp.uwa.edu.au](http://local.wasp.uwa.edu.au))

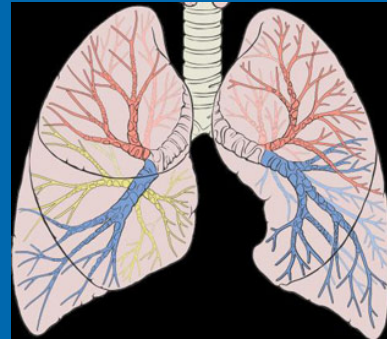


## SG203.1.12 Fractals in the Human Body: Lungs.

The human organs and systems are amazing feats of nature's technology. Fractal self-similarity pervades the human body, not in a mechanical, repetitive type of way but in a highly creative, adaptive, inter-active and dynamic way.

- The human circulatory system exhibits a more rapid branching than simple scaling would suggest. Studies have shown that the blood supply bifurcates between 8 and 30 times before reaching each particular location in the body, with a fractal dimension of 3.
- The human lung system also is an extraordinarily efficient distribution system.

*"The human lungs are the organs of respiration in humans. Humans have two lungs, with the left being divided into two lobes and the right into three lobes. Together, the lungs contain approximately the same length as 1500 miles (2,400 km) of airways and 300 to 500 million alveoli, having a total surface area of about 70 m<sup>2</sup> in adults — roughly the same area as one side of a tennis court. Furthermore, if all of the capillaries that surround the alveoli were unwound and laid end to end, they would extend for about 620 miles."* Wikipedia.



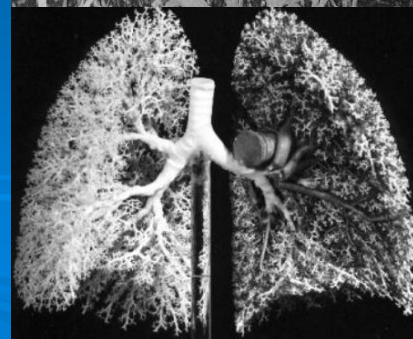
*The ratio of lengths in the first 7 generations of the human lung's bronchial tubes follow the Fibonacci scale. The diameters of the tubes are classical, that is Fibonacci, up to 10 generations. But after these initial steps, the scales change markedly.*

*Bruce West & Ary Goldberger have demonstrated that the lung incorporates a variety of fractal scales. This shifting of scales allows the lung greater efficiency. For example, after the 20th iteration, the branching takes place at a smaller scale of length but with the same windpipe diameter as the previous iteration. West & Goldberger say: "The final product, which we have dubbed 'Fractional/Fibonacci Lung Tree', provides a remarkable balance between physiological order and chaos".*

Briggs & Peat. Turbulent Mirror.



↑ Vascular structure of lungs.  
[www.microphotonics.com](http://www.microphotonics.com)



← Rubber cast of lungs.

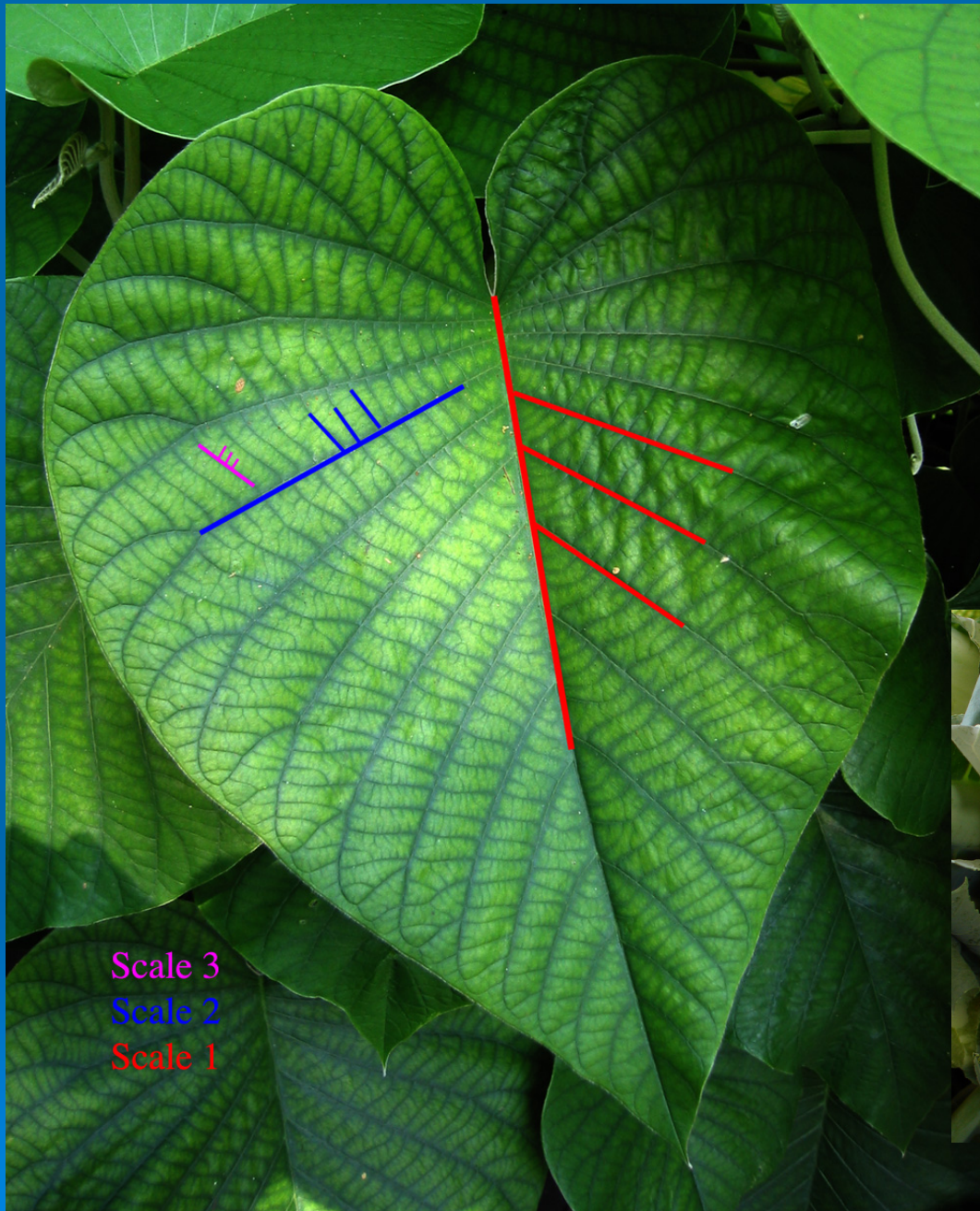


## SG203.1.13 Fractals in Nature

Many examples of fractals in plants & natural phenomena will be given and discussed in [◆SG205].

We are showing here this beautiful leaf structure from [Paul Bourke](#) (University of Western Australia)

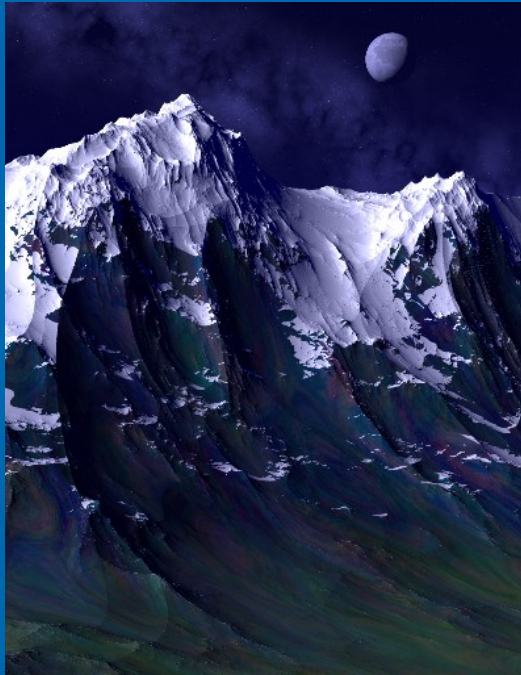
... ↓ and the star fractal of all gardens: the Romanesco broccoli.



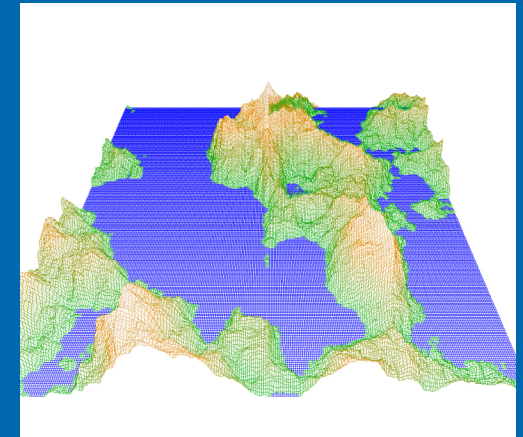
Scale 3  
Scale 2  
Scale 1



# SG203.1.14.1 Fractals Models (1) Landforms



These could be photographs, couldn't they?

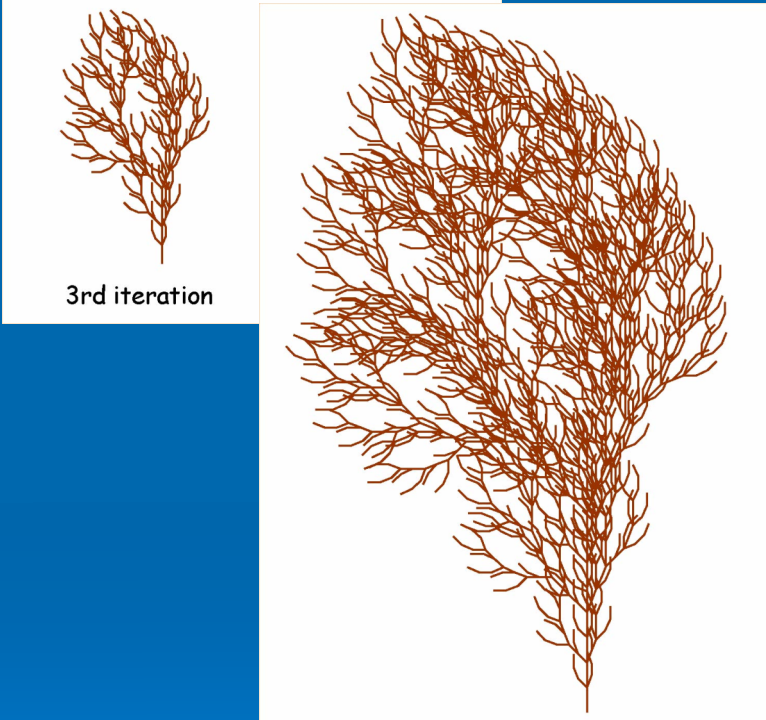
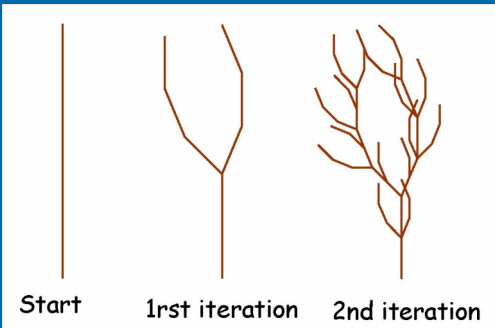


↑ Fractal landscapes are generated by "spatial subdivision".

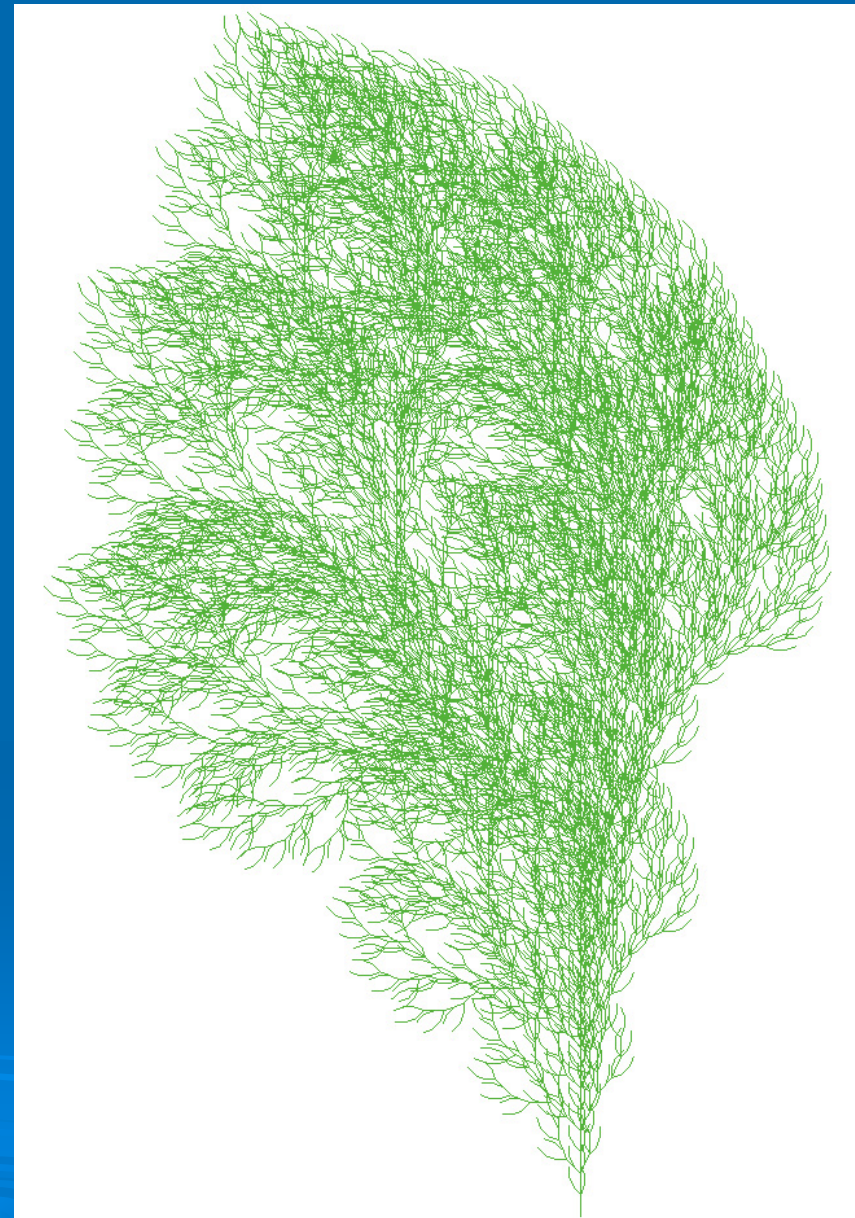


(All fractal models by Adam Brown. 2002-2003.)

## SG203.1.14.2 Fractals Models (2) Plants



↑ 4th iteration



(Images: Frank Timmes. [www.cococubed.com](http://www.cococubed.com))

5th iteration →

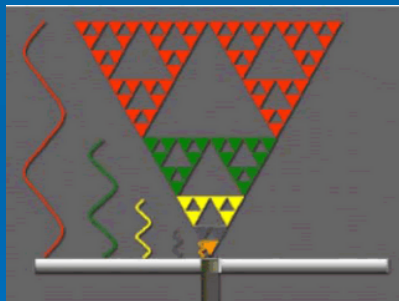
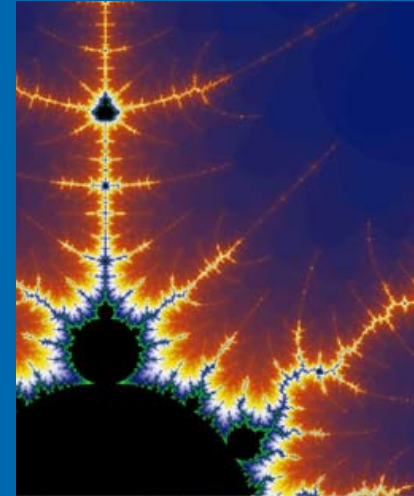
# SG203.1.15.1 Fractals in Technology (1) Antennas

A fractal antenna is an antenna that uses a fractal, self-similar design to maximize the length, or increase the perimeter (on inside sections or the outer structure), of material that can receive or transmit electromagnetic radiation within a given total surface area or volume.

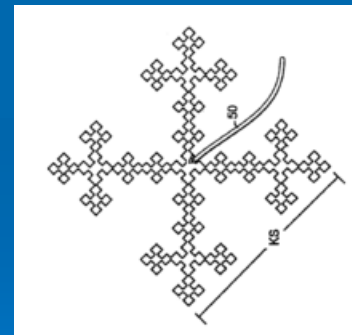
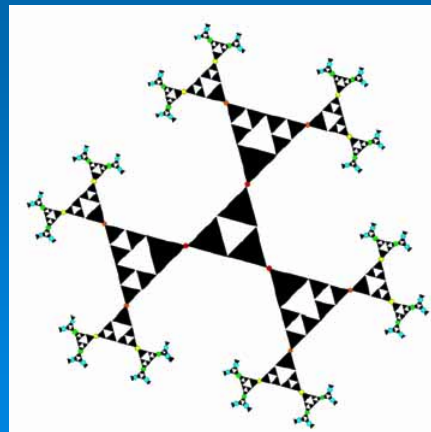
Such fractal antennas are also referred to as multilevel, and space filling curves, but the key aspect lies in their repetition of a motif over two or more scale sizes, or 'iterations'. For this reason, fractal antennas are very compact, are multiband or wideband, and have useful applications in cellular telephone and microwave communications.

A good example of a fractal antenna as a space filling curve is in the form of a shrunken fractal helix. Here, each line of copper is just small fraction of a wavelength.

A fractal antenna's response differs markedly from traditional antenna designs, in that it is capable of operating with good-to-excellent performance at many different frequencies simultaneously. Normally standard antennas have to be "cut" for the frequency for which they are to be used—and thus the standard antennas only work well at that frequency. This makes the fractal antenna an excellent design for wideband and multiband applications.



↑ In fractal antennas, the pattern is repeated several times at different scales.

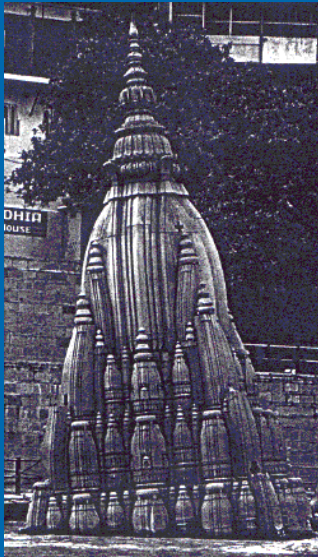


↑ Fractal antenna using a space-filling curve called a *Minkowski island*. (Wikipedia)

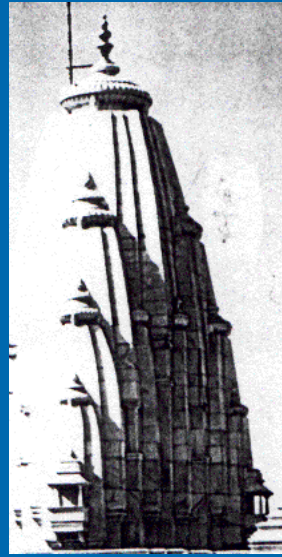


↑ Fractal antenna inlay on a military helmet. (Wikipedia)

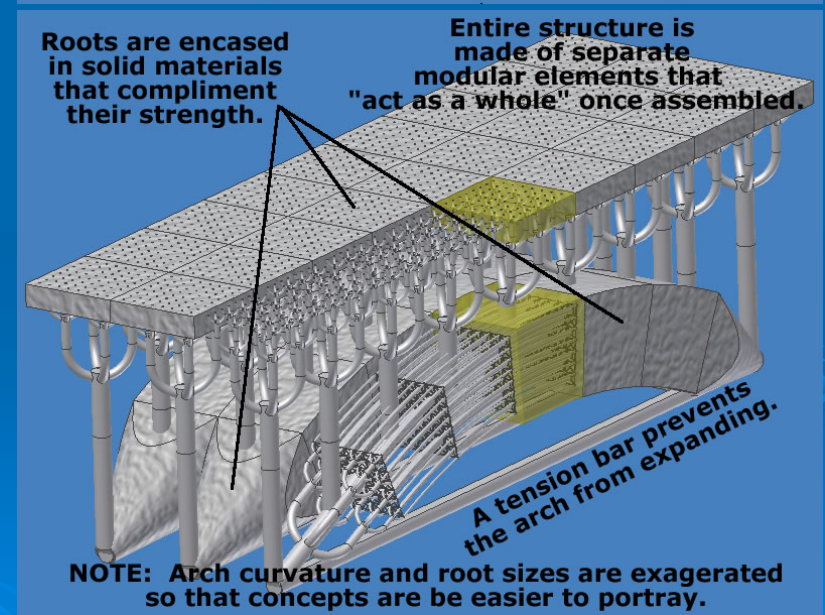
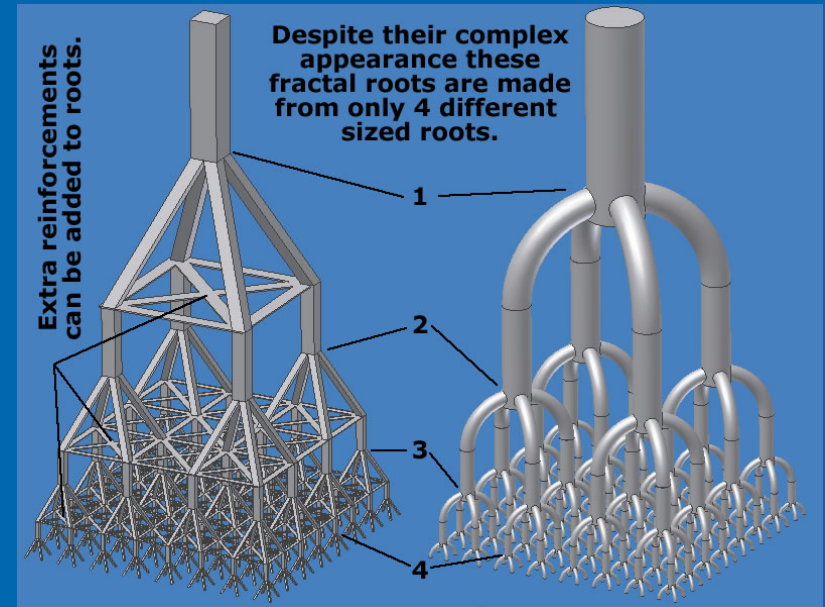
# SG203.1.15.2 Fractals in Technology (2) Architecture



↔  
Fractal  
Temples,  
India.



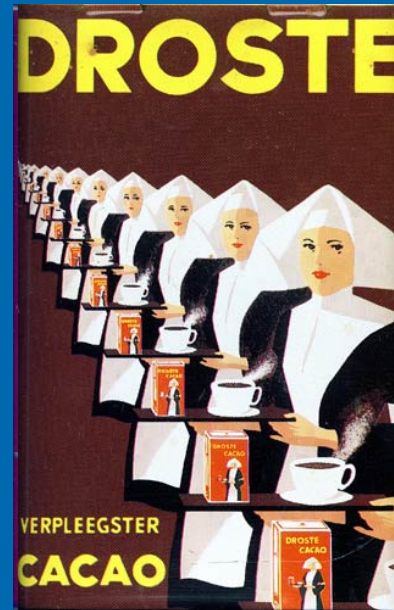
← Web image.  
(Unknown origin)



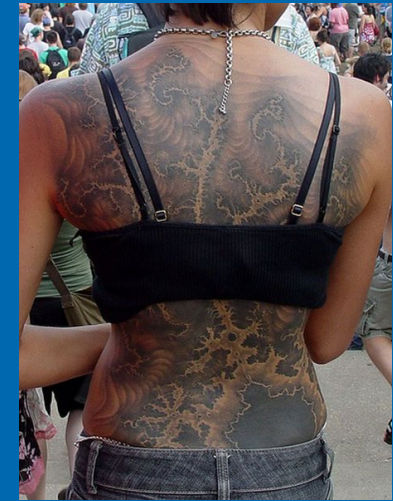
## SG203.1.16 Fractals in Cultures



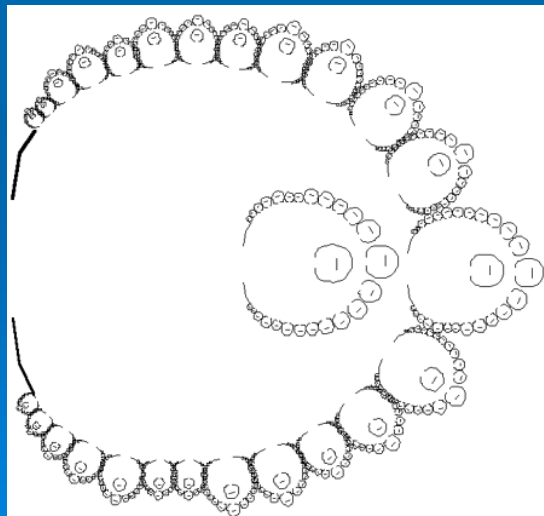
← Cathar mosaic in the form of the Sierpinsky gasket.



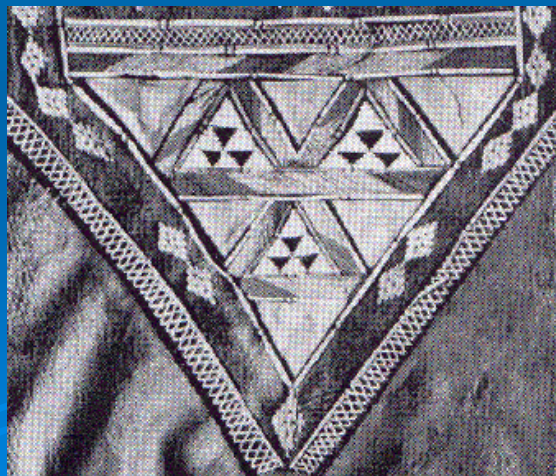
↑ Fractal design in advertising.



↑↓ [//talklikeaphysicist.com/](http://talklikeaphysicist.com/)



↑ Fractal plan of African village. (Ron Eglash)



↑ African design



↑ "The tattoo is in the shape of a Mandelbrot set, which to me, is symbolic of infinity, regeneration, life, knowledge & enlightenment."

## SG203.1.17.1 Fractals in Art (1)

The artistic developments made possible by the combination of fractal mathematics and the increasing power of personal computers are truly phenomenal... A new global artistic Renaissance has been spawned... Through the passionate direct exploration of exciting fractal visuals or the ubiquitous exposition to them, a whole generation has not only recovered faith in mathematics & science, at least in their New Paradigm forms, but has also regained a reverence for the mysterious and infinite beauty of the universe, realizing that humanity is only on the threshold of a magnificent adventure of expansion into larger dimensions of consciousness.

The concept of FRACTALITY has now pervaded the whole global culture and with it the overall feeling that a harmonious order is inherent in the wildest and most complex “chaotic” systems, including the most puzzling of them: human life. Yes, humanity is a fractal organism, let us celebrate it! Fractal science is pointing again to a true knowledge: the universal codes are revealed when we have the wisdom of Oneness, when we can inter-connect the parts with the whole, the zoomed-in details of individuality with the main cosmic highways of fractal harmonics, nature with technology, and love & imagination with a global culture of efficiently shared resources. “As Above, so Below”.

I will say this: the fractal art that has sprung up in the last 20 years is the current equivalent of the ancient *mandalas* or cosmological diagrams knowingly & purposely designed to create vibrational self-contained vehicles, frequency bubbles capable of re-activating inter-dimensional navigation for the subtle plasma-souls travelers that we are. When fractality is refined with the transparency power of the Golden Ratio, we are cosmic artists co-creating the universe!

We are short of space here to display even a fair sampling of the enormous production of fractal explorative art in physical or virtual existence. A simple Mandelbrot software program literally contains an infinity of artworks. And so the obsolete distinctions between art, science, life, story telling, yoga & technology... are rapidly disappearing as we make our lives the very canvas and the very art of being.

We suggest you go on a journey to explore the riches & myriads galleries of fractal art available through the web. More, we urge you to start playing with some simple fractal software. Google, borrow from friends, educate yourself... but play as a co-creator of your own Self... Remember to always see the fractal window of the hidden beauty that you only can see... and give it a form for the world to enjoy & celebrate!

*“Strange attractors and fractals evoke a deep recognition, something akin to the haunting recognition afforded by the convoluted and interwoven figures of Bronze Age Celtic art, the complex designs of a Shang ritual vessel, visual motifs from the West Coast American Indians, myths of mazes & labyrinths, the iterative language games of children or the chant patterns of so-called ‘primitive’ people”.*

*Briggs and Peat. Turbulent Mirror.*



↑ Internet fractal. Unknown origin. Thank you to this cosmic artist!

Note the mandala-like feeling of harmonic symmetry, the “golden eggs” (phi-based) and the perspectives opening into infinity.

## SG203.1.17.2 Fractals in Art (2)

As the honorary Chairman of the 2007 Fractal Art Contest, Benoit Mandelbrot made these comments:

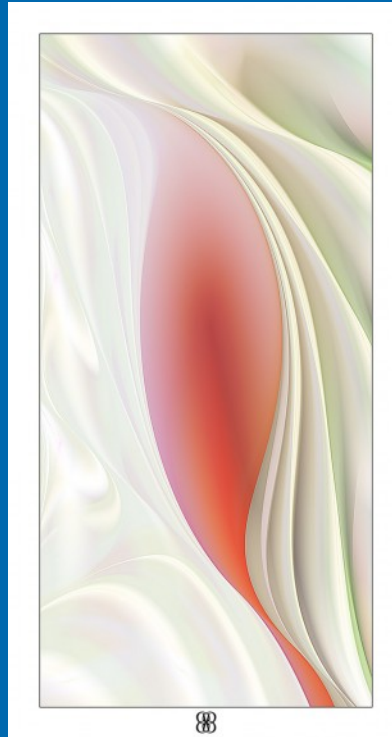
*"I am an inveterate optimist, but never expected to see a crowd standing in a long line to be allowed to admire mathematics in any of its forms. The organizers, sponsors, and producers of that exhibit have every reason to be proud. They deserve high praise and the waiting line deserves some thought and comment."*

*What distinguishes fractal geometry within mathematics is an exceptional and uncanny characteristic. Its first steps are not tedious, hard, and unrewarding, but playful and extraordinarily easy, and provide rich reward in terms of stunning graphics. To the mathematician, they bring a bounty of very difficult conjectures that no one can solve. To the artist, they provide backbones around which imagination can play at will. To everyone, a few steps in about any direction bring extraordinary pleasure. Nothing is more serious than play. Let's all play."*

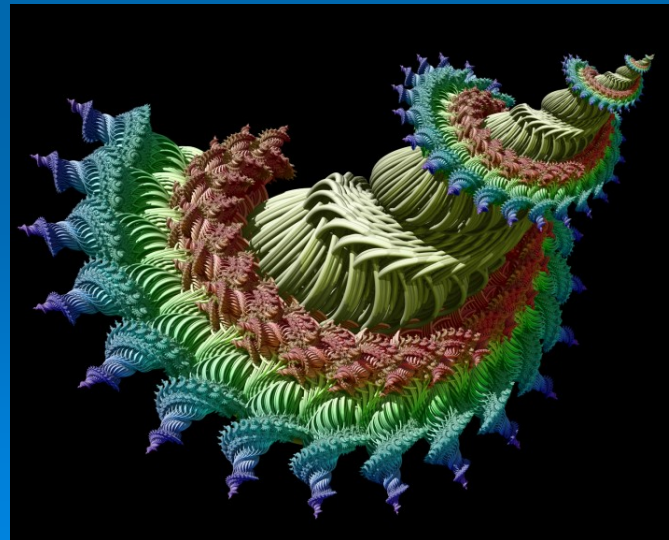


Go to the site of the Contest and look up some of the many hundred entries :

<http://www.fractalartcontests.com/2007/entries.php>



← Tender Moments.  
Lenora Clark.  
Winner 2007



← "Spiral Fantasy".  
Alfred Laing.  
Winner 2007



↑ Air. Ewa Stryza.  
Winner 2007



Upon completion of our short journey through Fractals, we offer the following comments:

- “Fractals”, a word coined & publicized only one generation ago, has now taken over the scientific world and the global public awareness. Fractals carry a revolutionary power that is just beginning to manifest itself.
- Fractals are now fulfilling the traditional function of Sacred Geometry: teaching the wisdom of “As Above, So Below”. Their inner and outer structures are identical : they can be turned inside out. And so can we. They teach us how to be completely transparent, non-resistive and how to allow for super-conductive transfer of energy/information.
- Do not let FRACTALITY be trivialized. Fractality does not belong to the old mindset of fractionation, fragmentation or explosion. It is reconnection, the gateway to Oneness. Fractality is IMPLOSION, centripetal embrace. It is SACRED.
- Fractals are mathematically & scientifically proving the essence of Sacred Geometry: the grand inter-connectivity of the universe and the illusion of “chaos” when looked past the appearances, and, shall we say, past the fears.
- Fractals describe the gap-less geometric nesting or embeddedness of energy/charge wave patterns weaving the fabric of the universe. This nesting is a zooming-in and out of Self or Soporuce.
- The self-similarity of fractals, when optimized by the Golden Ratio, is re-discovered, at an exponential rate, in all domains and on all scales of nature, as well as in biology, psychology and consciousness. “I am your own fractal”.
- PHI FRACTAL SYMMETRY is the key to go beyond the old beliefs in separation and bridge again between the structures of the very small and the structures of the very large, between physics & metaphysics, and between all beings: between you and me and also you and YOU.
- The GOLDEN ROAD of infinite and unified focus upon UNITY, as steps of quantum musical overtones, is an ineluctable scientific key as well as a spiritual key. No longer are we living in the war of duality. The Field of Life is unified again.
- Because a fractal creates compression and centeredness, and a Golden Mean Fractal creates infinite centered compression, we now have a mandala map for the new global awareness of a Golden Cosmos and for navigation of the multiples dimensions of manifested consciousness. The cosmic mandala is hubbed again. We can happily practice highway tunneling through space-time, dimensions & stargates and we CANNOT be lost. The universe is a fractal labyrinth: one way in and out.
- Phi fractal-based technologies are now reinventing life-enhancing, regenerative, bio-mimicry types of applications for sustaining a harmonious & happy life style all over the planet. Welcome to the inventive creativity of our cosmic children!
- PHI is the HARMONIC FRACTAL SCALING constant of the universe.
- Not only Phi is FRACTAL but it is DOUBLY FRACTAL as it embeds into itself by addition AND by multiplication. It is the only doubly, reciprocally symmetric universal code allowing for infinite cascading from Source to Manifestation and back, in the non-local, instantaneous way of enlightened consciousness.
- Manifesting the perfect positive interference of phase-conjugation, PHI fractality rises above mortality and all politics to provide the Golden Thread encoding all spirals of evolution, from sub-atomic vortices to DNA to galactic clusters, in a scale-invariant and dimension-invariant way. No longer are we body-bound.

The universe was already suspected to be based on SYMMETRIES.  
Now, it is described as functioning with FRACTAL SYMMETRY.  
Ultimately, it will be known to be based on PHI FRACTAL SYMMETRY.

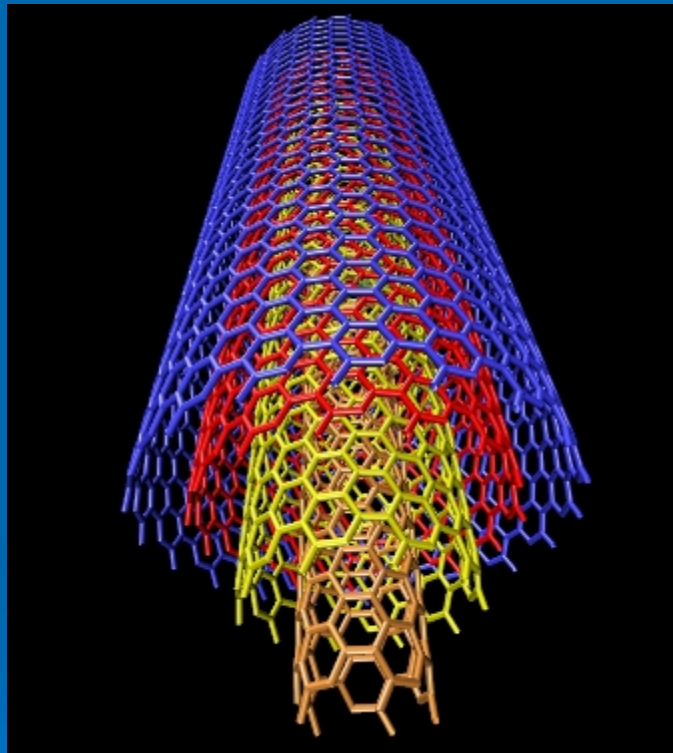
It’s going fast! We need this cosmic science to back up  
the New Paradigm higher evolutionary spiral of human consciousness.

## SG203.1.18 A Golden Fractal Universe.



[www.breakthru-technologies.com](http://www.breakthru-technologies.com)

## SG203.2 Chapter 2. New Penta-Symmetry Discoveries



Science is rapidly uncovering, in all domains of research and on all scales, the presence of the Phi/Fibonacci function as the most coherent, simple and efficient way of nature to self-organize. We have already had a brief introduction with the fractal-Phi connection.

In this chapter, we will focus on the specific re-discoveries of five-fold symmetries or Penta-Symmetries: Penrose tiling, Quasicrystals, water micro-clustering and the family of molecules called “Fullerenes”.

Five-fold symmetries are based, in 2D, on the Pentagon/pentagram geometries and, in 3D, on the dual Platonic Solids: the dodecahedron & icosahedron.

# SG203.2.1.1 Penrose Tiling (1)

## What is Tiling?

Tilings have been an on-going expression of human cultures for millenia. They are also known as *tesselations* (Latin *tessella* = *element of mosaic*) or pavings.

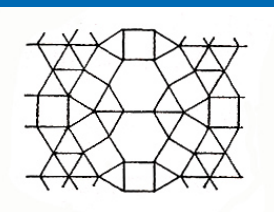
The geometries of tiling played a central role in the art, science & culture of Islam, from the citadel-palace of Alhambra in Garnada, Spain to the Great Mosque of Isfahan in Iran. The 10th century mathematician Abu'l-Wafa, Albrecht Dürer (1471-1528) and Johannes Kepler (1571-1630) were all tiling aficionados specially interested in 5-fold symmetry patterns. In the 20th century, Dutch graphic artist M. C. Escher (1898-1972) created exquisitely precise examples of tiling and in the 1970's, Roger Penrose publicized his "Penrose Tiles".

## Semi-Regular Tilings

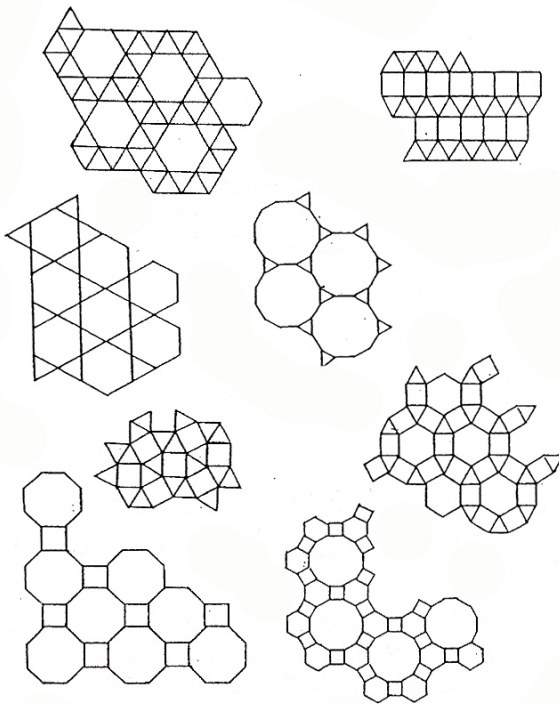
In semi-regular tilings, more than one kind of polygon surrounds each vertex. There are 8 kinds of basic semi-regular tilings (see patterns of left).

## Transformed Regular Tilings

By applying transformations (vertex motion, distortion, augmentation/deletion, parquet deformation...), many new tilings can be generated. Below a 7-uniform tiling.



## 3 Regular Tilings



## 8 Semi-Regular Tilings

## Regular Tilings

In regular tilings, the same number & order of a single kind of polygon surrounds each vertex. There are 3 regular tilings:

- with triangles: 6 per vertex [3,6]
- with squares: 4 per vertex [4,4]
- with hexagons: 3 per vertex [6,3]

Note: 3-fold symmetries (triangles with 120° rotation), 4-fold (squares with 90° rotation) and 6-fold (hexagons with 60° rotation) were the only "permissible" tiling symmetries... until the advent of Penrose Tiling and Quasi-Crystals.

Regular & semi-regular tilings are **uniform** and **periodic**, i.e. the entire configuration can be translated (without rotation) to a new position which reproduces the original tiling. Pentagonal symmetries rediscovered by Penrose and al. are **non-periodic**, i.e. they require translations and rotations.

## SG203.2.1.2 Penrose Tiling (2)

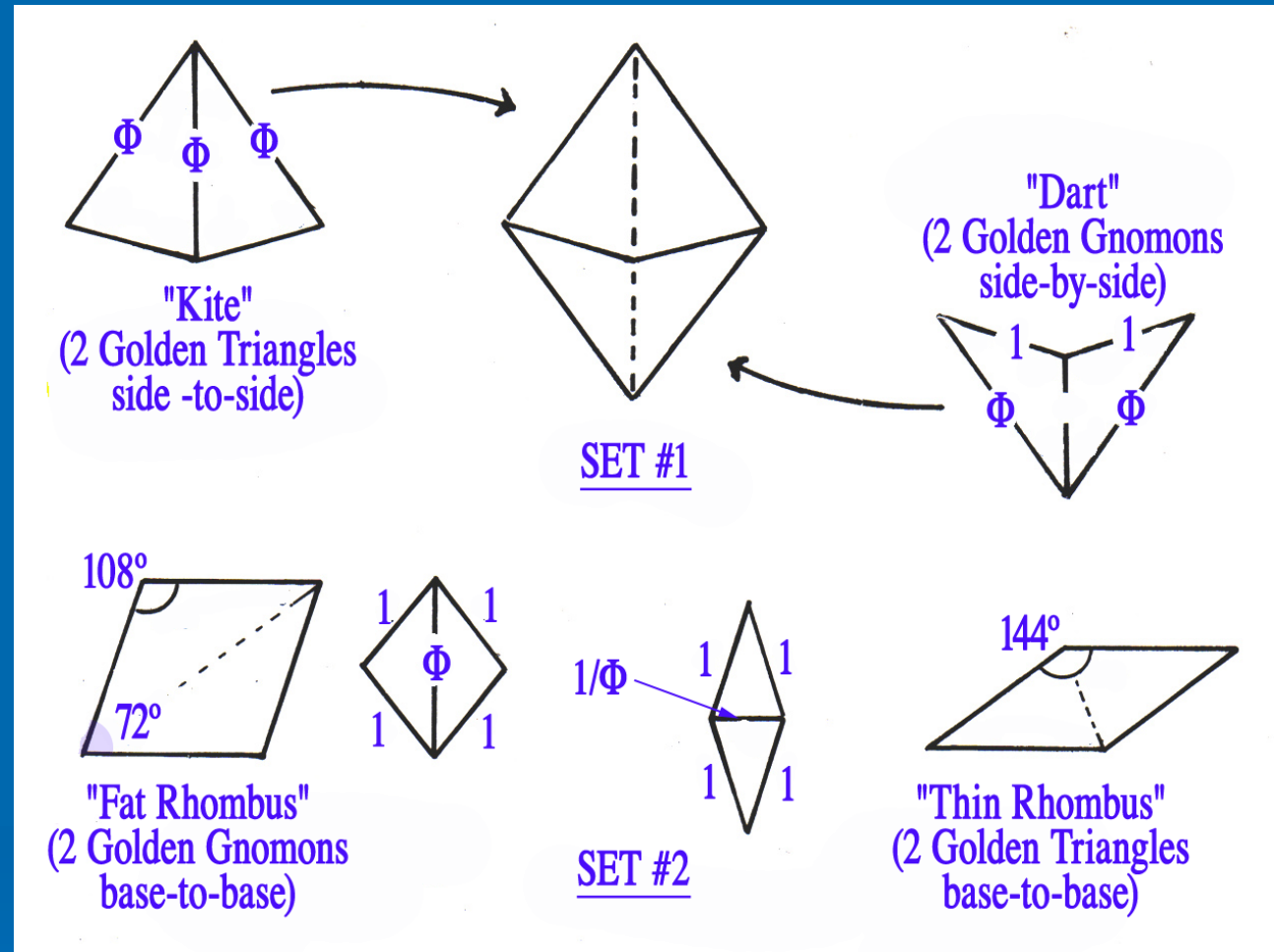
Although regular pentagons cannot tile the plane, Roger Penrose, doing “recreational mathematics” with tiling problems, rediscovered in 1974 two sets of tiles that nicely do the job.

They are called the “*dart*” and the “*kite*” (set #1) and the “*fat rhombus*” and “*thin rhombus*” (set #2) or colloquially the “*two rhombs*”.

If certain matching rules are followed, the two sets tile the plane non-periodically.



↑ Roger Penrose standing on a floor made with penrose tiling. (Texas A&M University). Wikipedia.



A unique “golden” property of the Penrose Tiling is:

$$N_{\text{kites}} / N_{\text{darts}} = \Phi \quad \text{and} \quad N_{\text{fat}} / N_{\text{thin}} = \Phi$$

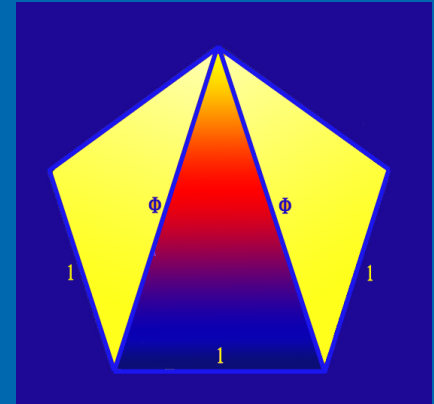
- The number of “*kites*” is 1.618 (=  $\Phi$ ) times the number of “*darts*”.
- The number of “*fat rhombi*” is 1.618 (=  $\Phi$ ) times the number of “*thin rhombi*”.

# SG203.2.2 The Two Penta-Modules

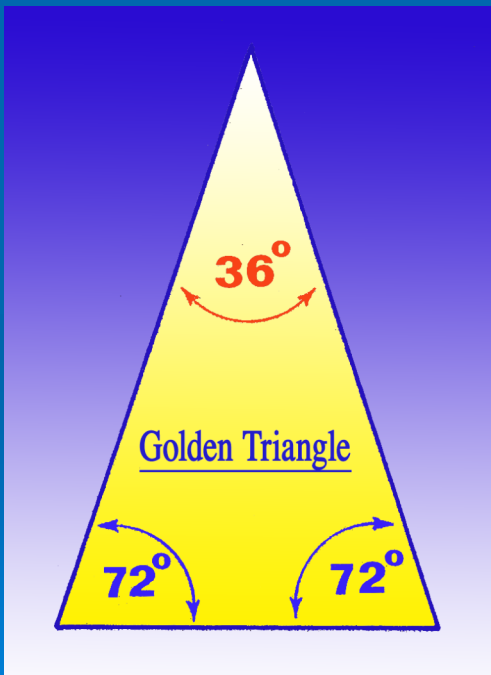
The two sets of tiles “invented” by Roger Penrose are in fact configurations of the Pentagon’s primary Penta-Modules: the Golden Triangle and the Golden Gnomon which have been known to Sacred Geometry for ages. This is a beautiful example of contemporary scientists treading on the footprints of ancient knowledge and naively believing they are the first ones on the path. (♦ SG104.4.2.2 for the 13th century Islamic manuscript featuring "penrose tiling" as well as ♦SG207 for exquisite examples of penta-symmetric mosaic in Islamic architecture).

The two Penta-Modules are the two basic triangular components of the Pentagon/Pentagram:  
The Golden Triangle & The Golden Gnomon

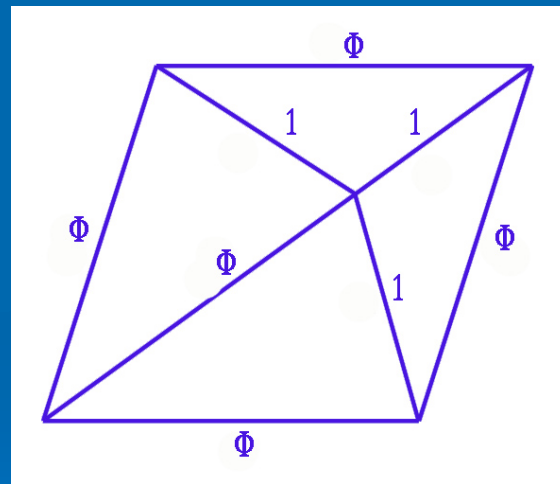
The two Penta-Modules are all you need to build & design Pentagons & Pentagrams, as they are complementary and can be divided (or expanded) into smaller (or larger) proportional triangles, up and down the scale of magnitudes. They allow for the pentagonal tiling of the 2D plane (Penrose Tiling) and for beautiful puzzle games.



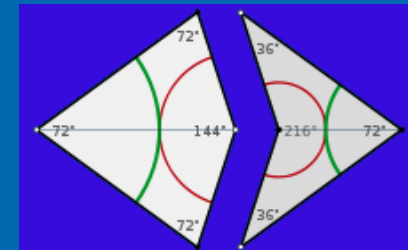
↑ One center Golden Triangle with two lateral Golden Gnomons form a pentagon.



↑ The isosceles Golden Triangle with base = unit 1 and sides = Phi.



↑ Two Golden Triangles side-to-side form a “dart” and fit into two Golden Gnomons base-to-base. The overall shape is a “fat” rhombus, one of the two elements of Penrose’s set #2.

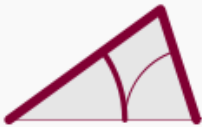




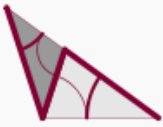





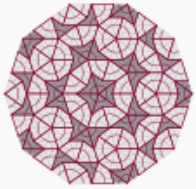



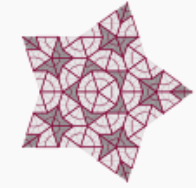


↑ Inner angles of the “kite” and “dart”.

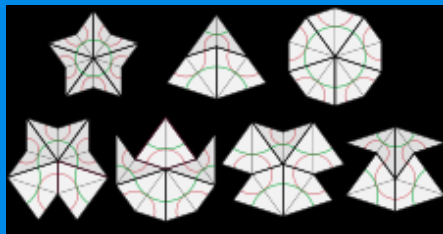


↑ The isosceles Golden Gnomon with base = Phi and sides = unit 1.

## SG203.2.3.1 Penrose Tiles in 2D (1)

Name	Generation 0 (or axiom)	Generation 1	Generation 2	Generation 3
Kite (half)				
Dart (half)				
Sun				
Star				

↑ Successive generations of “deflation”. Wikipedia.



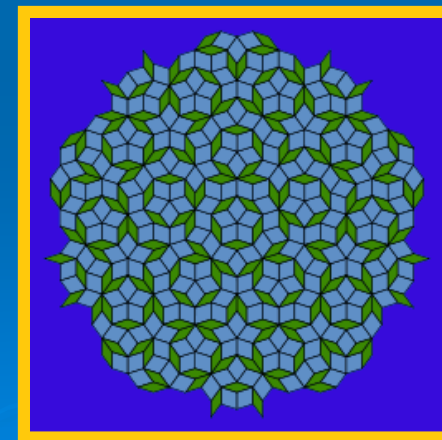
← The 7 possible vertex figures with the kite and the dart.

We can see here the endless, infinite possibilities of fractal self-similarity or recursiveness inherent in the sacred geometry of the pentagon/pentagon.

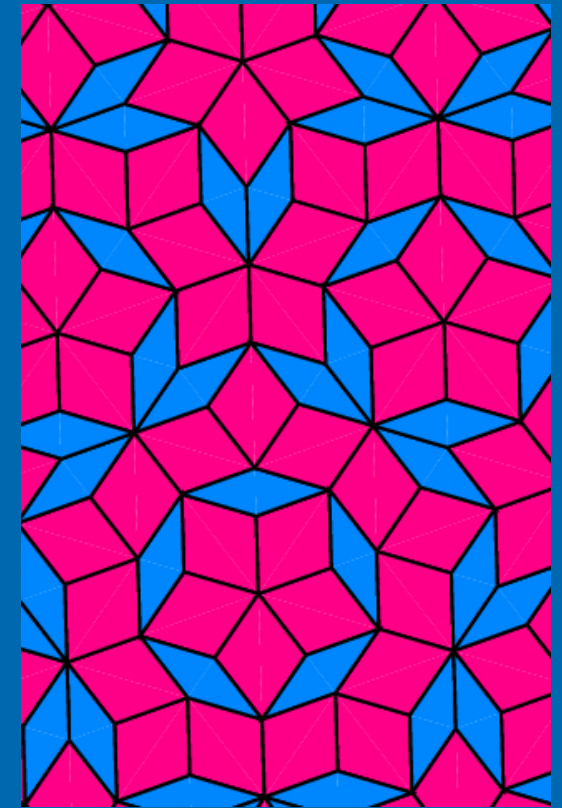
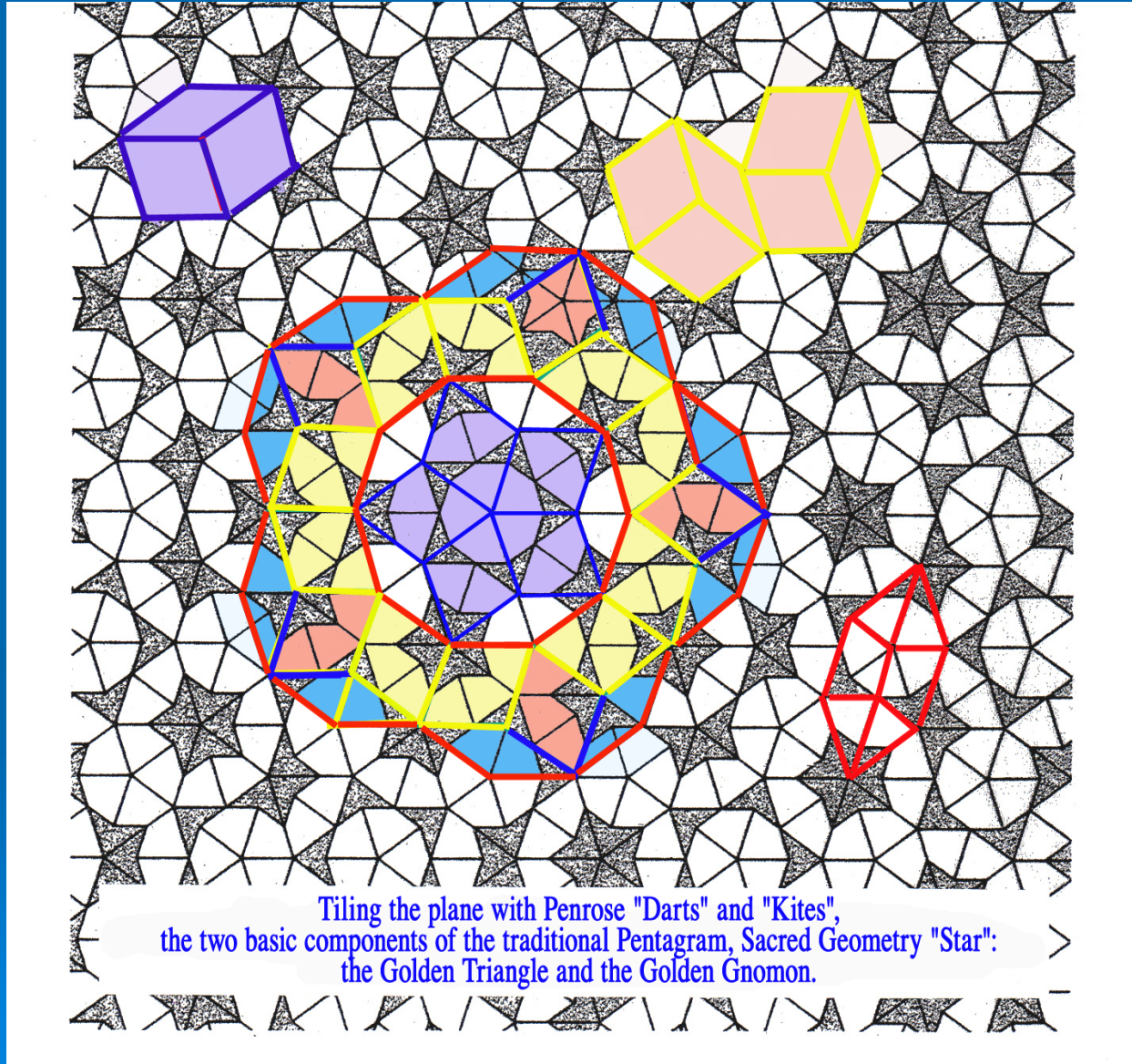
Science calls these operations “deflations”. We prefer to say: harmonic nesting cascades into the Above and into the Below - ad infinitum.

This is why nature has chosen the Golden Ratio as the organizing function of the universe.

It provides a scale-invariant, safe highway of information transfer.



## SG203.2.3.2 Penrose Tiling in 2D (2)



[www.physicstogo.org](http://www.physicstogo.org)  
Credit: Ianiv Schweber.

← Penrose Tiling  
with Darts & Kites.

Notice the overlaid 3D  
macro-patterns  
matching the golden  
geometry proportions.

## SG203.2.4.1 Penrose Tiling in 3D (1)



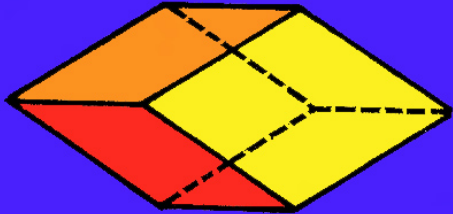
← A colored version of the 3D macro-pattern emerging from “Penrose Tiling”.

These pentagonal geometries are found in quasi-crystals.



## SG203.2.4.2 Penrose 3D (2) Golden Rhombohedra

In 1976, Robert Ammann extended Penrose's work into 3D and (re)discovered two golden volumes that "tile in 3D", and fill up space without any gap. They are called the Golden Rhombohedra. Their faces are the same as the 2D rhombi of Penrose set #2.



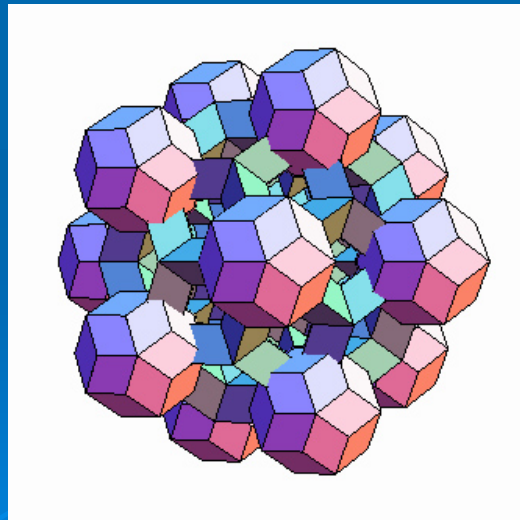
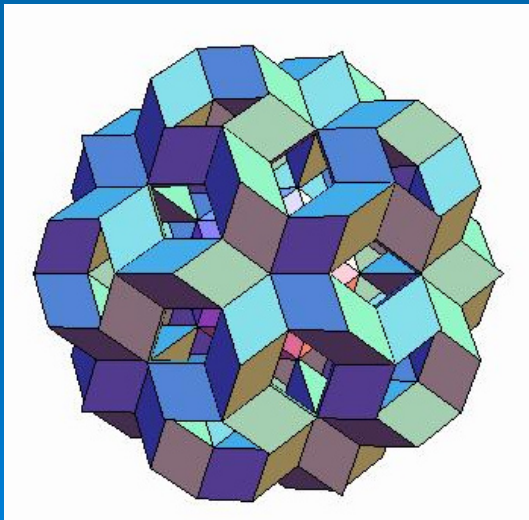
The two golden  
rhombohedra



Google "Penrose Tiling" for many examples & applications.



[www.wecanchangetheworld.wordpress.com](http://www.wecanchangetheworld.wordpress.com)



↑ Two forms of rhombic golden polyhedra based on the Penta-Modules.

"Penrose tiling" in 2D or 3D has become very popular in art, architecture and interior design. The Golden ratio is re-entering the global culture.



[www.eschertile.com](http://www.eschertile.com)

## SG203.2.4.3 Penrose 3D (3) Golden Rhomb

A DIY (Do It Yourself) net or fold-out pattern for the Golden Rhombohedron with the following properties:

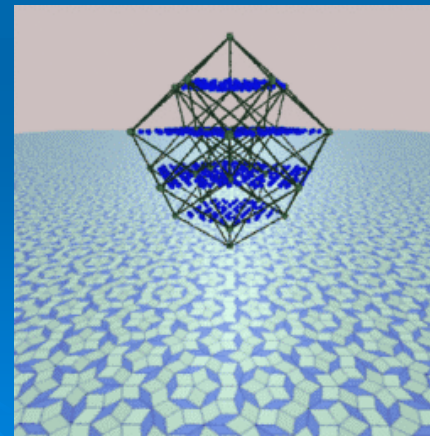
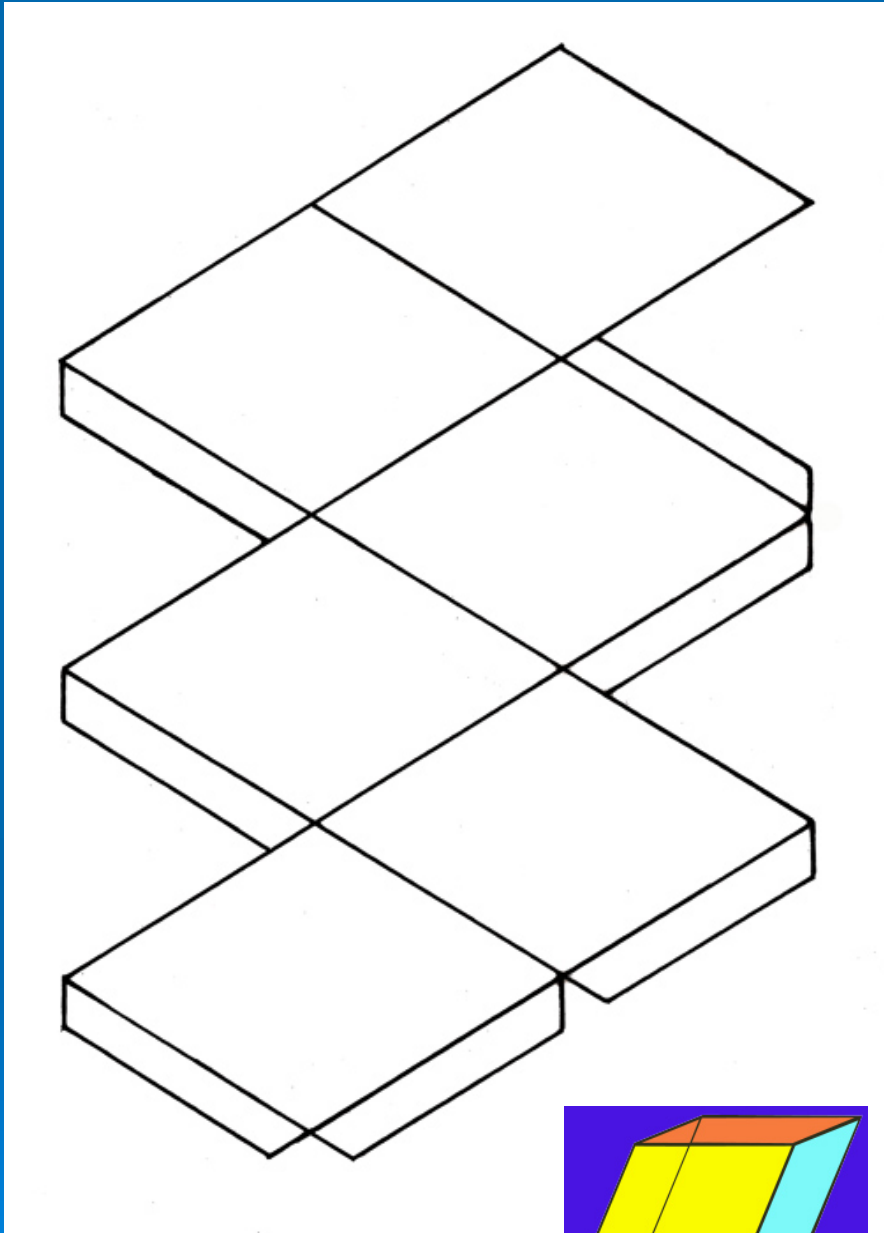
Long axis =  $\Phi$   
Short axis = 1

The 6 faces are identical to the “fat rhombus” Penrose tile made of two Golden Gnomons base-to-base.

### Instructions:

- Print out the net on stock paper (colored).
- Cut the sides of all the flaps.
- Put together, using tape if needed.
- Play with your rhomb and enjoy its Phi harmonic proportions & beauty. You are holding one of the interrelated shapes encoded in the universal law of Phi harmonic scaling.

Note that this 3D Golden Rhomb will reappear in the Quasi-crystals.



← QuasiTiler logo.

*QuasiTiler* is a software program allowing you to design your own quasi-crystal bathroom tiles and more...

[www.geom.uiuc.edu/apps/quasitiler/about.html](http://www.geom.uiuc.edu/apps/quasitiler/about.html)

## SG203.2.4.4 Penrose 3D (4) Penrose Toilet Paper

The story of the Penrose Toilet paper is an instructive tale about intellectual "property".

**The Case.** Sir Roger Penrose, a British Knight holding a chair of mathematics at Oxford University and credited with the discovery of "Penrose Tiling", came unexpectedly face to face with his own copyrighted penta-symmetric pattern in Kleenex quilted toilet paper. Mrs. Penrose first recognized the pattern on the "loo paper" in a store and, returning from shopping, brought it to her husband's attention. "He wasn't pleased," said Penrose's lawyer, as quoted in *The Wall Street Journal*. So Penrose and Pentaplex Ltd., the Yorkshire, England, company that owns the licensing rights to the Penrose pattern, filed a lawsuit against Kimberly-Clark, the British division of the Dallas-based Kleenex corporate empire, for breach of copyright. Penrose charges that Kimberly-Clark unlawfully appropriated an important geometric pattern of his creation and imprinted it on rolls of Kleenex Quilted bathroom tissue.

*"So often we read of very large companies riding rough-shod over small businesses or individuals,"* said David Bradley, director of Pentaplex. *"But when it comes to the population of Great Britain being invited by a multi-national to wipe their bottoms on what appears to be the work of a Knight of the Realm without his permission, then a last stand must be made."*

The quilted British tissue is embossed with the pentagonal pattern to make it "thicker and softer," according to Kimberly-Clark literature. Penrose's writ argues that making the tissue fluffier enables manufacturers to reduce the amount of paper used on each roll. "But, if the pattern repeats itself, the tissue would likely bunch up, looking unattractive," the suit claims. "That can be corrected using a Penrose-type pattern that lets the paper sit evenly on the roll."

Penrose is among the luminaries of modern science, known for researching the origins of the universe in association with Stephen Hawking, and for analyzing the nature of consciousness in such works as "The Emperor's New Mind." Penrose has had a longstanding interest in the recreational mathematics of tessellation geometry and thus proved that nowadays, *recreational* maths can not only become serious but can also be breakthrough science.

**The Outcome.** At first, talks between Kimberly-Clark and Pentaplex about a legal settlement apparently broke down, with Kimberly-Clark denying any violation of copyright laws. Nevertheless, the company has announced that Kleenex Quilted will be "re-launched" with a new look - certainly a less "mathematically significant" one. Then, the news were that Penrose and Pentaplex have resolved their differences with the British company who is the current holder of the Kleenex toilet tissue: both sides have now developed a working relationship, described as "cordial and constructive." No doubt, good business in perspective and maybe the Penrose Toilet Paper becoming a brand name.

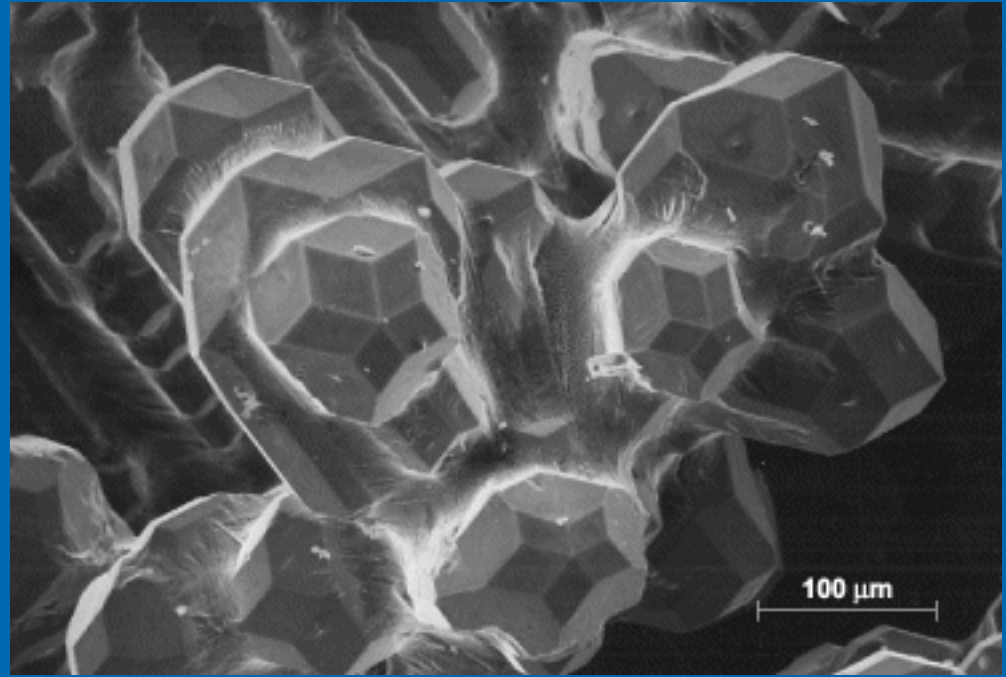
### Comments.

- Let us remember that the knowledge of the Penta-Modules and pent-symmetry has been around for many centuries, lately (13th century) in the Islamic culture. Closer, Johannes Kepler and Albrecht Dürer played with it. Penrose's discovery is at most a RE-discovery. Who owns what exactly?
- In 1995, computer expert Roger Schlafly received a patent on two extremely large prime numbers. Among the chorus of protesters against the idea of someone claiming ownership to a number: Sir Roger Penrose. *"It's absurd,"* Penrose said of the Schlafly case. *"Mathematics is out there for everybody."*
- A journalist commented: *"There is a lesson here for anyone who stumbles across an undiscovered part of the universe and tries to claim it as his own".*

## SG203.2.5.1 Quasicrystals - The Story (1)

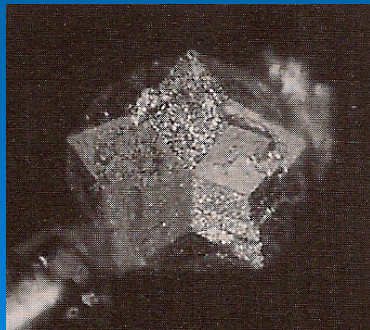
November 12th, 1984: the Physical Review Newsletter published a ground-breaking paper by Dan Schachtman and others. The paper announced the discovery of a “*quasicrystal*” with a 5-fold symmetry.

This was an “*impossible*” crystal. According to the standards of crystallography, there were only two types of crystals: either amorphous (like glass) or highly ordered & symmetric (like table salt). Furthermore, periodic crystals only exhibit symmetries of 2, 3, 4 and 6: a 5-fold (icosahedral) symmetry was not “*allowed*”.



↑ Alloy of aluminum, copper and iron showing icosahedral symmetry.

Well, the International Union of Crystallography has since been obliged to redefine the term “crystal” as the newly discovered quasicrystals indeed existed, although “forbidden”, and proved to share Sacred Geometry PHI properties with the Penrose Tiles.



Quasi-crystal Al-Li-Cu

Quasicrystals are somewhere in between amorphous & periodic structures:  
they exhibit fold symmetry and long-range order (periodicity)  
but lack absolute translational symmetry  
(a shifted copy will never exactly match with its original).

Thus Quasicrystals display order within disorder: they bridge order & chaos.  
Remember the Goddess Harmony in the Greek Mythology? [◆SG102]  
She was born from Ares, the God of War and Aphrodite, the Goddess of Love.

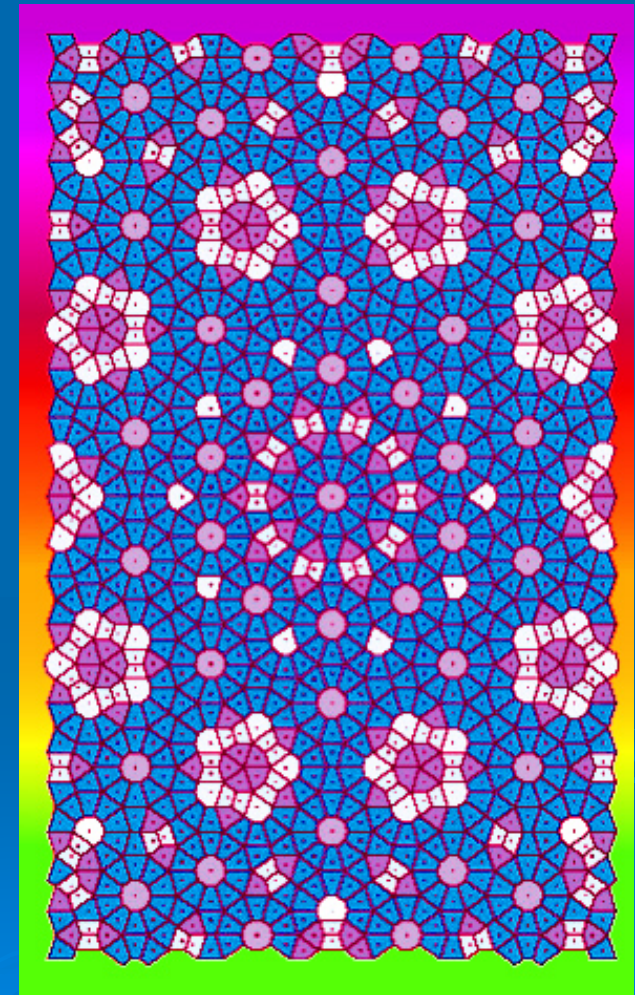
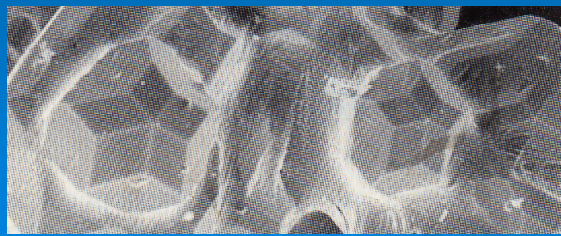
## SG203.2.5.2 Quasicrystals - The Story (2)

Since the original discovery of Shechtman hundreds of quasicrystals have been reported and confirmed. Undoubtedly, the quasicrystals are no longer a unique form of solid: they exist universally in many metallic alloys and some polymers. Quasicrystals are found most often in aluminum alloys (Al-Li-Cu, Al-Mn-Si, Al-Ni-Co, Al-Pd-Mn, Al-Cu-Fe, Al-Cu-V, etc.), but numerous other compositions are also known (Cd-Yb, Ti-Zr-Ni, Zn-Mg-Ho, Zn-Mg-Sc, In-Ag-Yb, Pd-U-Si)

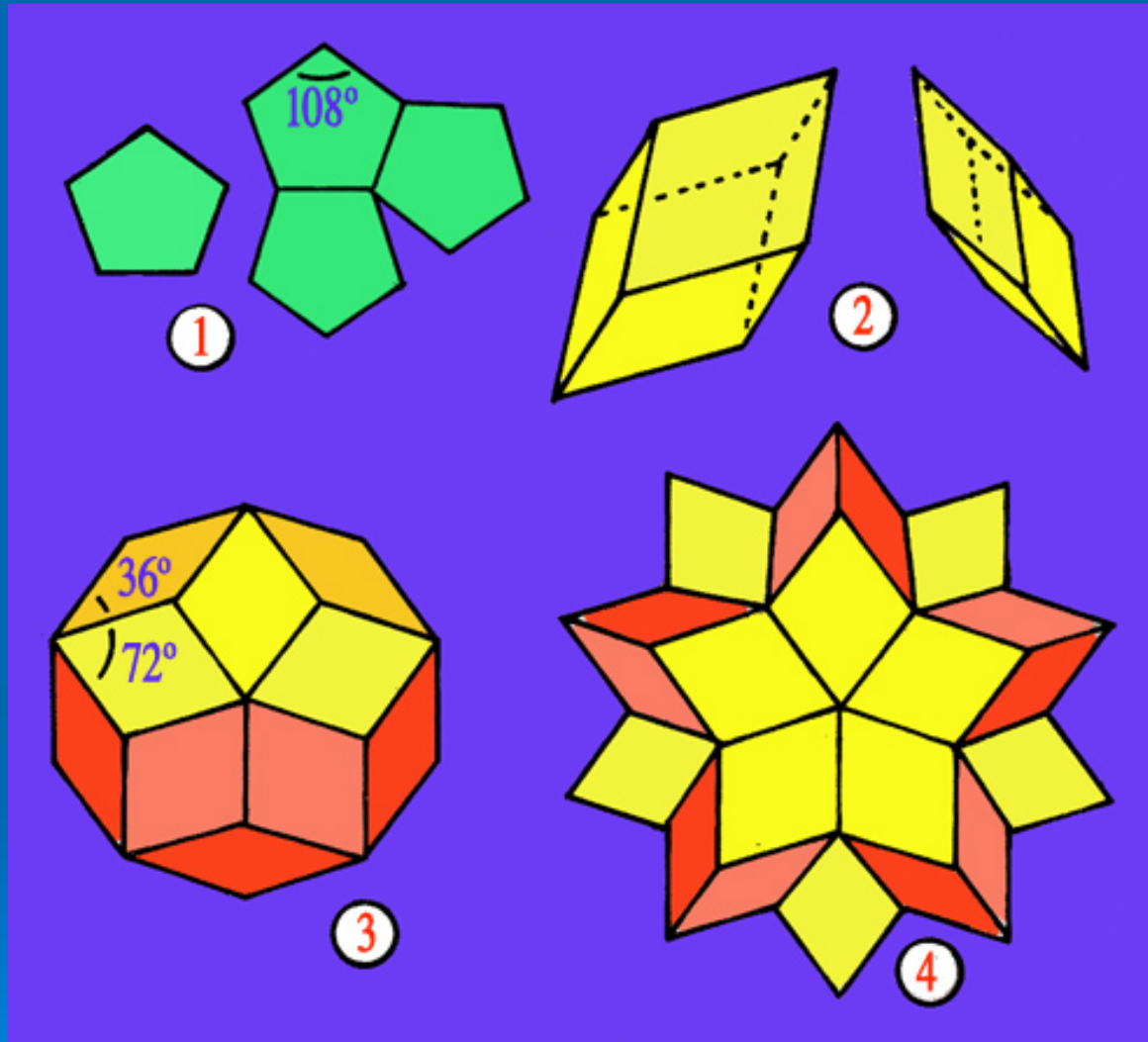
Except for the Al-Li-Cu system, all the stable quasicrystals are almost free of defects and disorder, as evidenced by x-ray and electron diffraction revealing peak widths as sharp as those of perfect crystals such as Si (silicon) used for computer chips. Diffraction patterns exhibit 5-fold, 3-fold and 2-fold symmetries, and reflections are arranged quasi-periodically in three dimensions.

*“The origin of the stabilization mechanism is different for the stable and meta-stable quasicrystals. Nevertheless, there is a common feature observed in most quasicrystal-forming liquid alloys or their undercooled liquids: a local icosahedral order. The icosahedral order is in equilibrium in the liquid state for the stable quasi-crystals, whereas the icosahedral order prevails in the undercooled liquid state for the meta-stable quasicrystals.”* (Wikipedia: Quasi-crystals)

Thus, not only the long-standing topic of “recreational mathematics”, i.e. “tiling with 5-fold symmetry” has found nature-made and man-made counterparts, but models of quasiperiodic tiling have now entered highly theoretical fields (particle physics, mathematical topology and astro-cosmology) as well as very practical fields of material sciences & applied technology.

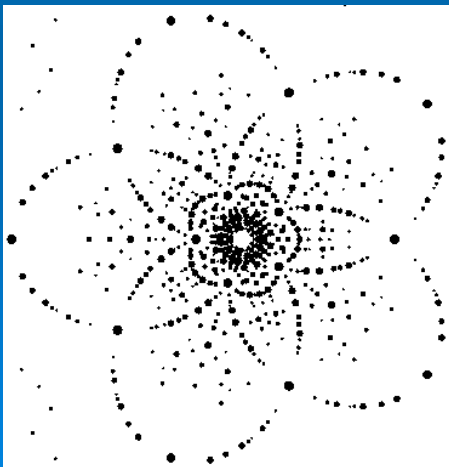
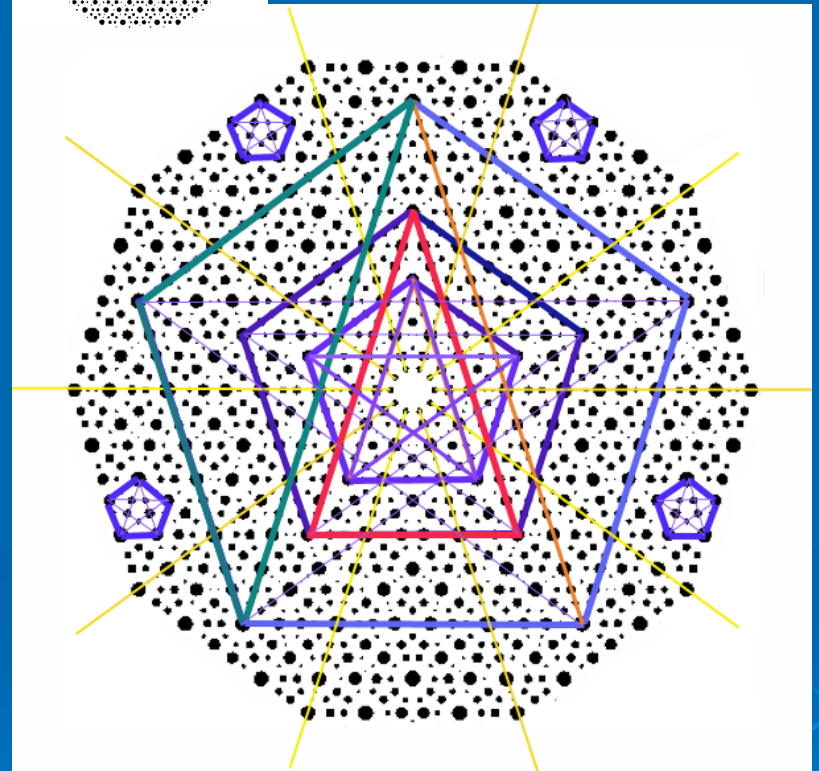
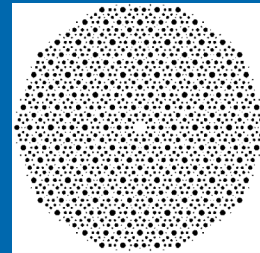
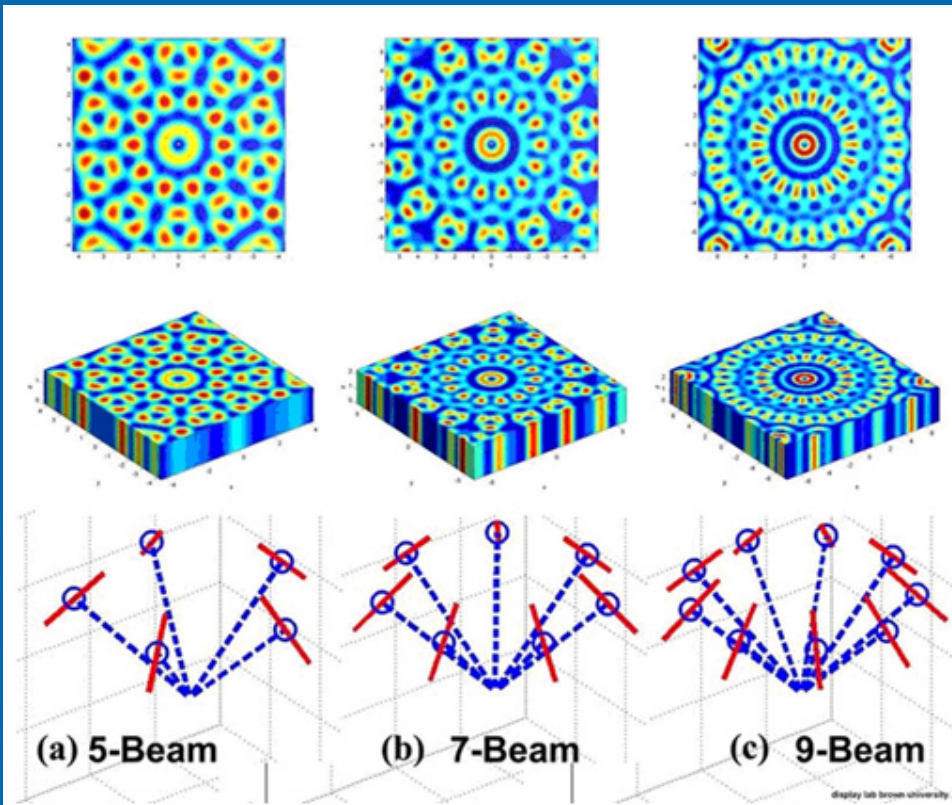


## SG203.2.5.3 Quasicrystals - The Story (3)

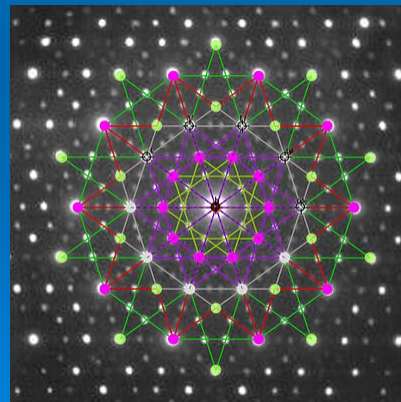


1. Tiling the plane is impossible with regular crystals. The mathematical reason is simple: the summit angle of the pentagon is not a sub-multiple of  $2\pi$ .
2. The two basic rhombohedra allow tiling in 3D.
3. Combined, they form the “yang” or *convex triacontahedron* (30 faces).
4. A “yin” or *concave polyhedron* (60 faces) fits between the triacontahedra. This arrangement forms a quasicrystal.

# SG203.2.6 Quasicrystals - Diffusion Patterns



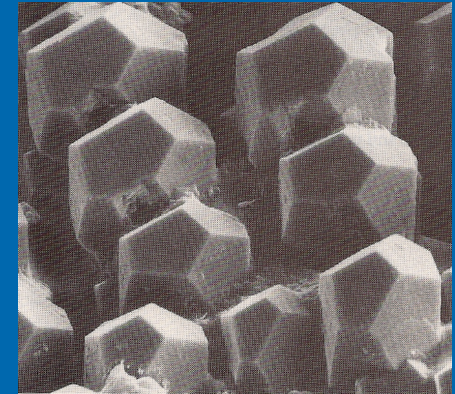
Laue Diffusion Pattern



Electron diffraction pattern of an icosahedral Zn-Mg-Ho QuasiCrystal  
(en.wikiversity.org)

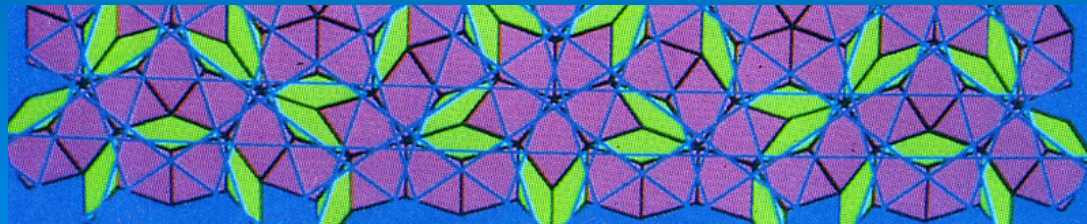
↑ Some of the 5-fold symmetry geometric patterns in a quasi-crystal.  
In blue: pentagons & pentagrams.  
In red: Golden Triangle.  
In Green: Golden Gnomon.

## SG203.2.7.1 Quasicrystals - R & D (1)



Quasi-crystal Al-Cu-Ru

- Quasicrystals have been called “*a new state of matter*”, as they share some properties of regular crystals as well as properties of non-crystalline (amorphous) matter. They present novel atomic arrangements as well as unique surface properties.
- The pentagonal symmetries of quasicrystals have opened up a vast range of new research and understanding about ordered non-periodicity. Crystallography got projected into quantum physics and the geometry of solids has now welcomed (back) the penta-symmetrical rotation of the atomic patterns forming these quasi-crystalline alloys. Let us be reminded that penta-symmetry is based on the Golden Ratio as the key to Harmonic Vortex energy transmission, as enacted in the DNA.
- Technologists have taken advantage of the special properties of quasicrystals for industrial applications. Quasicrystals are now a high focus of interest because of the interesting surface properties they exhibit, such as low friction, low adhesion, corrosion resistance, catalytic activity and, most important for technology, STABILITY. For instance, the hardness of quasicrystals comes from their metallic bonding: unlike metals with translational periodicity, quasicrystals will not shear along plane boundaries. Razor blades and surgical instruments are now produced with quasicrystals alloys.
- Most stable quasicrystalline phases have icosahedral symmetry and exhibit 3 high-symmetry surfaces: fivefold, threefold and twofold. The study of the relative stability of the 3 hi-symmetry surfaces shows that, on the 2-fold symmetry surfaces, no facets are observed. This suggests that the 2-fold surface is as stable as the other high symmetry surfaces.
- Quasicrystals have also been associated with hydrogen storage: a storage capability of up to two atoms of H per quasicrystal atom has been found.
- Roger Penrose and other researchers see in quasicrystals a way to link the classical physics of the human scale to the quantum physics of the atomic scale. John W. Cahn explains that quasicrystals imply a higher-dimensional hyperspace, as they seem to act as a bridge by partaking of two spaces simultaneously.
- Indeed, whereas natural crystals are formed atom by atom without reference to the remote position of other atoms, in the case of quasicrystals the non-periodicity of their pentagonal tiling presupposes that the atoms need to know the position of remote “atomic tiles” to know the type and 3D position of new tiles... what does that say about the oneness of humanity’s global consciousness? So we have here the non-local effects encountered in quantum physics.  
Johannes Kepler who was playing with 5-fold tiling patterns as well cosmological harmony would be (is?) quite pleased!

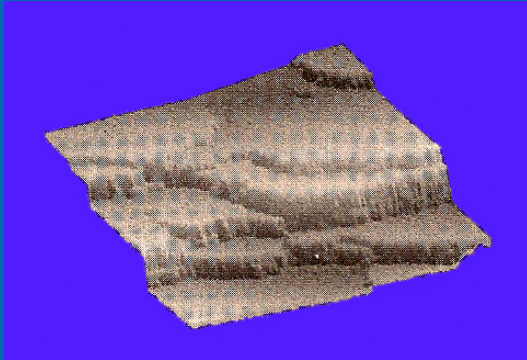




## SG203.2.7.2

# Quasicrystals - R & D (2)

Another aspect of the intimate link between quasicrystals and the Golden Ratio PHI was uncovered recently and described by Mario Livio, in his information-filled book: *The Golden Ratio, The Story of Phi, the World's Most Astonishing Number* (2002).

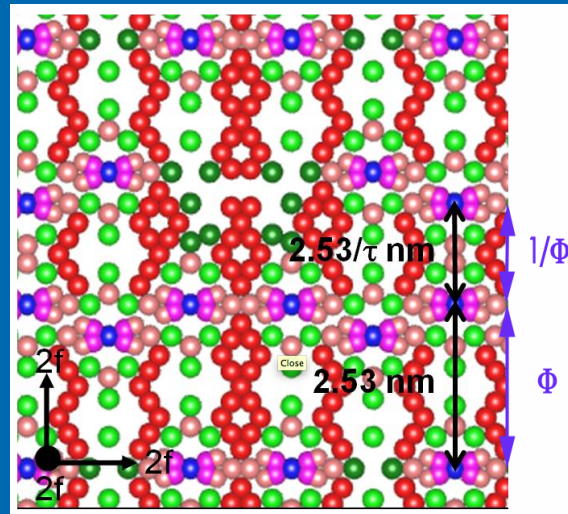


“Using scanning tunneling microscopy (STM), scientists... were able to obtain hi-res images of the surfaces of an aluminum-copper-iron alloy and an aluminum-palladium-manganese alloy, both of which are quasicrystals.

The images show flat “terraces” terminating in steps that come primarily in two heights, “high” and “low” (both measuring only a few hundred-millionths of an inch).

*The ratio of the two heights was found to be equal to the Golden Ratio!”*

→ Line plots show the atomic density of i-Cd-Yb (which is isostructural to i-Ag-In-Yb) along the three high symmetry surfaces. Terraces are formed at the position intersecting the rhombic triacontahedral (RTH) cluster centers. [Note:  $0.48 / 0.30 = 1.6 \sim \Phi$ ]

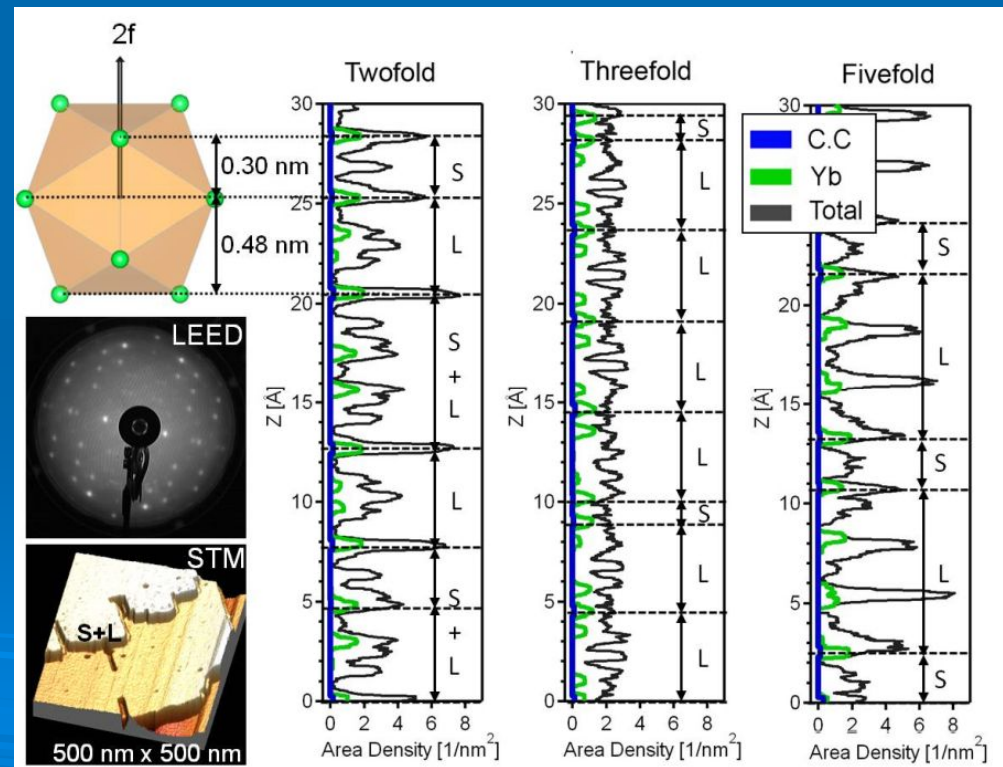


← An atomic plane intersecting the rhombic triacontahedral (RTH) cluster centers (colored in blue) in the i-Cd-Yb quasicrystal model.

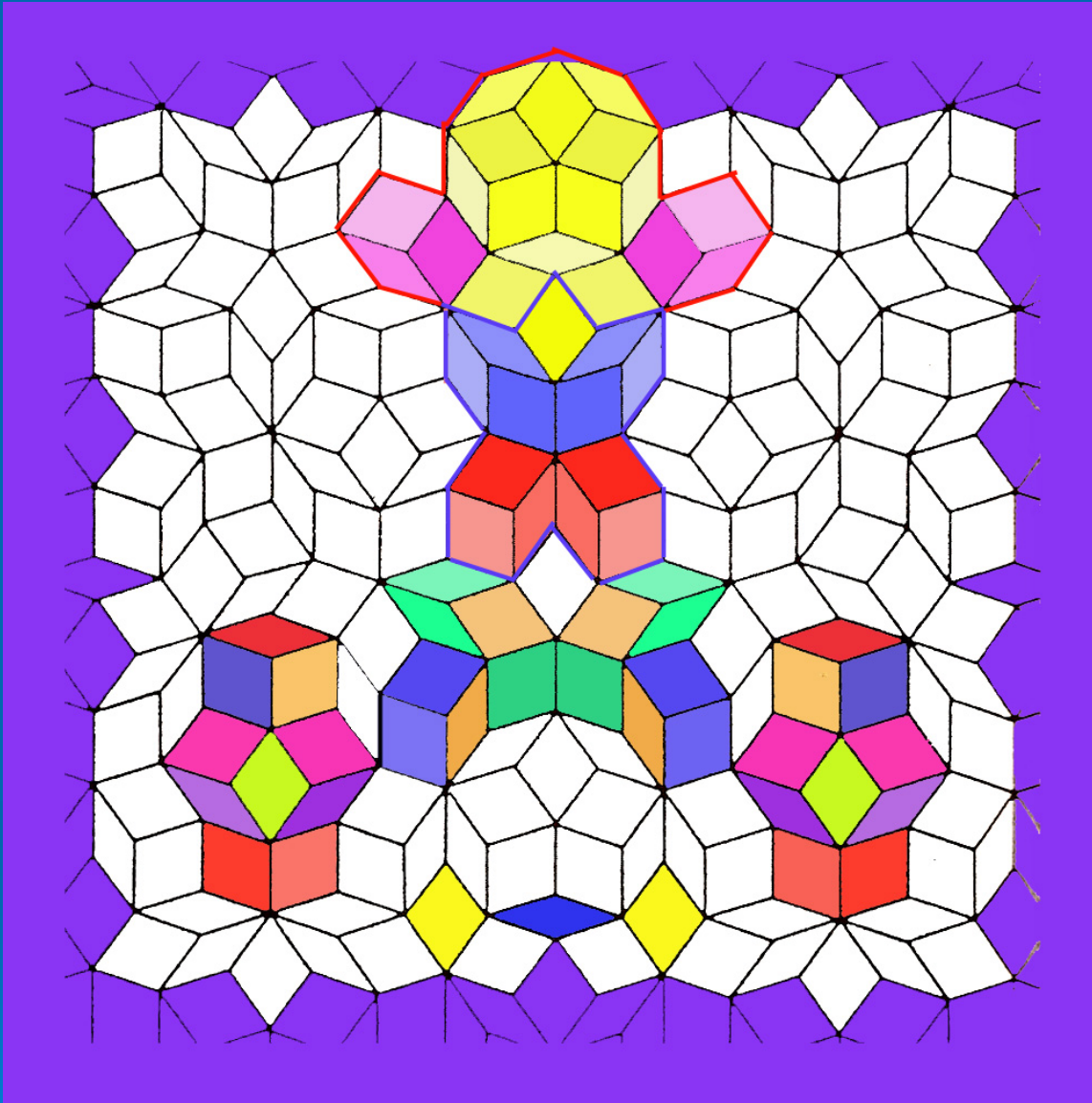
[Note:  $\tau$  is another mathematical notation for  $\Phi$ , the Golden ratio.]

Here we have  $AB = 2.53 \text{ nm} = \text{unit } 1$ ;  
and  $BC = 2.53 / \tau = 1 / \Phi = 1.56 \text{ nm}$

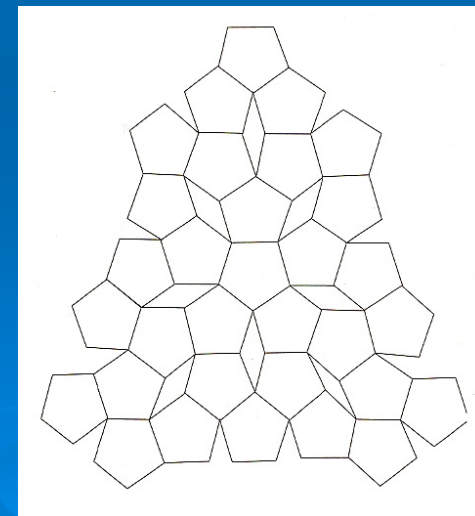
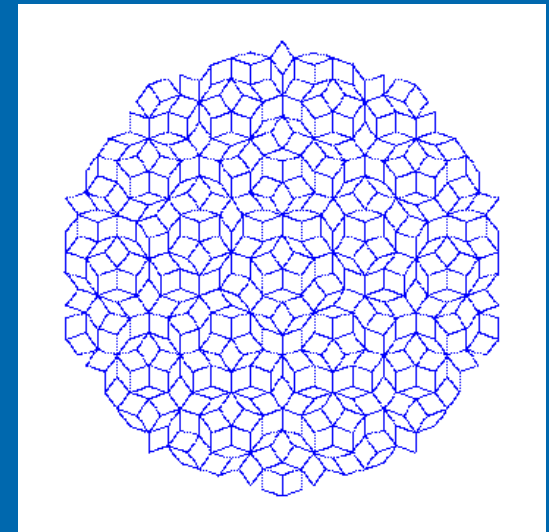
(© Journal of the Institute of Physics.  
<http://journals.iop.org/cws/article/jpcm/55497>)



SG203.2.8.1  
Quasicrystals -  
Design (1)



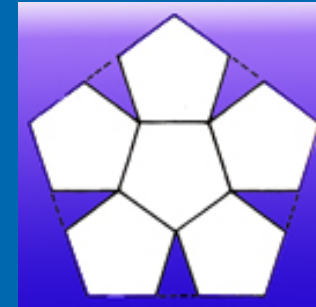
Nano Gnome playing  
in the Quasicrystal Field



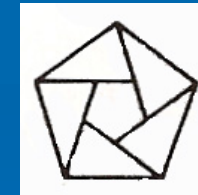
↑ Albrecht Dürer's  
Design, 1500's

## SG203.2.8.2 Quasicrystals - Design (2)

Step #1: Take 7 regular pentagons and combine 6 of these to make a larger pentagon:

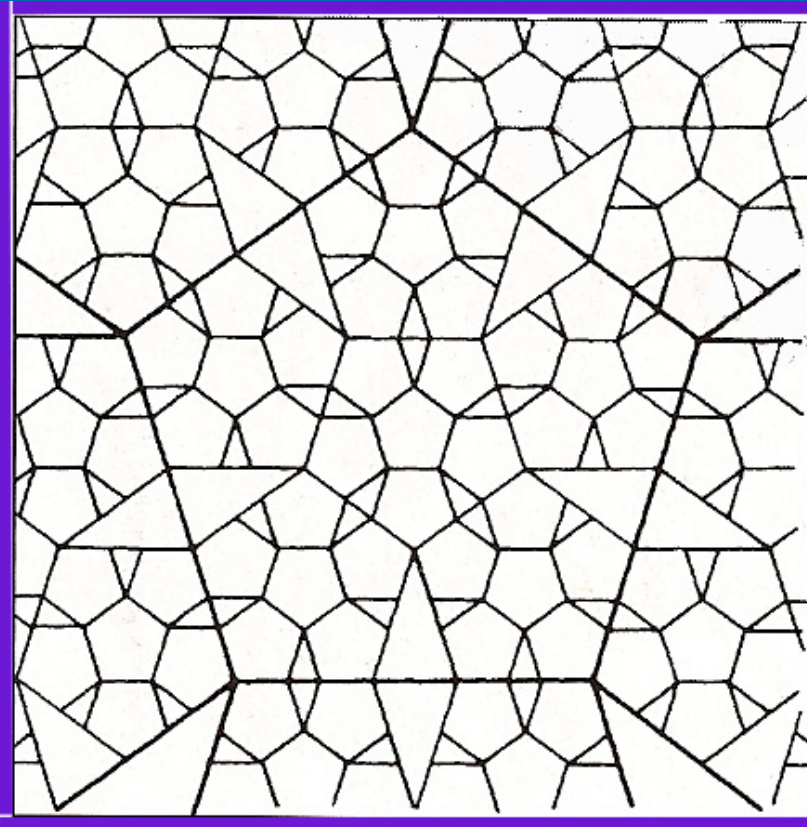


Step #2: Take the 7th pentagon and divide it thusly:



Step #3: Take the 5 triangles from the 7th pentagon and use them to fill the gaps in the large pentagon.

Step #4: Take 6 more large pentagons and continue the same procedure.



From:  
Istvan Hargittai.  
*Symmetry, A Unifying Concept.* 1994.

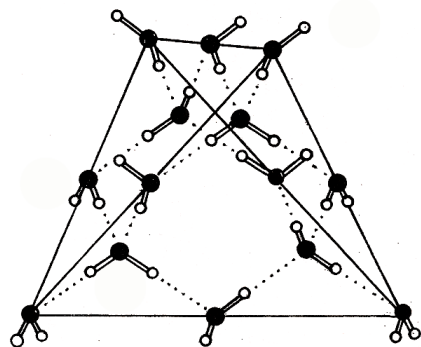
## SG203.2.9.1 Water Micro Clusters (1)

Water (H<sub>2</sub>O) is:

- the 2nd most common molecule in the Universe (after H<sub>2</sub>)
- the most abundant substance on earth
- the only naturally occurring inorganic liquid. 1 billion cubic kilometers of water reside in our oceans and 50 tons of water transit through our bodies in our lifetimes.
- Water is also the most extra-ordinary substance.

Researchers at the London South Bank University, led by Martin Chaplin, have studied the supra-molecular clustering of water. This brought them to describe water as a “network of icosahedral clusters”.

[www.lsbu.ac.uk/water/clusters.html](http://www.lsbu.ac.uk/water/clusters.html) 

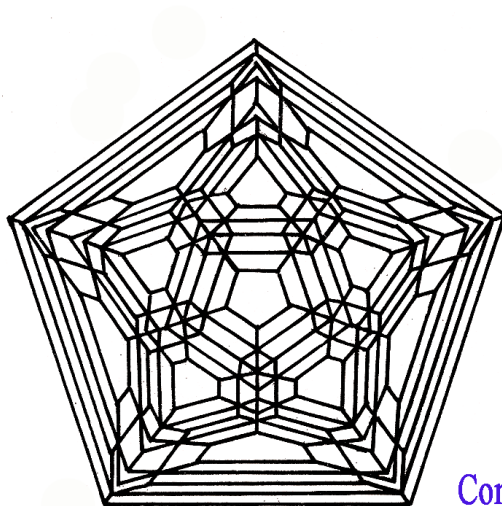


**Tetrahedral Water Cluster**

← Tetrahedral cluster.

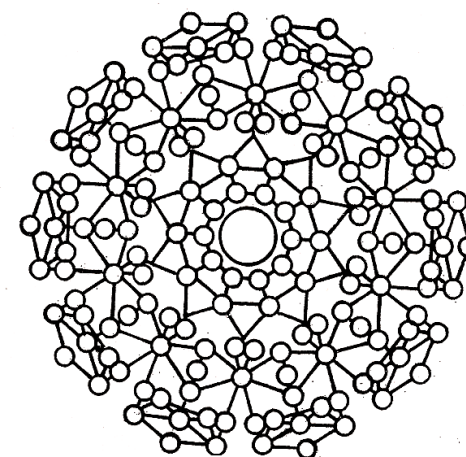
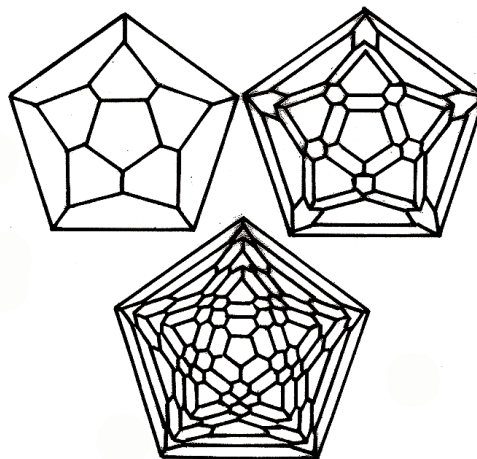
This cluster consists of 14 water molecules. “The central 10 molecules form a strong cluster and the remaining 4 form pentagons. There are 6 water molecules on each face and 3 on each edge.”

(All diagrams redrawn from the LSBU website)



**Connectivity map of the Water Icosahedron with inner, middle & outer shells**

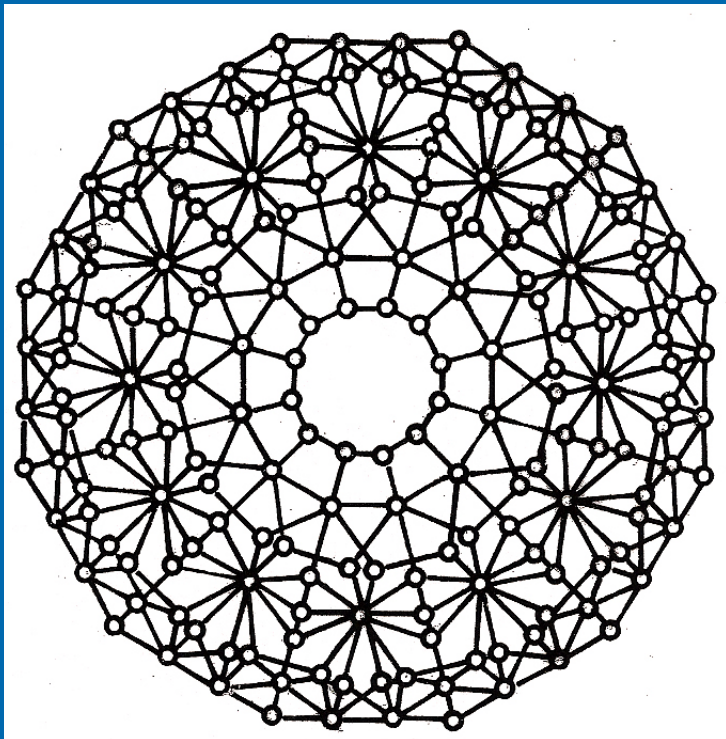
↑ Connectivity map of the 280-molecule icosahedron water cluster



**Icosahedral Water Cluster**

↑ Icosahedral Cluster.

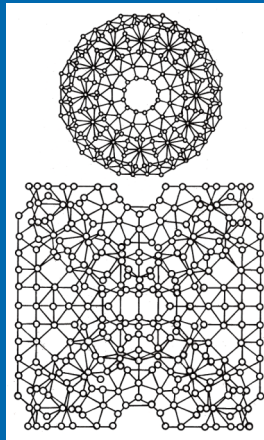
Twenty of the 14-molecules tetrahedral units (= 280 molecules of water) form a 3 nanometer diameter icosahedral unit.



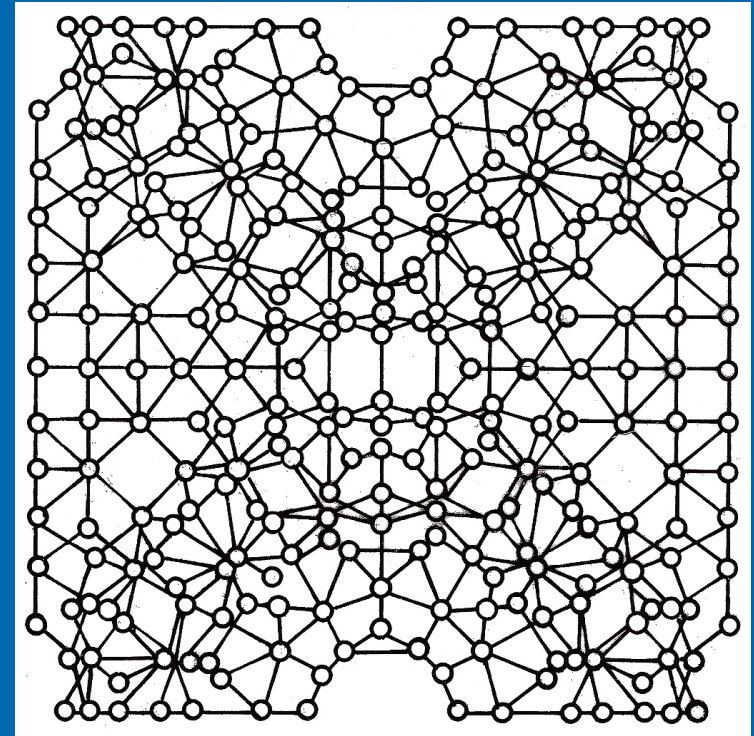
↑ Tetrakaidecahedral water cluster.

Twenty-four of the 14-molecules tetrahedral units can be arranged in a tetrakaidecahedral geometry (only the 336 oxygen atoms are shown).

## SG203.2.9 .2 Water Micro Clusters (2)



↑ Composite collage of the two diagrams on this page.

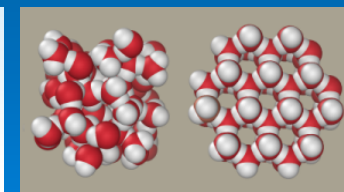


↑ A fully tessellated structure has been hypothesized from a combination of the 280-molecules icosahedra and the 336 molecules tetrakaideca clusters (only the oxygen atoms are shown). Note: this is starting to look just like "lattice-pattern grids" in Islamic art.

### In the words of Martin Chaplin (LSBU):

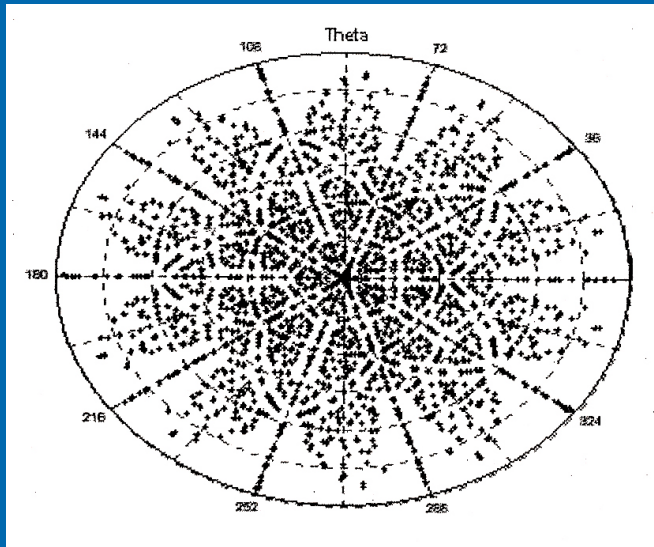
"Water is the most studied material on Earth but it is remarkable to find that the science behind its behavior and function are so poorly understood (or even ignored), not only by people in general, but also by scientists working with it every day. It can be extremely slippery and extremely sticky at the same time. The small size of its molecule belies the complexity of its actions and its singular capabilities. Many attempts to model water as a simple substance have failed and still are failing. Liquid water's unique properties and chameleonic nature seem to fit ideally into the requirements for life as can no other molecule."

[See **◆SG203B.5** about V. Schaubberger, the "Water Wizard"]



↑ Electrolysis alters the alkalinity of water by ionization. This gives the water several unique properties: smaller water molecule clusters (micro-clustering), hexagonal (orderly) structure, lower surface tension, negative electrical charge (anti-oxidant) and higher availability of minerals for absorption.

## SG203.2.9.3 Water Micro Clusters (3)

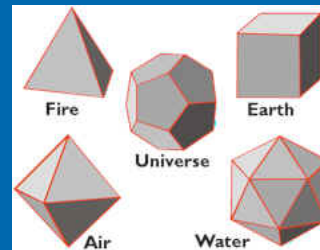
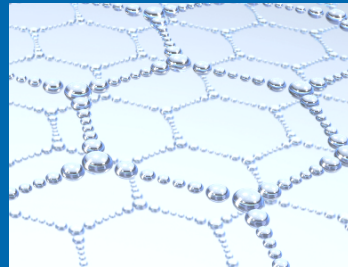


↑ Spherical coordinates of the water oxygen atoms in a supercluster of 13 icosahedral water clusters.

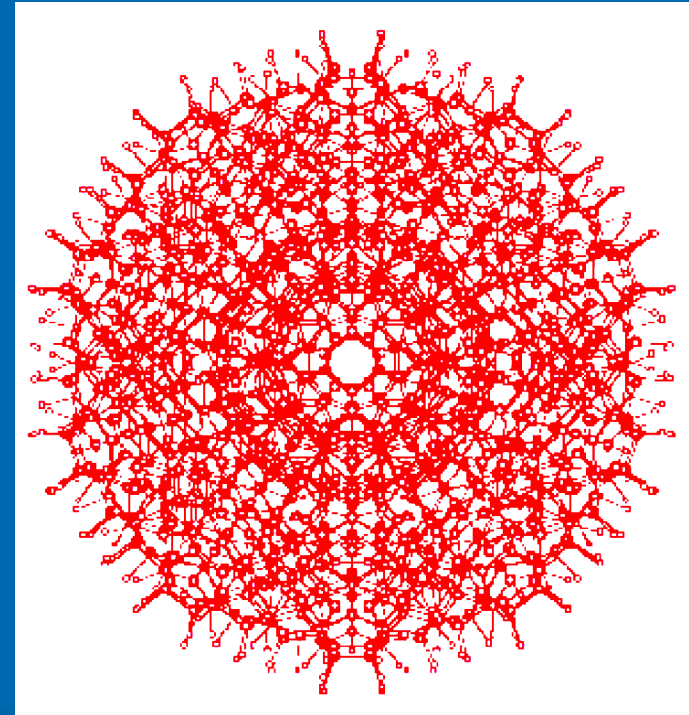
Use of spherical coordinates is nicely applicable to symmetrical systems with molecules positioned on nested concentric spherical surfaces, such as the icosahedral water cluster.

Note the pentagonal and decagonal geometries.

(<http://www1.lsbu.ac.uk/water/spher.html>)



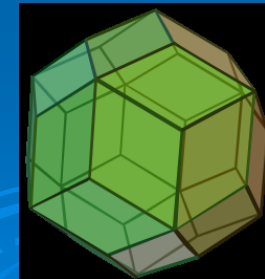
↑ For Plato, Water is associated with the Icosahedron.



↑ Model of one type of possible superclustering.

This consists of “13 complete but overlapping icosahedral clusters forming an icosahedron of interpenetrating icosahedra” i.e. a triacontahedron.

This supercluster contains 1820 water molecules. This type of superclustering is called a “nano-bubble formation”.



↑ Triacontahedron

Conclusion from Martin Chaplin, Emeritus Professor at LSBU:

*“The icosahedral water cluster is a highly symmetrical and aesthetically pleasing structure... There is a sufficient and broad evidential base for its existence, including the ability to explain all the ‘anomalous’ properties of water. The icosahedral cluster model offers a structure, not possible with other models, on which large molecules can be mapped in order to investigate their interaction with water within a three-dimensional hydrogen-bonded network, and offers new insights into the ways biological and non-biological ions and macromolecules interact with each other in aqueous solution. It also offers explanations concerning some strange dilution effects and the way some organisms produce low-density water to protect against desiccation and high temperatures and pressures.*

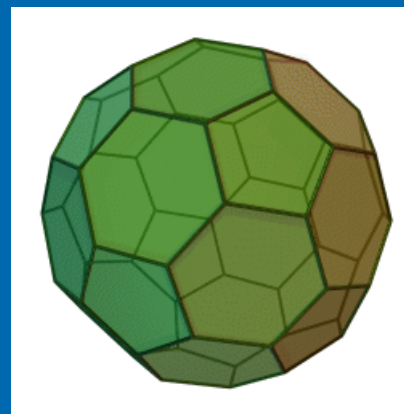
*Plato thought that water could be represented by an icosahedron. So do I.”*

## SG203.2.10.1 Fullerenes - Buckyball (1)

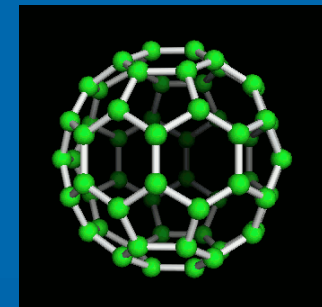
The Truncated icosahedron (one of the 13 Archimedean solids with 12 pentagonal faces, 20 hexagonal faces, 60 vertices and 90 edges uSG107) became famous, in 1985, with the discovery of the C<sub>60</sub> Buckminsterfullerene (in short “*Buckyball*”), named after geodesic dome inventor, mathematician, and Spaceship Earth captain Buckminster Fuller (1885-1983). Fuller’s ideas guided the chemists who discovered the substance to theorize that its shape was a truncated icosahedron.

In 1985, Kroto, Curl, and Smalley, using mass spectrometry, observed discrete peaks corresponding to molecules with the exact mass of sixty or seventy or more carbon atoms. They called the C<sub>60</sub> molecule “buckminsterfullerene” and shortly thereafter they came to discover the family of fullerenes. Kroto, Curl, and Smalley were awarded the 1996 Nobel Prize in Chemistry for their roles in the discovery of this class of compounds. C<sub>60</sub> and other fullerenes were later noticed occurring outside the laboratory (e.g., in normal candle soot). By 1991, it was relatively easy to produce gram-sized samples of fullerene powder.

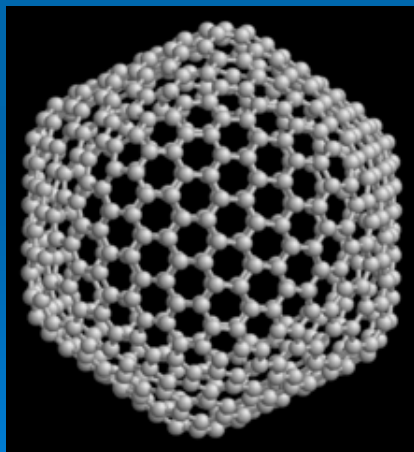
*“A fullerene is any molecule composed entirely of carbon, in the form of a hollow sphere, ellipsoid, or tube. Spherical fullerenes are also called buckyballs, and cylindrical ones are called carbon nanotubes or buckytubes. Fullerenes are similar in structure to graphite. The suffix -ene indicates that each C atom is covalently bonded to three others (instead of the maximum of four).”* [Note: that is 3 edges to each vertex, like the truncated icosahedron.] *Minute quantities of the fullerenes, in the form of C<sub>60</sub>, C<sub>70</sub>, C<sub>76</sub>, and C<sub>84</sub> molecules, are produced in nature, hidden in soot and formed by lightning discharges in the atmosphere. Recently, fullerenes were found in a family of minerals known as Shungites in Karelia, Russia.”* (Wikipedia)



↑ Truncated icosahedron



↑ C<sub>60</sub> molecule

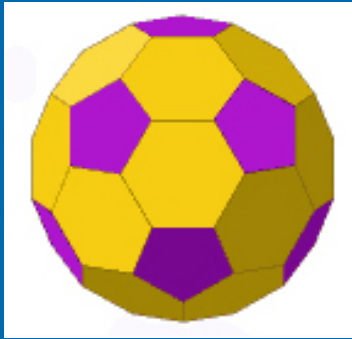


The Icosahedral fullerene C<sub>540</sub>

C<sub>60</sub>, the Buckminsterfullerene molecule, has a 5-fold rotational symmetry based on the pentagon structure. This species of carbon has been linked with soot formation and possible existence in interstellar space. C<sub>60</sub> is a very stable molecule maintaining the tetravalency of all carbon atoms. Many purely theoretical papers have already been published on this structure alone.

Since the discovery of fullerenes, structural variations on fullerenes have evolved well beyond the individual clusters themselves. Examples include:

- Buckyball clusters: smallest member is C<sub>20</sub> (unsaturated version of dodecahedrane)
- Nanotubes: hollow tubes of very small dimensions, having single or multiple walls (see next second page)
- Megatubes: larger in diameter than nanotubes potentially used for the transport of molecules.
- Polymers: chain, 2D and 3D polymers are formed under high pressure & high temperature conditions.
- Nano "onions": spherical particles based on multiple carbon layers surrounding a buckyball core.
- Linked "ball-and-chain" dimers: two buckyballs linked by a carbon chain.
- Fullerene rings.



## SG203.2.10.2 Fullerenes - Buckyball (2)

Buckminster Fuller arrived at the concept of the Buckyball through his life-long study of the "Geometry of Thinking" (the subtitle of his landmark book "Synergetics" - 1975).

The geodesic dome was one of his many gifts to humanity. The Buckyball was his posthumous gift.

### The Geodesic Dome (US patent June 1954 - #2,682,235)



*"When I invented and developed my first clear-span, all-weather geodesic dome, the two largest domes in the world were both in Rome and were each 150 feet in diameter. They are St. Peter's, built around A.D. 1500, and the Pantheon, built around A. D. 1. Each weighs approximately 30,000 tons.*

*In contrast, my first 150 ft diameter geodesic dome installed in Hawaii weighs only 30 tons - 1/1000th the weight of its masonry counterpart. An earthquake would tumble both the Roman 150-footers, but would leave the geodesic unharmed."*

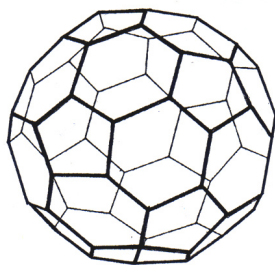
*"At no time during my last 56 years have I paid any attention to conventional architecture's 'orders' about the superficial appearance of my structures...*

*When the whole installation and assembly is complete and tested, and I can stand off and look at it as an operating reality, if it does not look beautiful to me, I know that I have failed..."*

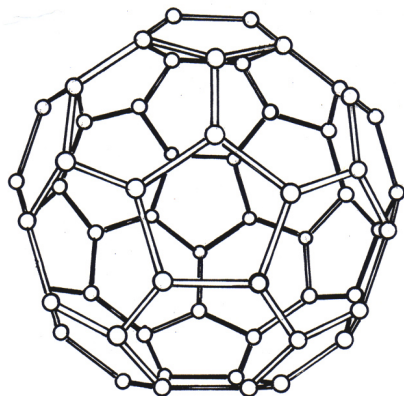
(R. Buckminster Fuller. *Inventions*. 1983)



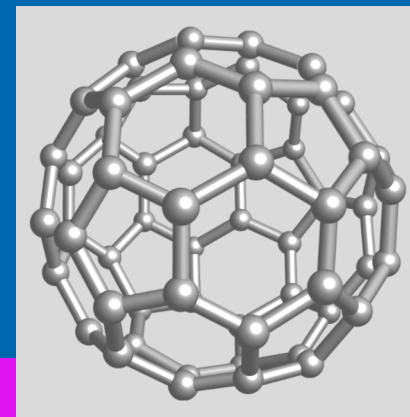
## SG203.2.10.3 Fullerenes - Buckyball (3)



**The C<sub>60</sub> geometry:**  
a truncated icosahedron



**Buckminsterfullerene**



The “Buckyball” Molecule:  
The Buckminsterfullerene C<sub>60</sub> has a 5-fold symmetry based on the pentagon geometry.

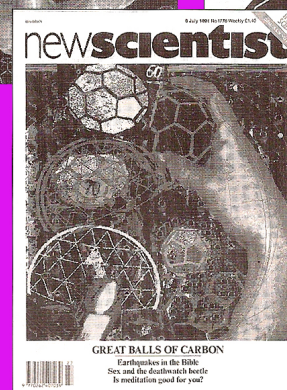
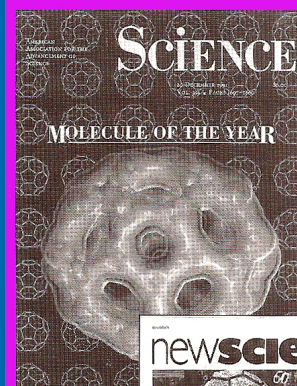
**nature**  
INTERNATIONAL WEEKLY JOURNAL OF SCIENCE  
Volume 231, Number 5233, 23 November 1983

**SIXTY-CARBON CLUSTER**  
AUTUMN BOOKS

Harry Kroto Transformation of MIT	Hendrik B. G. Casimir (Physics and philosophy)
R. N. Johnson-Laird China and mind	Gordon Thompson Dimensional nuclear proliferation
Anthony V. Clark Psychicists in religion	Jürgen Neils Belgium of 1947
A. Ch. Langer Empire down	Edward Harrison Light through the canopy

**Buckminster Fuller**

**Famous Molecule:**  
In 1991, C<sub>60</sub> was named  
"molecule of the Year"  
by Science magazine.



### Science History note:

The same hypothesis was published in 1970 by Japanese scientist Eiji Osawa and in 1973 by Russian scientists Bochvar & Gal'pern, but not in English.

Also in 1970, R.W. Henson proposed the structure and made a model of C<sub>60</sub>. The evidence for this new form of carbon was very weak and was not accepted, even by his colleagues. The results were never published.



Visit the Buckminster Fuller Institute: [www.bfi.org](http://www.bfi.org)

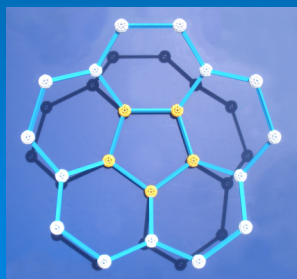
(↑ Images from Hargittai. *Symmetry*. 1994)

# SG203.2.10.4 Fullerenes - Buckyball (4) - 5 and 6



The Geodesic Dome at the 1967 Montreal International Expo. Interspersed among the hexagons are some pentagons to allow for curvature.  $5 + 6 =$  spherical geometry.

The special stability of  $C_{60}$  arises because the 12 pentagons are held apart by the hexagonal matrix. This led to the "Pentagon Isolation Rule": when shaping a fullerene (spherical) structure, one should try to minimize the contact between the 12 pentagons.



← The shallow bowl formed by one pentagon and 5 hexagons. This is the bottom part of the spherical  $C_{60}$  geodesic structure. (made out of Zometool components).

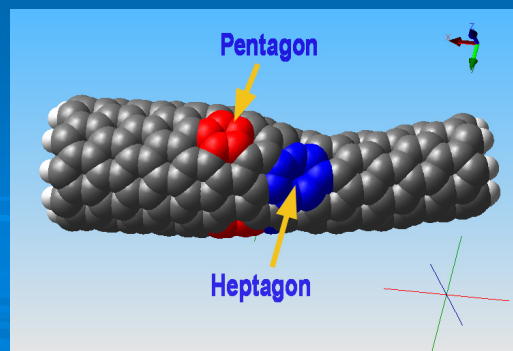
$C_{60}$  has been named the "Most Beautiful Molecule" (title of the book by H. Aldersey-Williams, 1995) because it has the highest symmetry of any known molecule.

Science history has it that two of the discoverers of  $C_{60}$ , Rick Smalley & Harry Kroto, visited the Montreal Expo in 1967 and were impressed, like many visitors, by the US pavilion housed in the giant geodesic dome designed by Buckminster Fuller. Both noticed the overall hexagonal symmetry but, according to Aldersey-Williams, failed to notice the "handful of pentagons insinuated among the hexagons of the gigantic orb". These pentagons would come to be of crucial importance when the two scientists met again to start working on the shape of their newly discovered  $C_{60}$  molecule.

Indeed, 18 years later, the  $C_{60}$  discovery team had determined that their carbon cluster required a structure with sixty vertices and was looking for a self-folding shape that could accommodate them. At that time, they still thought a geodesic dome was only composed of hexagons. However, back home, Rick Smalley started to cut off hexagons and fit them together - but that didn't work, but he felt the solution was close.

*"Reluctantly, he began to reconsider the pentagons... $C_{60}$  was already odd enough. Pentagons would just make it odder. But in desperation he began to draw his first pentagon. As soon as this was cut out, he placed five of the hexagons around it. It formed a shallow dish. The structure now curved with no cheating at all. The pentagon was the way to curvature... Rick quickly cut out more pentagons... His heart leaped. There would be sixty carbon atoms in the sphere..."* (H. Aldersey-Williams)

By some loop of subconscious history, the half-forgotten but potent visit to the Montreal geodesic dome may have guided the discoverers to determine the structure of their carbon molecule and to decide to name it "Buckminsterfullerene".



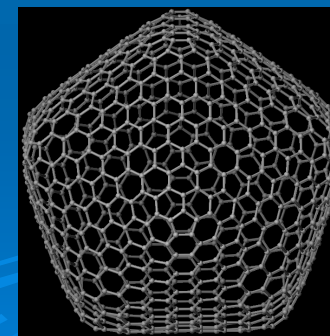
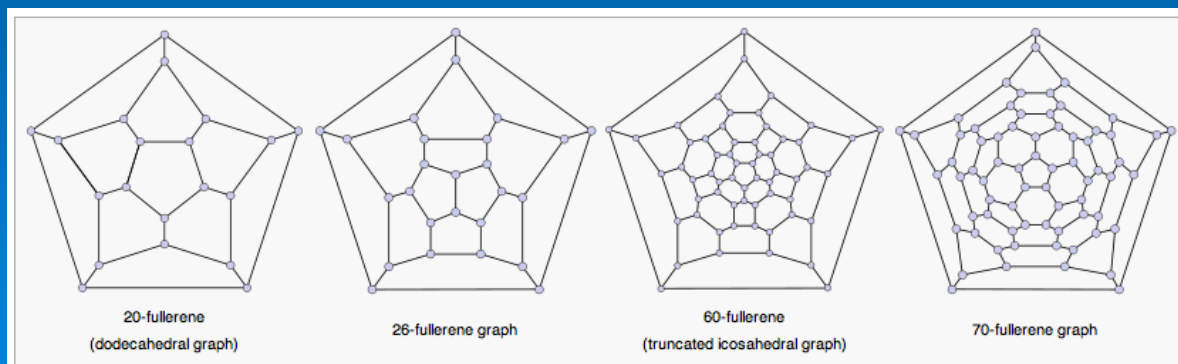
← Carbon nanotubes (rolled around graphite sheets) can be made to taper, flare or bend by adding pentagons or heptagons. This is "nano-architecture".

[//machine-phase.blogspot.com](http://machine-phase.blogspot.com)

## SG203.2.11.1 Fullerenes - Variations (1)

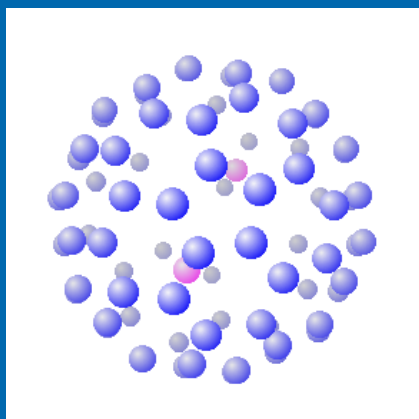
- Besides  $C_{60}$ , another fairly common buckminsterfullerene is  $C_{70}$ , but fullerenes with 72, 76, 84 and even up to 100 carbon atoms are commonly obtained.
- The smallest fullerene is the dodecahedron, the unique  $C_{20}$ . There are no fullerenes with 22 vertices. The number of fullerenes  $C_{2n}$  grows with increasing  $n = 12, 13, 14, \dots$ . For instance, there are 1812 non-isomorphic fullerenes  $C_{60}$ . Note that only one form of  $C_{60}$ , the buckminsterfullerene alias truncated icosahedron, has no pair of adjacent pentagons (the smallest such fullerene). To further illustrate the growth, there are 214,127,713 non-isomorphic fullerenes  $C_{200}$ , 15,655,672 of which have no adjacent pentagons.
- Trimetaspere carbon nanomaterials were discovered by researchers at Virginia Tech. This class of novel molecules comprises 80 carbon atoms ( $C_{80}$ ) forming a sphere which encloses a complex of three metal atoms and one nitrogen atom. These fullerenes encapsulate metals which puts them in the subset referred to as metallofullerenes. Trimetaspere has potential for use in diagnostics (as safe imaging agents), therapeutics and in organic solar cells.
- Other atoms can be trapped inside fullerenes to form inclusion compounds known as *endohedral fullerenes*. An unusual example is an egg shaped fullerene which violates the isolated pentagon rule. Recent evidence for a meteor impact at the end of the Permian period was found by analyzing noble gases so preserved. Metallofullerene-based inoculates are beginning production as one of the first commercially-viable uses of buckyballs.
- In 1999, researchers from the University of Vienna demonstrated that wave-particle duality applied to molecules such as fullerene. Science writer Marcus Chown stated on the CBC radio show *Quirks and Quarks* in May 2006 that scientists are trying to make buckyballs exhibit the quantum behavior of existing in two places at once (“*quantum superposition*”).
- A fully developed theory of  $C_{60}$  solids superconductivity is still lacking, but it has been widely accepted that strong electronic correlations and the Jahn-Teller electron-phonon coupling produce local electron-pairings that show a high transition temperature close to the insulator-metal transition.

(←↑ Data & images:  
Wikipedia)

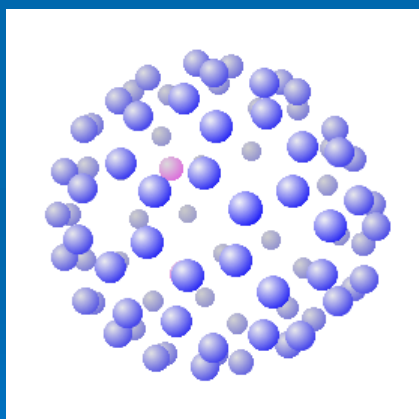


See giant fullerenes here [www.ccs.uky.edu](http://www.ccs.uky.edu)  
Animated C540 here [www.3dchem.com](http://www.3dchem.com)

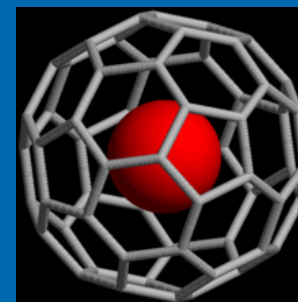
# SG203.2.11.2 Fullerenes - Variations (2) Endohedral



A pair of Lanthanum atoms are rotating inside a fullerene “cage”.



↑ Image credits:  
[www.photon.t.u-tokyo.ac.jp](http://www.photon.t.u-tokyo.ac.jp)



↑ Rendering of a molecule of  $C_{60}$  containing a noble-gas atom. (Wikipedia)

*Endohedral fullerenes* are fullerenes that have additional atoms, ions, or clusters enclosed within their inner spheres. The first lanthanum  $C_{60}$  complex was synthesized in 1985 and was called  $\text{La}@C_{60}$ .

Note: The '@' symbol in the formula indicates that the atom(s) are encapsulated inside the cage, rather than being chemically bonded to it.

Two types of endohedral complexes exist: endohedral metallofullerenes and non-metal doped fullerenes:

- Endohedral metallofullerenes are characterised by the fact that electrons will transfer from the metal atom to the fullerene cage and that the metal atom takes a position off-center in the cage.

- In non-metal doped fullerenes, the central atom in these endohedral complexes is located in the center of the cage. While other atomic traps require complex equipment, e.g. laser cooling or magnetic traps, endohedral fullerenes represent an atomic trap that is stable at room temperature and for an arbitrarily long time.

Atomic or ion traps are of great interest since particles are present free from (significant) interaction with their environment, allowing unique quantum mechanical phenomena to be explored. For example, the compression of the atomic wave function as a consequence of the packing in the cage could be observed with special spectroscopy. The nitrogen atom can be used as a probe, in order to detect the smallest changes of the electronic structure of its environment.

## Reported Species

I	II	IIIb	IVb	Vb	VIb	VIIb	VIIIb	Ib	IIb	III	IV	V	VI	VII	0		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
H																	He
Li	Be										B	C	N	O	F		Ne
Na	Mg										Al	Si	P	S	Cl		Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La*	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac**	Rf	Db	Sg	Bh	Hs	Mt	Uun	Uuu	Uub		Uuq		Uuh		
Lanthanides *		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
Actinides **		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

← A Periodic Table of the reported endohedral fullerenes, as of 2007.

[//homepage.mac.com](http://homepage.mac.com)

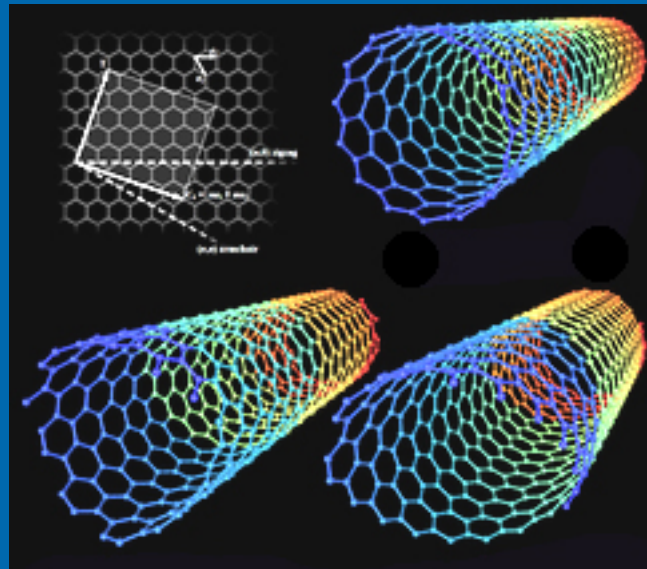
SG203.2.12.1  
**Fullerenes -  
Applications (1)  
Nanotubes**

*“Carbon nanotubes (CNTs) are allotropes of carbon. Their name is derived from their size, since the diameter of a nanotube is on the order of a few nanometers (approximately 1/50,000th of the width of a human hair), while they can be up to several millimeters in length, with a nanostructure that can have a length-to-diameter ratio of up to 28,000,000:1.*

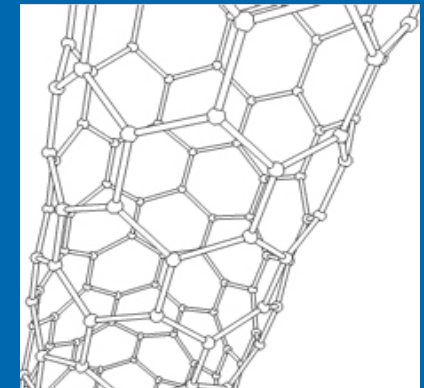
*Nanotubes are members of the fullerene structural family, which also includes the spherical buckyballs. The ends of a nanotube might be capped with a hemisphere of the buckyball structure.*

*These cylindrical carbon molecules have novel properties that make them potentially useful in many applications in nanotechnology, electronics, optics and other fields of materials science, as well as potential uses in architectural fields. They exhibit extraordinary strength and unique electrical properties.”*

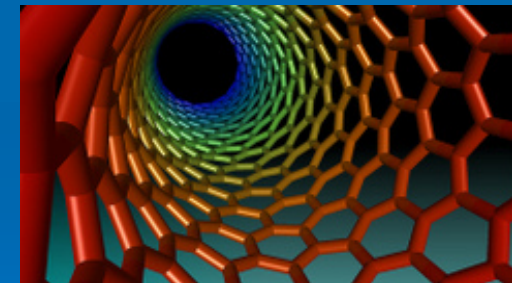
(Wikipedia)



The question of the toxicity of nanotubes has been raised as NTs can cross cell membrane barriers.



↑ A Nanobud combining a nanotube and a fullerene. Nanobuds are good field emitters.

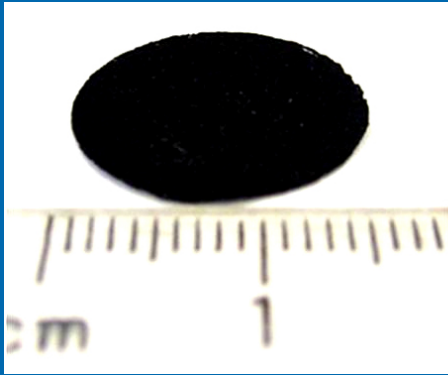


**Potential applications of NTs:**

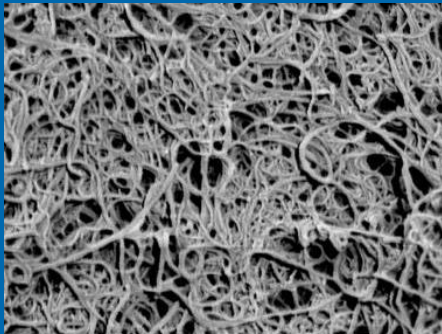
GDVs (gene delivery vehicle) - Nano batteries - Nano flowers & meadows - Nano radios - Nanotori (NTs bent into a torus shape) - Nano transistors - Solar cells - Space elevators - Ultra capacitors...

For simulations of nano-components:  
[//machine-phase.blogspot.com](http://machine-phase.blogspot.com)

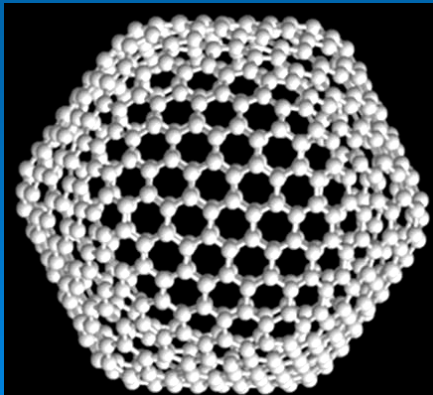
## SG203.2.12.2 Fullerenes - Applications (2) Buckypaper



↑ Naked-eye size Buckypaper

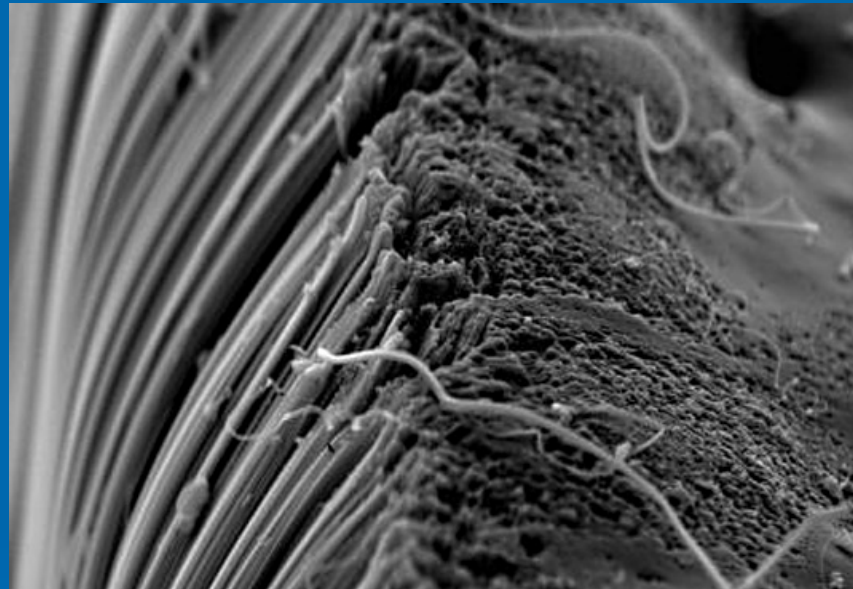


↑ Intermediate size

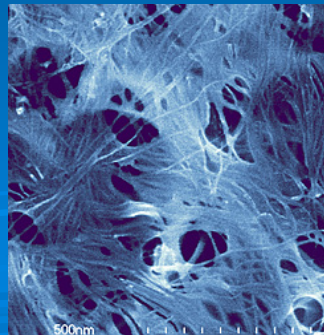


↑ Nano size

*Buckypaper* is a thin sheet made from an aggregate of carbon nanotubes. The nanotubes are approximately 50,000 times thinner than a human hair. Buckypaper is one tenth the weight of steel yet potentially 500 times stronger when its sheets are stacked to form a composite. It can disperse heat like brass or steel and it can conduct electricity like copper or silicon.



[www.inhabitat.com](http://www.inhabitat.com)

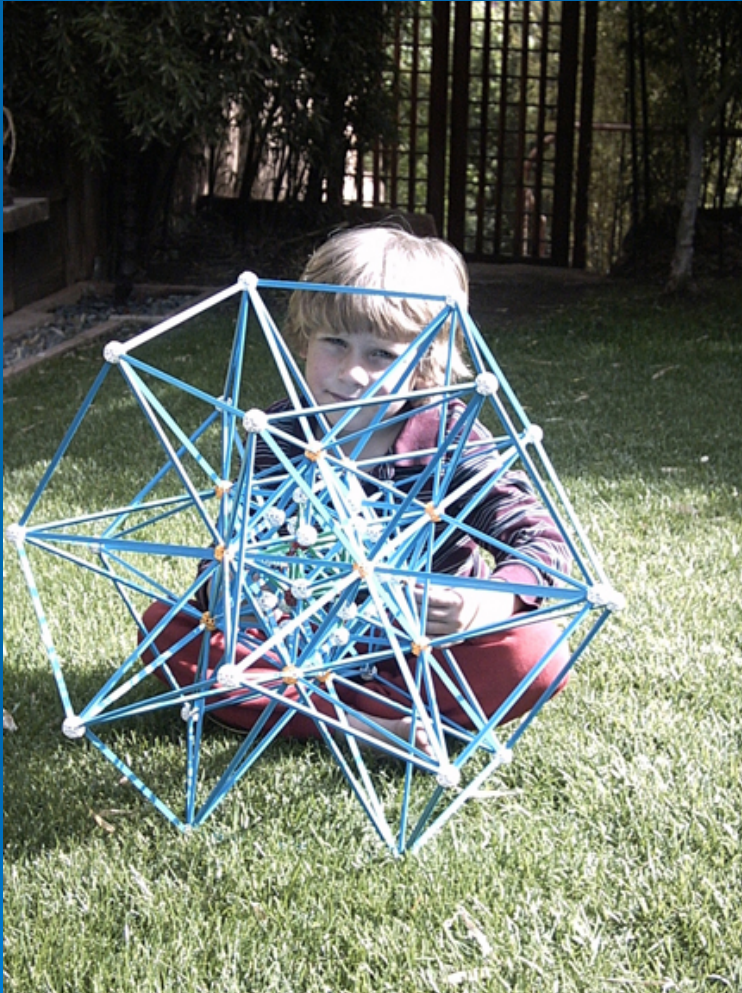


← The nanotubes here have an average length 820 nm and make a continuous, electrically conducting network overall in spite of obvious gaps.

On a macroscale this material would be nearly transparent. Color added for clarity.

[//cdn.physorg.com](http://cdn.physorg.com)

## SG203.3 Chapter 3. Platonic Models in Science



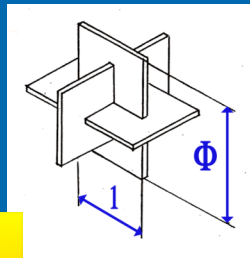
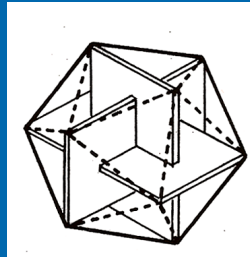
This chapter is a survey  
of the Platonic Models  
making a comeback in Science,  
from the atomic nucleus to chemistry to cosmology...

We have just reviewed the story  
of the truncated ico-  
sa, raised from being a ground-level soccer-ball  
to the crown-level status  
of being the "most beautiful molecule".  
This is a sign of the times.

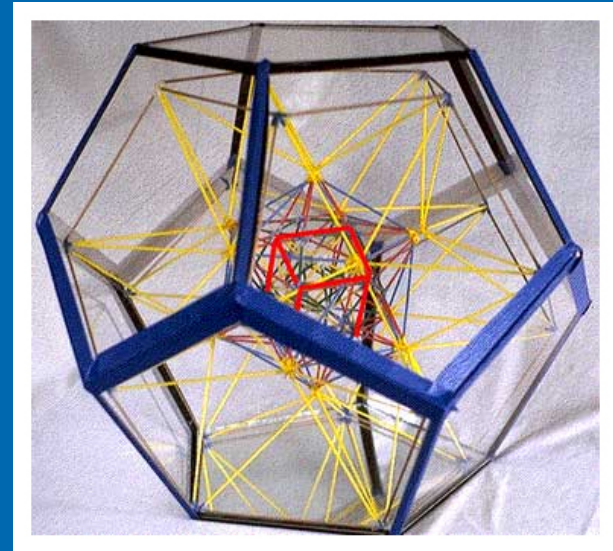
# SG203.3.1 The Nesting of the Cosmic Figures

Let us recall that the 5 Platonic Solids, due to their common Phi seed (icosa & dodeca), are nesting together in the Phi Ratio.

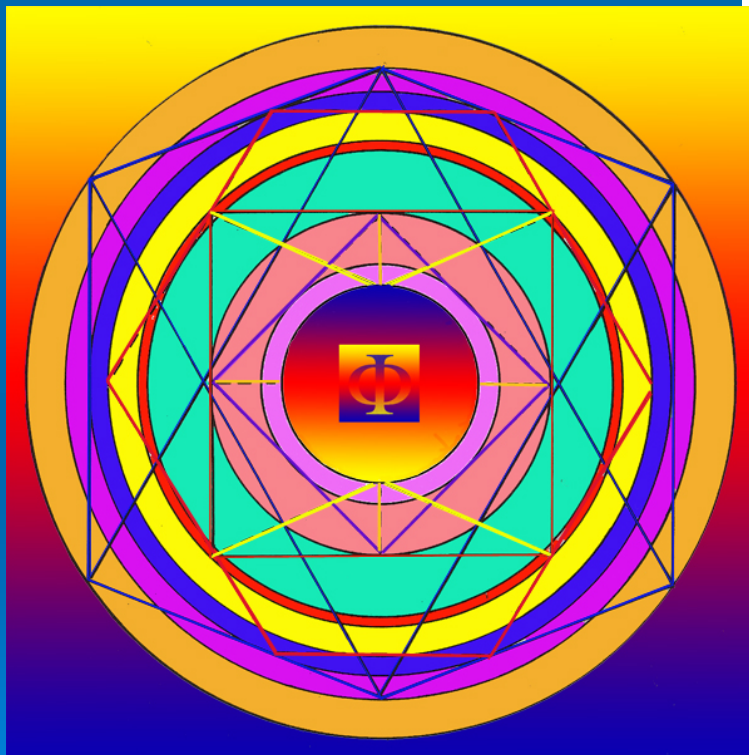
The Golden Frame (made out of 3 Golden Rectangles) is the inner seed or framework for constructing the icosahedron and its dual the dodecahedron - from this golden standard, the other Platonic Solids are proportionally built. [◆SG107.4.4]



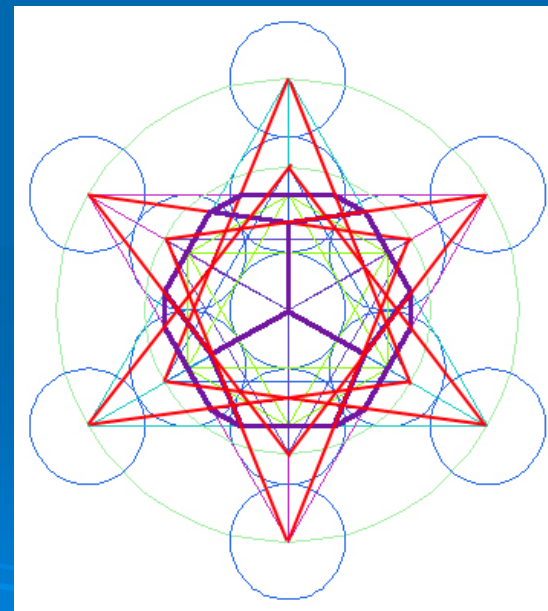
↑ Golden Frame



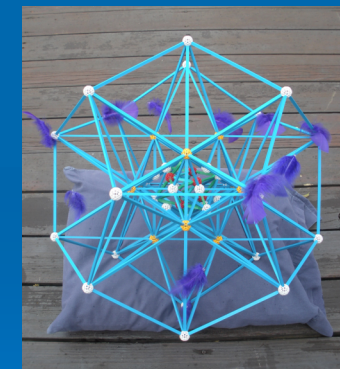
↑ Platonic Nesting with an older construction kit.



↑ Geometries of the 5 Platonic Solids collapsed in 2D.



↑ Metatron's Cube is a template for all 5 Platonic Solids. [◆SG107.1.8]



↑ A feathery Star-Mother



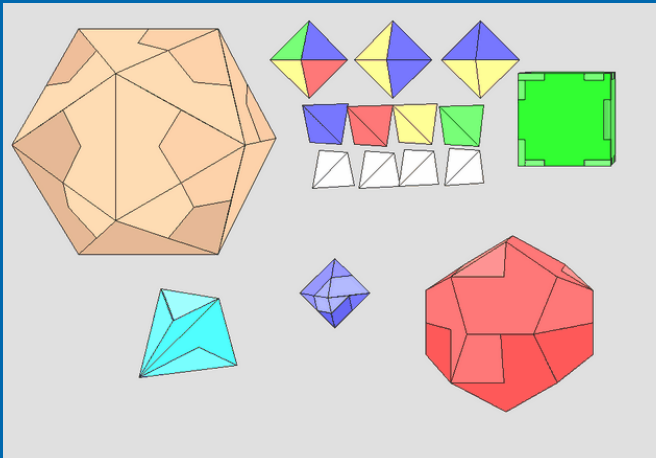
# SG203.3.2 The All Five Puzzle

Wayne Daniel, a retired physicist who had a career working on engine design for General Motors Research Labs, is also a puzzle expert. After retiring, he set out to create a nested puzzle model of the 5 Platonic Solids.

The "All Five" model is an interlocking wooden puzzle made out of 37 pieces, without any void in between. From the outside in, the sequence is: icosahedron, dodecahedron, cube, tetrahedron and finally, at the core, octahedron. Like a *matryoshka doll*, each layer peels off to reveal a smaller Platonic Solid inside.

The "All Five" is a miniature cosmos on its own. Each form is its own puzzle and the whole constitutes a 'tiny symphony of Platonic play'.

Quoting a New York Times article: *"The major hurdle was to design pieces with one the solids outside and another on the inside. Most difficult, he said, was the outermost shell, 'in which a set pieces had to assemble into an icosahedron with a dodecahedral hole in the middle. An expert woodworker, Mr. Daniel makes all his puzzles himself, shaping each piece out of woods like peroba rosa, zebrawood, jatoba and chechen. This is the first puzzle with all 5 Platonic Solids in a concentric, integrated and solid form."*



↑ Final image of the Quicktime movie [www.waynedaniel.net](http://www.waynedaniel.net)



↑ The 37 components of the All Five Puzzle

# SG203.3.3 The Dodeca-Icosa Doctrine

As we have seen in the history of Sacred Geometry [◆SG102], Plato's cosmology is based on the Platonic Solids. Each Solid was embodying one of the 5 elements:

- Tetrahedron - Fire
- Cube - Earth
- Octahedron - Air
- Icosahedron - water
- Dodecahedron - ether or the body of the universe.

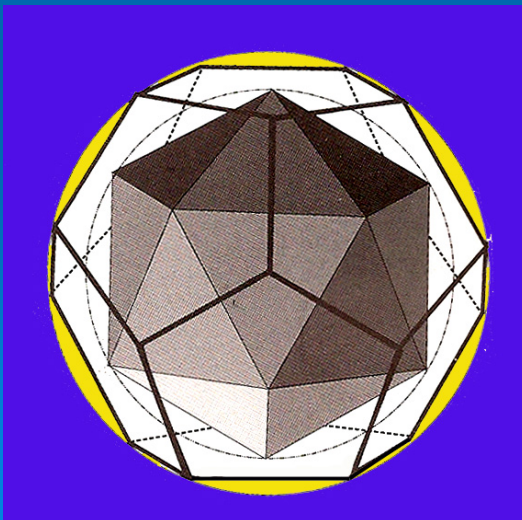
The general harmony of the universe was traditionally represented by the combination of these 5 elements. This became also known as the "Dodeca-Icosa Doctrine" because the two larger Solids (dodecahedron & icosahedron) are directly based on the Phi Ratio AND happen to be typical forms of nature, on all scales. Actually, the more we extend our observations in the micro-scale and the macro-scale, the more these two shape-structures (or their combination) appear.

*"The Dodeca-icosa doctrine is a golden thread passing through all human science."*  
(A. Stakhov)



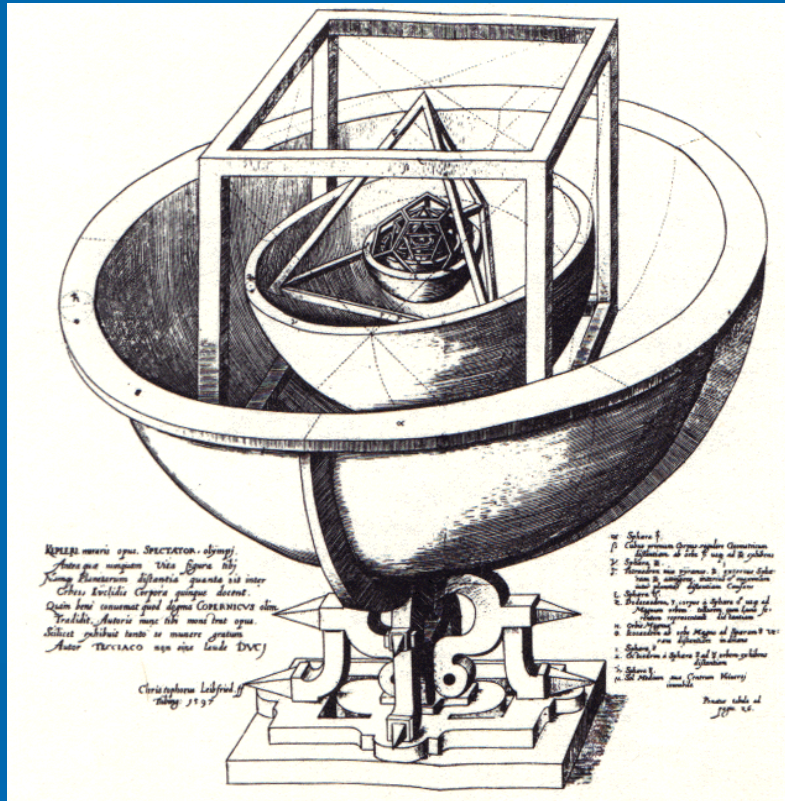
↑ StarWheel #84 "Sri Bindu".

→ Each of the 12 vertices of the icosahedron is centered in the middle of the 12 faces of the dodecahedron.



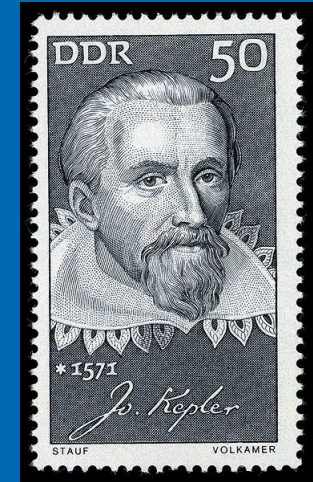
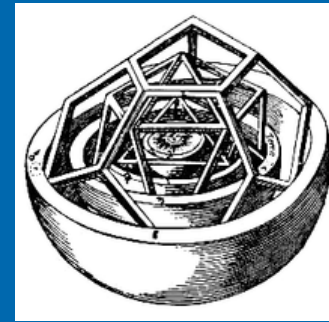
As pointed by A. Stakhov: "According to Proclus (412 – 485 CE), the commentator on Euclid's Elements, Euclid did consider a theory of geometric construction of the Platonic Solids, the 'Main Geometric Figures of the Universe', as a central goal of his Elements. And therefore, Euclid placed this major mathematical information in the final (and most honorable) book of the Elements." (= Book 13).

# SG203.3.4 Kepler's Cosmic Cup



In his *Mysterium Cosmographicum* (1596), written at the age of 25, Johannes Kepler began experimenting with the 3-dimensional Platonic regular polyhedra.

He found that each of the five Platonic solids could be uniquely inscribed and circumscribed by spherical orbs. Nesting these solids, each enclosed in a sphere, within one another would produce six layers, corresponding to the six known planets - Mercury, Venus, Earth, Mars, Jupiter, and Saturn.



*"Earth's orbit is the measure of all orbits. We place the dodecahedron around this orbit. The orbit around the dodecahedron is Mars' orbit. We place the tetrahedron around Mars' orbit. The orbit around the tetrahedron is Jupiter's orbit. We place the cube around Jupiter's orbit. The orbit around the cube is Saturn's orbit. Then we insert the icosahedron inside Earth's orbit. The orbit inside the icosahedron is Venus's orbit. We insert the octahedron inside Venus's orbit. The orbit inside the octahedron is Mercury's orbit."*

In 1621, Kepler issued a second edition of *Mysterium* explaining that the first edition of 1596 had been written *"as if an oracle from the heavens dictated through me the chapters ... During 25 years, (this first book) illuminated my way in astronomy many times."*

However Kepler admitted that the new data he had obtained were not supporting a spherical model for planetary motion and thus, in the re-edition of *Mysterium*, he incorporated the seminal 3 astronomical laws he had discovered and published earlier in *Astronomia Nova* (1609) and *Harmonices Mundi* (1619).

Although the observational data did not exactly fit, Kepler's overall insight (explaining the number & properties of the planets by the symmetries of the Platonic Solids) opened the way to understand the harmonic geometry of the solar system as proven by contemporary data.

In more scientific terms, Kepler understood that the rotational energy of the sun is distributed throughout the solar system in a quantized way, according to the Golden Ratio principle.

# SG203.3.5 Felix Klein & the Icosahedron

The German mathematician Felix Klein (April 25th 1849 - 1925) delighted in pointing out that each of the month ( $2^2$ ), day ( $5^2$ ) and year ( $43^2$ ) of his date birth was the square of a prime. In 1884, Klein published his *Lectures on the Icosahedron and Solution of the 5th Degree Equations* dedicated to a Geometric theory of the Icosahedron.

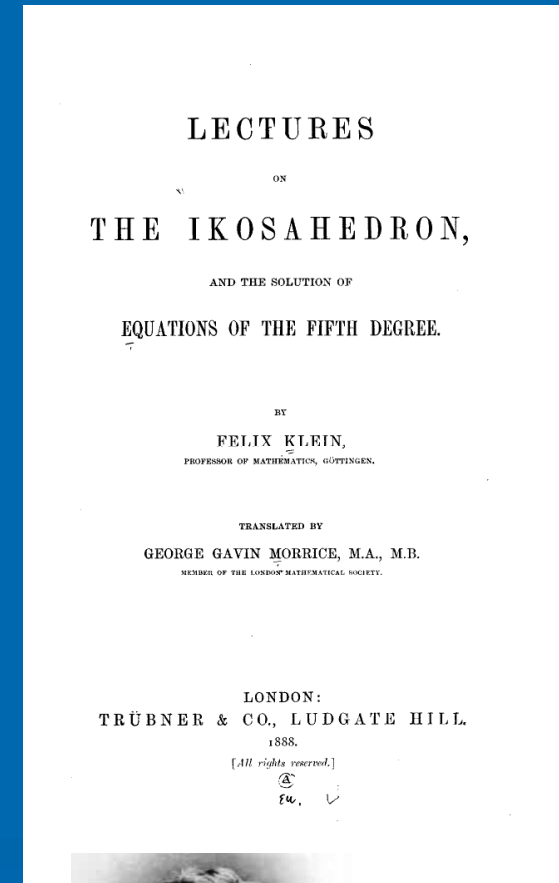
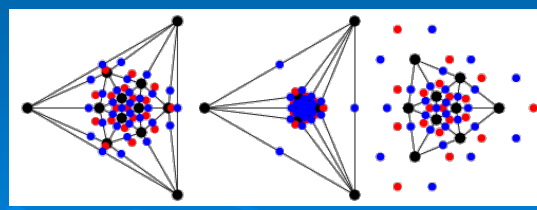
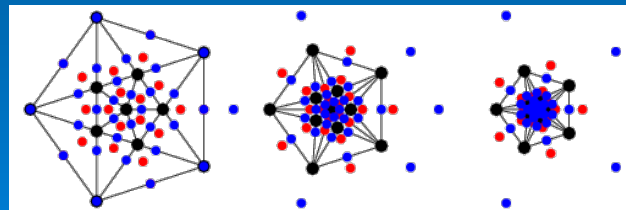
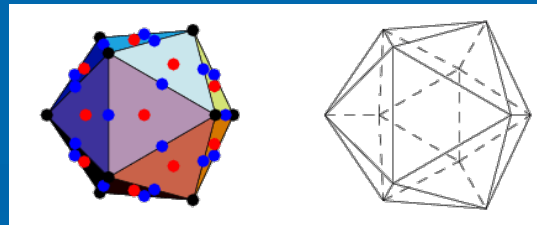
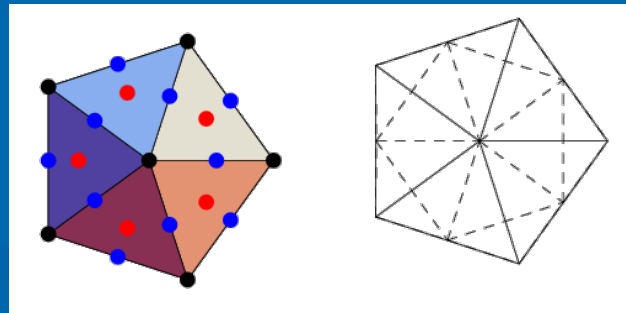
*"According to Klein, the fabric of mathematics extends widely and freely with the threads of the different mathematical theories. However, there are geometric objects in which many mathematical theories converge... The icosahedron, in Klein's opinion, is just such a mathematical object. Klein treats the regular icosahedron as the central mathematical object from which the branches of the 5 mathematical theories follow, namely: Geometry, Galois' Theory, Group Theory, Theory of Invariants and Differential Equations..."*

*Following after Pythagoras, Plato, Euclid & Kepler, Klein realized the fundamental role of the Platonic Solids, in particular the icosahedron, for the development of science and mathematics.*

*Klein's idea is extremely simple: each unique geometrical object is somehow or other connected to the properties of the regular icosahedron."*

(A. Stakhov. The Mathematics of Harmony. p. 162)

A century later, the icosi-dodeca pair is becoming the most famous couple in science.



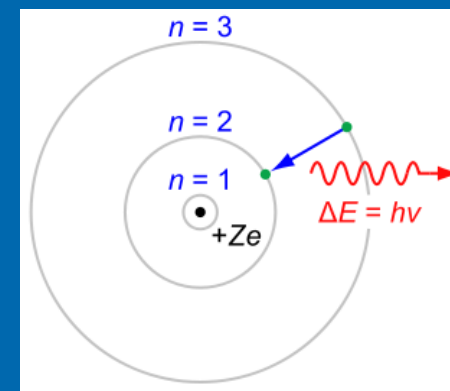
↑ Felix Klein

↑ There are a number of algebraic equations known as the *icosahedral equation*, all of which derive from the projective geometry of the icosahedron. [mathworld.wolfram.com](http://mathworld.wolfram.com)

# SG203.3.6.1 New Atom Model (1) Wave Function

"The Bohr model is a primitive model of the hydrogen atom... It may be considered to be an obsolete scientific theory." So says Wikipedia. However, because of its simplicity and its correct results for selected systems, the Bohr model is still commonly taught to school students and still widely held as a "scientific model" in the general culture. The main objections are that "orbiting" electrons are fixed (non-interactive) and not compatible with the evidence from stereochemistry and crystallography.

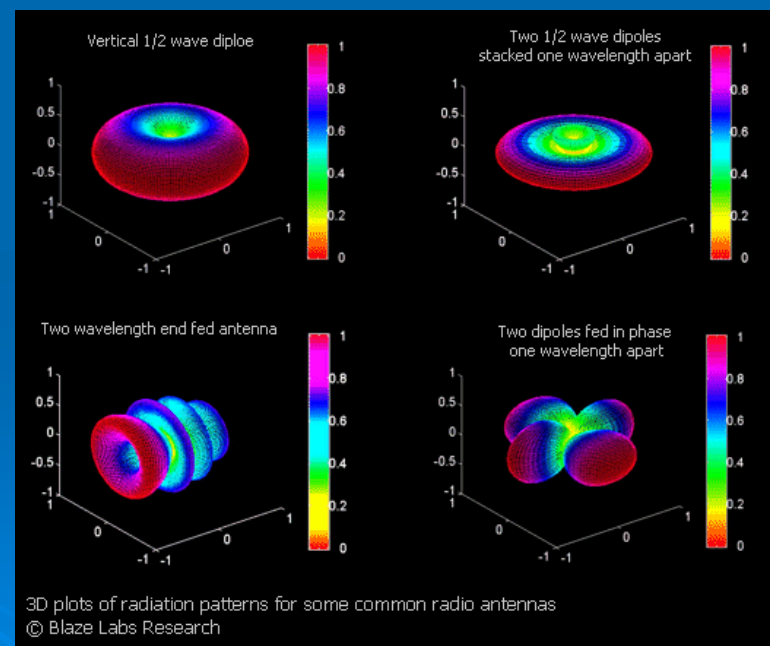
In the words of engineer Xavier Borg (Blaze Labs): "There are no Electron Orbits! Bohr's model, which started the notion of electrons traveling around the nucleus like planets has misled a lot of people and scientists. If you have learned such an idea, forget about it immediately. Instead, all calculations and all experiments show that there are standing wave patterns, very similar indeed to the polar plots of antenna radiation patterns... The electron path is NOT around and far off the nucleus, nor is the atom made up of 99.999% empty space! Instead, the center of the electron pattern is also the center of the proton pattern. This is the normal situation of the H atoms in the universe: they have spherical symmetry, not orbits... Particulate matter is not a requirement to generate the effects known to define matter." ([www.blazelabs.com](http://www.blazelabs.com))



↑ In the outdated Bohr model, the electrons are described as "satellites" orbiting the nucleus.

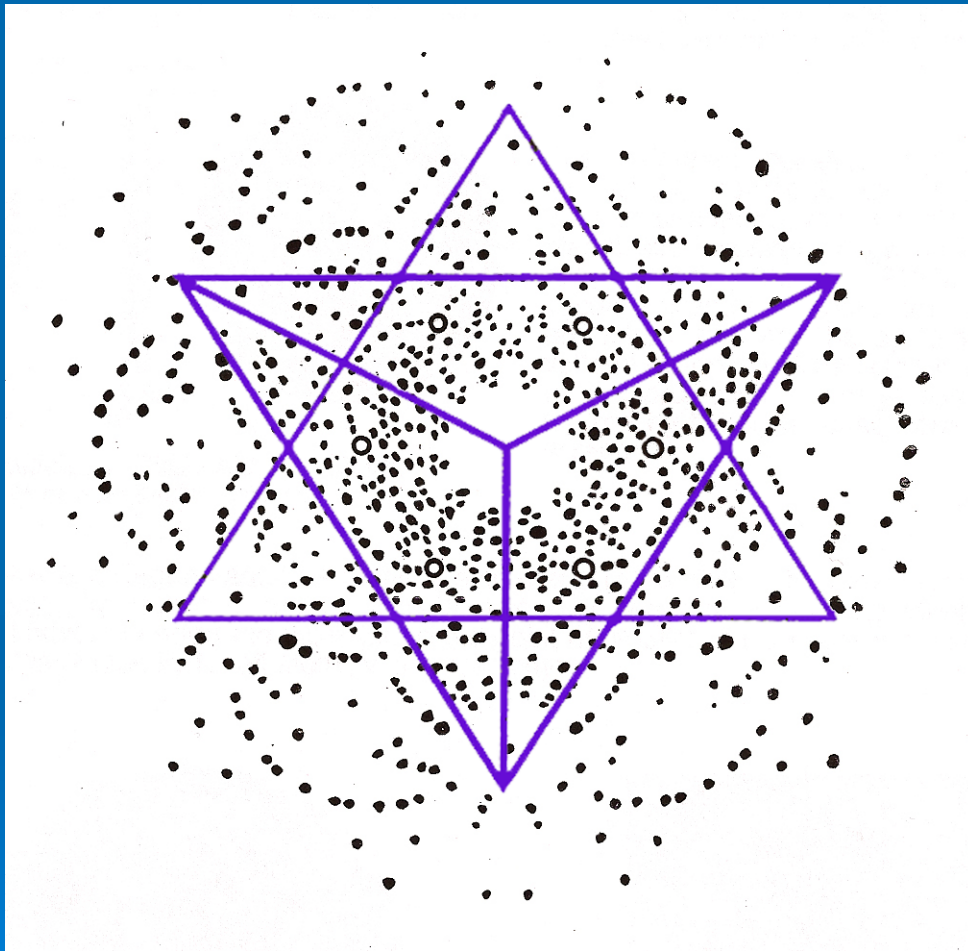
	s	p <sub>z</sub>	p <sub>x</sub>	p <sub>y</sub>	d <sub>z<sup>2</sup></sub>	d <sub>xz</sub>	d <sub>yz</sub>	d <sub>xy</sub>	d <sub>x<sup>2</sup>-y<sup>2</sup></sub>
n=1	•								
n=2	•								
n=3	•								
n=4	•								
n=5									
n=6					...	...	...	...	...
n=7		...	...	...	...	...	...	...	...

↑ Some of the Spherical Harmonics showing the probability density of electrons in the atomic structure. (Wikipedia).



↑ 3D plots of antenna radiation patterns obtained for some common antenna configurations.

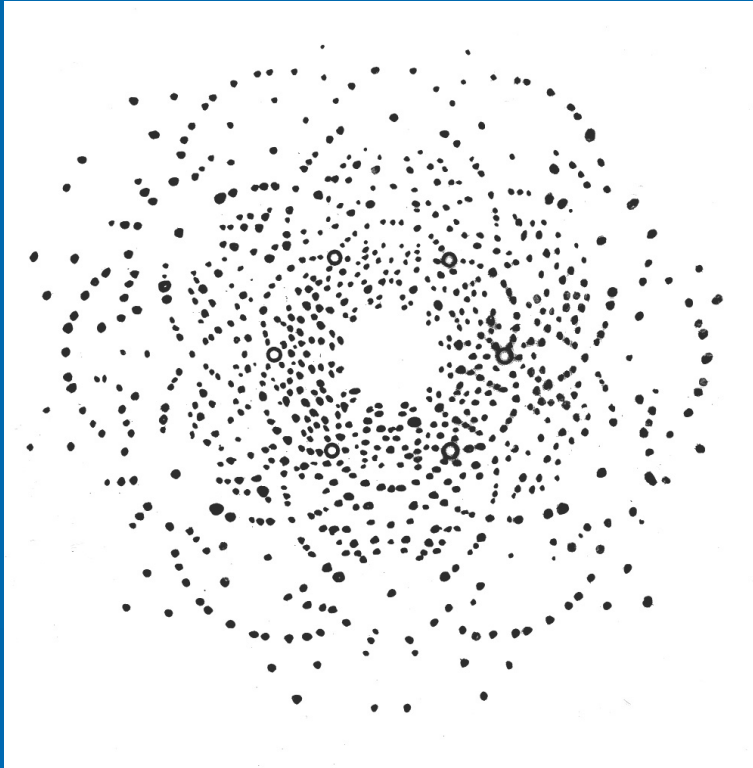
## SG203.3.6.2 New Atom Model (2) Harmonic Symmetry



Star-tetrahedron overlay  
on X-ray diffraction pattern

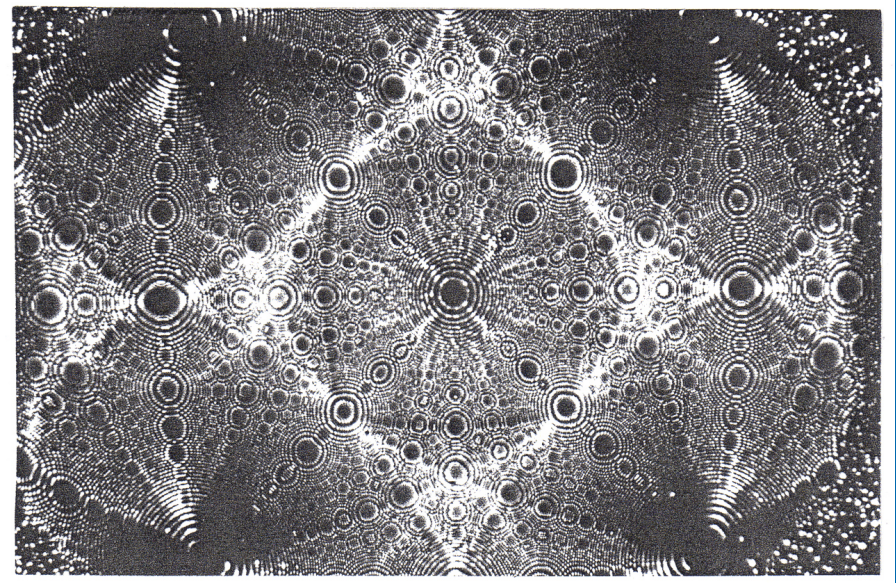
**Matter is an  
exquisite  
choreography  
of  
harmonic  
waveforms**

### SG203.3.6.3 New Atom Model (3) Harmonic Symmetry

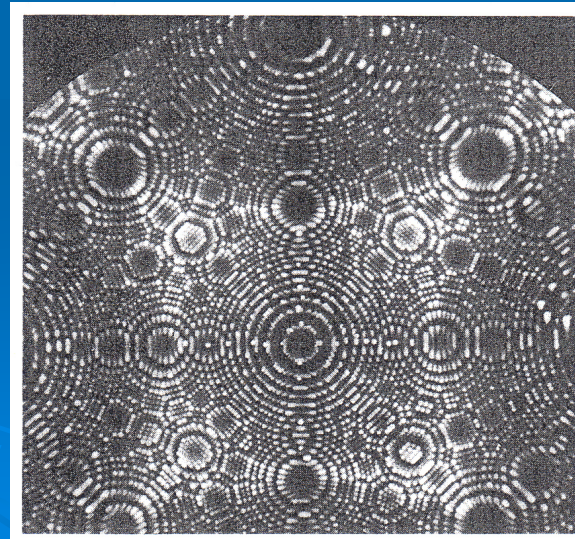


↑ X-ray diffraction pattern in Beryl

The universe  
is a immense dance of wave resonance,  
optimized by PHI.



↑ Platinum crystal (x 750,000 times)



← Iridium crystal  
(x 1,000,000 times)

# SG203.3.6.4 New Atom Model (4)

## Spherical Resonance

In the wave function model of matter, electro-magnetic waves can be 'materialized' within a volume of space (provided that their dimensions are exact multiples of Planck's half wavelength.) The nodes within the standing wave thus obtained exhibit the structure.

As explained by Xavier Borg: *"All objects have a frequency or set of frequencies with which they naturally vibrate when struck, plucked, or somehow given an impulse... So the natural frequencies of an object are merely the harmonic frequencies at which standing wave patterns are established within the object. These standing wave patterns represent the lowest energy vibrational modes of the object. While there are countless ways by which an object can vibrate (each associated with a specific frequency), objects favor only a few specific modes or patterns of vibrating. The favored modes (patterns) of vibration are those which result in the highest amplitude vibrations with the least input of energy. Objects are most easily forced into resonance vibrations when disturbed at frequencies associated with these natural frequencies.*

*The wave pattern associated with the natural frequencies of an object is characterized by points which appear to be standing still; for this reason, a pattern in 2D is often called a 'standing wave pattern', whilst we may call a pattern in 3D, a 'standing wave structure'. The points in the structure which are at stand-still are referred to as nodal points (in 2D) or vertex positions (in 3D). These positions occur as the result of the destructive interference of incident and reflected waves. Each nodal point is surrounded by anti-nodal points, creating an alternating pattern of nodal and anti-nodal points."*

A classical demonstration is the *cymatic plate* activated by a violin bow [◆SG201].

The new understanding of the standing wave structure of matter was launched in 1924 by Louis de Broglie proposed that all objects have properties of waves. *"His hypothesis was soon confirmed in 1927 by the observation of diffraction patterns in the scattering of electrons from crystals.*

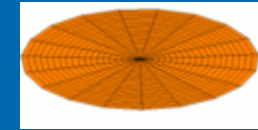
*De Broglie's model was the last of the accepted "physical" models, since just 5 years later, Werner Heisenberg derived his 'Uncertainty Principle' which states that it is impossible to determine simultaneously the momentum and position of an electron. Such a principle has been widely accepted and to the present day, science gave up the search for a 'physical' model.*

*De Broglie's model is correct in principle, but is too simplistic and cannot account for all the experimental observations done on atoms. For this model to be complete, we first need to transform the 2D Broglie diagram into a 3D spatial equivalent". (Xavier Borg).*

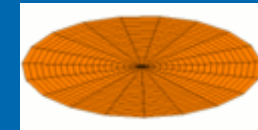


← 2D wave model of atom. 1924.

3D wave model → of atom. 1990s.

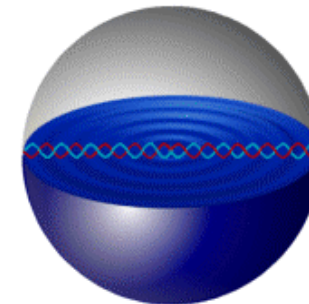


↑ A higher harmonic standing wave on a disk with a node at the center of the drum. (Animation)



↑ A two-dimensional standing wave on a disk. This is the fundamental mode. (Animation)

### Wave structure of a particle



The shown ripples are due to the incoming waves (cyan).  
The red waves are the reflected waves at a slightly lower frequency.  
The EM energy in the spherical volume is equal to the particles internal energy.  
The resulting standing waves create 'shells' of matter.  
Any doppler shift at the centre will show as energy radiated/absorbed by the particle.

(Credit: [www.blazelabs.com](http://www.blazelabs.com))



## SG203.3.7 Platonic Wave Resonance

Going beyond the blind acceptance of the "cartesian coordinates" by established physics, Buckminster Fuller proposed a "synergetic" system of coordinates based on observing the close-packing of equal spheres: the Platonic Volumes are showing up, starting with the simplest, the tetrahedron. This synergetic geometry perfectly reflects the way crystals grow in their various forms [◆SG203 supra about Quasicrystals & Fullerenes]. And engineering applications have already proven the efficiency and the stability of platonic geometry structures.

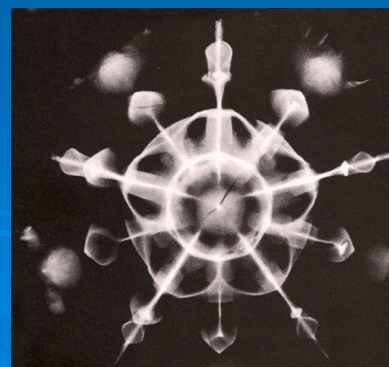
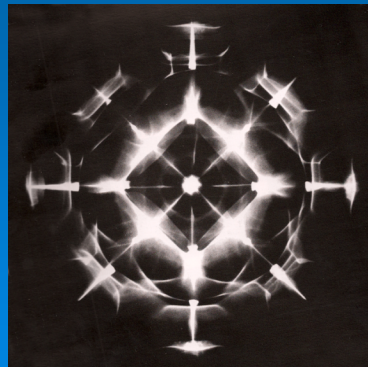
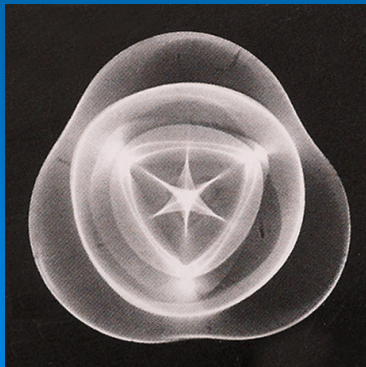
So we have a special type of wave resonance based on the geometric structure of the Platonic Volumes. Both the students of Buckminster Fuller and his protégé Dr. Hans Jenny (the founder of cymatics [◆SG201]) devised experiments to show how the geometries of the Platonic Solids would form within a vibrating / pulsating 3D sphere.

### The Fuller-inspired experiments

*"In the experiment conducted by Fuller's students, a spherical balloon was dipped in dye and pulsed with pure sine wave sound frequencies. A small number of evenly-distanced nodes would form across the surface of the sphere, as well as thin lines that connected them to each other. If you have four evenly spaced nodes, you will see a tetrahedron. Six evenly spaced nodes form an octahedron. Eight evenly spaced nodes form a cube. Twelve evenly spaced nodes form the icosahedron and twenty evenly spaced nodes form the dodecahedron. The straight lines that we see on these geometric objects simply represent the stresses that are created by the closest distance between two points for each of the nodes as they distribute themselves across the entire surface of the sphere."*

### The Jenny experiments

*"In Hans Jenny's life work about cymatics, Dr. Hans Jenny conducted a similar experiment, wherein a droplet of water contained a very fine suspension of light-colored particles, known as a colloidal suspension. When this spherical droplet of particle-filled water was vibrated at various diatonic musical frequencies, the Platonic Solids would appear inside, surrounded by elliptical curving lines that would connect their nodes together. As we shall see, these dark points, which are nothing but point of intersections of nodes, are the supposed 'point bits of matter'. In a 3D standing waves, a structure, with all characteristics of a Platonic solid, is formed for each standing wave mode. Within an atom, which is the building block of matter, the platonic solid is not formed by salt or known particles, but by electromagnetic waves in vacuum. The final result, the standing wave structure, is one which has a structure, an inertia, a reaction to other standing wave structures, and a reaction to external EM waves, all characteristics of what we use to call 'a particle', which can be felt and seen. 'Particles' are point effects of the standing wave nodes." (X. Borg).*



← Cymatic experiments showing standing waves in fluids vibrated by sound frequencies.

Note the tetrahedral, cubic and penta-dodecahedral geometries.

## SG203.3.8.1 Oscillons & Solitons (1)

Adding to our understanding of the wave structure of the universe are new forms of wave patterns that have been recently discovered: the *solitons* (1960's) and the *oscillons* (1989).

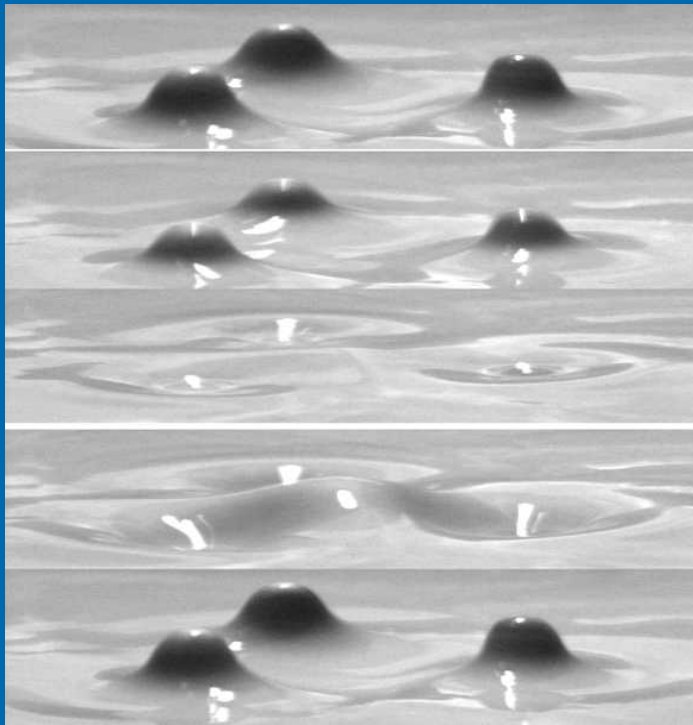
Both oscillons and solitons display self-sustainability (meta-stability).

The term "soliton" was introduced in the 1960's, but the scientific research about solitons had started in the 19th century when John Scott-Russell observed a large *solitary wave* in a canal near Edinburgh. Nowadays, many model equations of nonlinear phenomena are known to possess *soliton* solutions.

Solitons are very stable non-linear wave excitations (solitary waves) behaving like "particles". Solitons are evolving undistorted and remain undistorted even after mutual collision. When they are located mutually far apart, each of them is approximately a traveling wave with constant shape and velocity. As two such solitary waves get closer, they gradually deform and finally merge into a single wave packet; this wave packet, however, soon splits into two solitary waves with the same shape and velocity before "collision".

The stability of solitons stems from the delicate balance of "nonlinearity" and "dispersion" in the model equations. Nonlinearity drives a solitary wave to concentrate further. Dispersion is the effect to spread such a localized wave. If one of these two competing effects is lost, solitons become unstable and, eventually, cease to exist. In this respect, solitons are completely different from "linear waves" like sinusoidal waves. In fact, sinusoidal waves are rather unstable in some model equations of soliton phenomena. Computer simulations show that they soon break into a train of solitons.

From the modern perspective, the soliton model is used to formulate the complex dynamical behavior of wave systems throughout science: from hydrodynamics to nonlinear optics, from plasmas to shock waves, from tornados to the Great Red Spot of Jupiter, from the elementary particles of matter to the elementary particles of thought.



A triad of stationary wave structures ("oscillons") in a colloidal suspension. ([www.aip.org](http://www.aip.org))

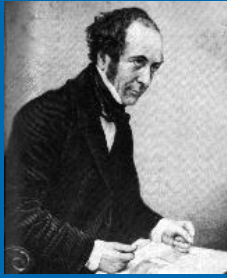
"Oscillons are stable interacting localized waves with sub-harmonic response. Oscillons in granular media result from vertically vibrating a plate with a layer of uniform particles placed freely on top. When the sinusoidal vibrations are of the correct amplitude and frequency and the layer of sufficient thickness, a localized wave, referred to as an *oscillon*, can be formed by locally disturbing the particles.

Oscillons are *meta-stable*: they will remain for a long time (many hundreds of thousands of oscillations) in the absence of further perturbation. An oscillon changes form with each collision of the grain layer and the plate, switching between a peak that projects above the grain layer to a crater like depression with a small rim. This self-sustaining state was named by analogy with the soliton, which is a localized wave that maintains its integrity as it moves. Whereas solitons occur as travelling waves in a fluid or as electromagnetic waves in a waveguide oscillons may be stationary." (Wikipedia)

### Historical art "oscillons"

In the 1950s & 60s, artist Ben Laposky used arrays of oscilloscopes with many controls to manipulate basic waves into elegantly rhythmic designs he called "oscillons." →





← John Scott Russell

Story from:  
[www.ma.hw.ac.uk](http://www.ma.hw.ac.uk)

## SG203.3.8.2 Oscillons & Solitons (2)

Here is a historical retrospective/prospective about the amazing discovery of the "Soliton" wave by John Russell.

"In 1834, John Scott Russell was observing a boat being drawn along 'rapidly' by a pair of horses. When the boat suddenly stopped Scott Russell noticed that the bow wave continued forward 'at great velocity, assuming the form of a large solitary elevation, a well-defined heap of water which continued its course along the channel apparently without change of form or diminution of speed'. Intrigued, the young scientist followed the wave on horseback as it rolled on at about eight or nine miles an hour, but after a chase of one or two miles he lost it.

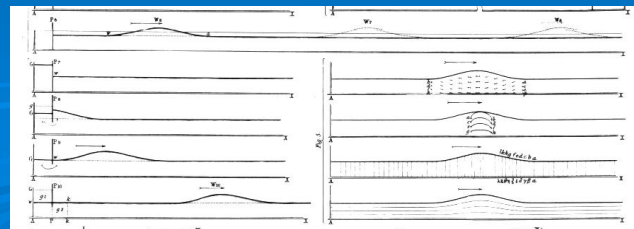


↑ Soliton on the Scott Russell Aqueduct on the Union Canal near Heriot-Watt University.

On Wednesday 12 July 1995, an international gathering of scientists witnessed a re-creation of the famous 1834 'first' sighting of a soliton or solitary wave on the Union Canal near Edinburgh. They were attending a conference on nonlinear waves in physics and biology at Heriot-Watt University, near the canal.

The 'Wave of Translation' itself was regarded as a curiosity until the 1960s when scientists began to use modern digital computers to study non-linear wave propagation. Then an explosion of activity occurred when it was discovered that many phenomena in physics, electronics and biology can be described by the mathematical and physical theory of the 'soliton', as Scott Russell's wave is now known. This work has continued and currently includes modeling high temperature superconductors and energy transport in DNA, as well as in the development of new mathematical techniques and concepts underpinning further developments.

After a delay which would probably be unacceptable to present day funding bodies, and in a field he could never have dreamed of, Scott Russell's observations and research of 160 years ago have hit the big time in the present day fiber-optic communications industry. The qualities of the soliton wave which excited him (the fact that it does not break up, spread out or lose strength over distance) make it ideal for fiber-optic communications networks where billions of solitons per second carry information down fiber circuits for cable TV, telephone and computers ("The secrets of everlasting life", New Scientist 15 April 1995). It is fitting that a fiber-optic cable linking Edinburgh and Glasgow now runs beneath the very tow-path from which John Scott Russell made his initial observations, and along the aqueduct which now bears his name."



← Following up on his discovery, Scott Russell built a 30' wave tank in his back garden and made further important observations of the properties of the solitary wave.

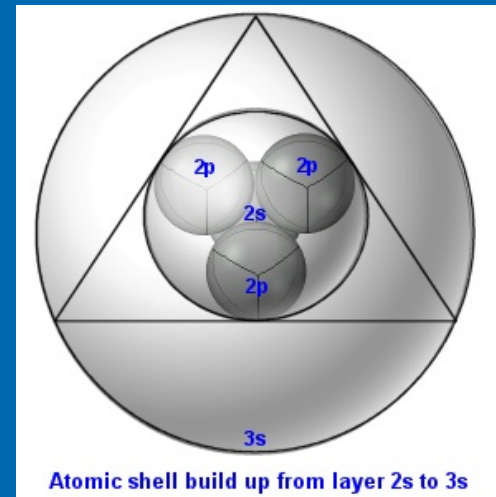
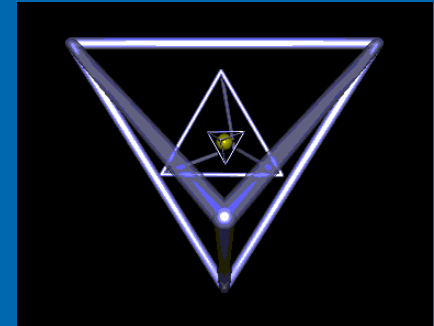
# SG203.3.9 Rotating Platonics Atom Model (1)

*"There are no Electron Orbits! Bohr's model, which started the notion of electrons traveling around the nucleus like planets has misled a lot of people and scientists... Instead, there is striking evidence that the atom structure is a standing wave... It is understood that everything that we apply for this shape will apply for the other four Platonics. Each spheres, one inscribed within its faces and one circumscribed by its vertices, as shown in the diagram. The inscribed sphere, will in turn be the circumscribed sphere of a smaller nested platonic structure, and so on, until a point is reached where the actual sides of the platonic equates to the smallest possible vibrating length in space, relating to Planck length.*

*The vertices of the internal nested platonic (the dual) will form at the centre of each face of the parent platonic. Curiously enough, this point is shown by dots on the 3,000 year old stones shown previously. This makes the inscribed sphere look very dense, in terms of standing wave structures. Unlike the conventional model, where the space between electron shells is described as a void and empty space, in our model it is the space in between the inscribed and circumscribed spheres, which contain the inward and outward going spherical waves forming up the 3D standing wave shape. Thus such a volume will be less opaque, and less dense than the standing wave shells. This volume, that is the volume trapped between the two spheres is what most call the 'electron cloud'. The internal inscribed sphere is as you might have guessed, what most call the nucleus. To reassure us of such an idea, we have to mention that one stable solution to Maxwell's equations is equivalent to a continuous standing electromagnetic wave arranged concentrically about a point.*

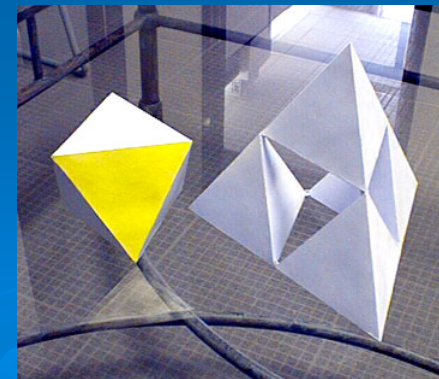
*It follows that the smallest entity which can have all characteristics of a particle should be one the simplest of the basic platonics described above. If this entity is unique, then it must be one whose dual is itself, and which has got its analogue existing in all dimensions.*

*There is just one platonic satisfying this criteria and this is the Tetrahedron (in 3D), called the Simplex in 4D." ([www.blazelabs.com](http://www.blazelabs.com))*

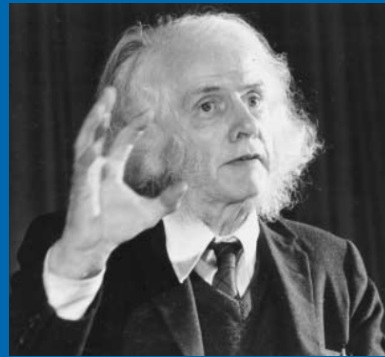
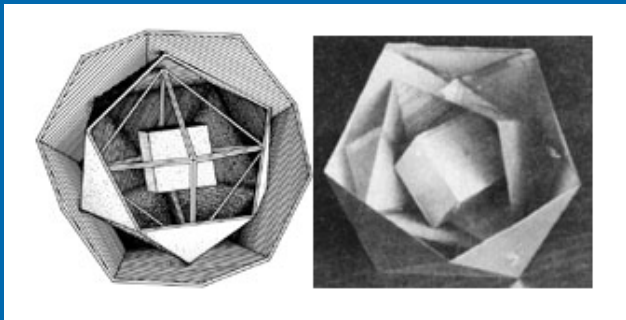


←↑  
Tetrahedral  
atomic models.  
(Blazelabs).

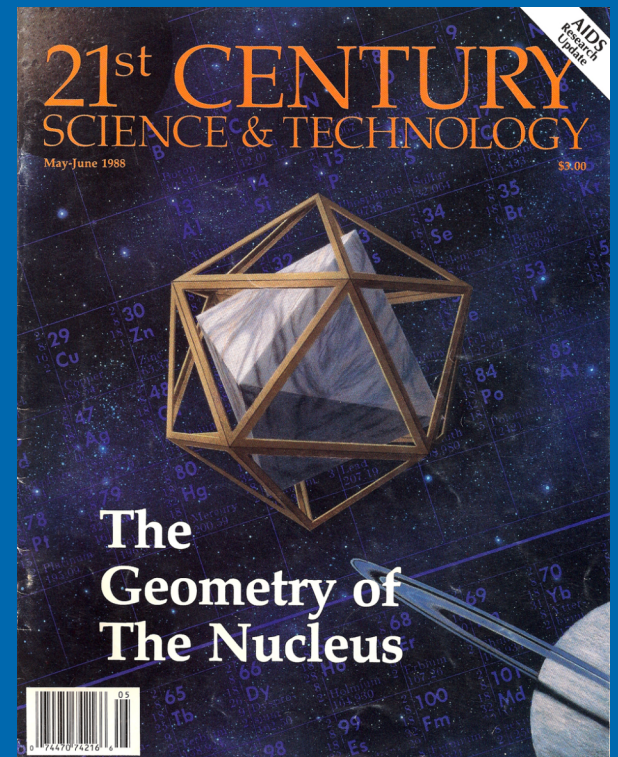
**At this point, we should realize how the macro and quantum worlds are easily unified when one considers the fact that the universe we live in, and of which we are part of, exists in a fractal hyper-dimension. All things we observe are just a small piece of this immense fractal function projected onto our 3D observation plane. When a fractal function 'separates' from another it is observed as a separate entity (in 3D), but actually each one of them still forms part of one unified function in higher dimension.**



## SG203.3.10.1 Atomic Nucleus - Moon Model (1)



↑ Dr. Robert Moon



The May-June 1988 issue of *21st Century Science & Technology* features a pioneering article by Laurence Hecht about the “Geometric Periodicity of the Elements”. In this and follow-up articles, Hecht highlights the nuclear model proposed by Dr. Moon. In the “Moon Model”, protons are considered to be located at the vertices of a nested structure of four of the five Platonic Solids.

Says Hecht: “*The five regular, or Platonic, solids are best conceptualized as the regular tilings of the surface of a sphere. They thus define a crucial boundary of what can be constructed in visual space. In nested arrangements, the solids and their implicit variations may represent a multiply connected manifold, which serves as a metaphor for the relationship of the individual to the whole universe in physical space. Construction of the solids, and exploration of their variations, has thus always been the foundation for creative work in science.*”

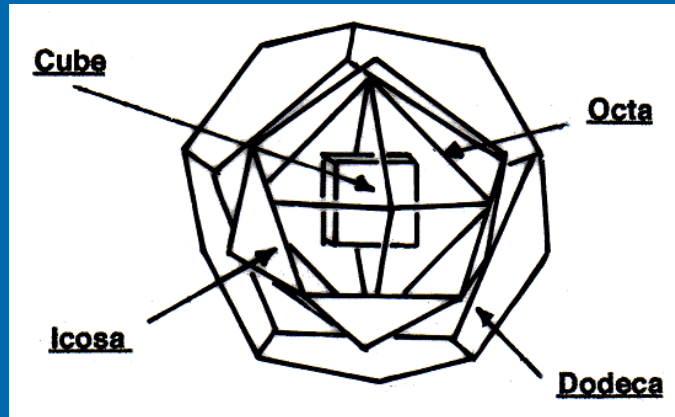
Around 1985, Dr Robert J. Moon (1911-1989) developed an atomic nucleus model based on the nesting structure of the Platonic Solids and inspired by Johannes Kepler’s conception of the solar system, as described in his work *Mysterium Cosmographicum*.

The “*Moon Model of the Nucleus*” considers protons to be located at the vertices of the solids. This model thus lays a geometric foundation for the periodicity occurring in the Table of the 92 naturally existing elements.

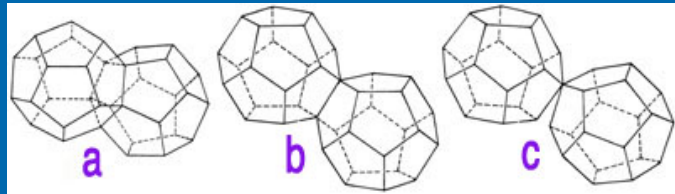
The **Moon Model** follows on the Sacred Geometry tradition pioneered by Plato and beautifully applied by Kepler to the solar system: the 5 regular solids explain the harmonic nature of the universe. For R. Moon, the “*vacuum*” has a quantized nested structure based on golden symmetries.

The next two pages present, in the words of Laurence Hecht, some aspects of the “Moon Model” as well as some advances recently proposed.

## SG203.3.10.2 Atomic Nucleus - Moon Model (2)



↑ The "Keplerian Atom" of Dr. Moon



↑ The Completed Uranium Atom

(a) To go beyond palladium (atomic number 46), which is represented by the completed dodecahedron, an identical dodecahedron joins the first one at a face. When the second dodecahedron is completed, it is seen that six positions on the common dodecahedral face are already occupied. This represents the nucleus of radon (atomic number 86).

(b) To go beyond radon, the twin dodecahedra open up, using a common edge as if it were a hinge.

(c) To create 91-protactanium, the hinge is broken at one end. To create 92-uranium, the position where two protons join must be slightly displaced, creating the instability which permits fission.

*"In the atomic nuclear structure hypothesized by Dr. Robert J. Moon in 1986, protons are considered to be located at the vertices of a nested structure of four of the five Platonic solids .*

*8 protons, corresponding to the Oxygen nucleus, occupy the vertices of a cube which is the first nuclear "shell." 6 more protons, corresponding to Silicon, lie on the vertices of an octahedron which contains, and is dual to, the cube. The octahedron-cube is contained within an icosahedron, whose 12 additional vertices, now totalling 26 protons, correspond to Iron. The icosahedron-octahedron-cube nesting is finally contained within, and dual to, a dodecahedron. The 20 additional vertices, now totalling 46 protons, correspond to Palladium, the halfway point in the periodic table.*

*Beyond Palladium, a second dodecahedral shell begins to form as a twin to the first. After 10 of its 20 vertices are filled at Lanthanum (atomic number 56), a cube and octahedron nesting fill inside it, accounting for the 14 elements of the anomalous Lanthanide series.*

*Next, the icosahedron forms around the cube-octahedron structure, completing its 12 vertices at Lead (atomic number 82), which is the stable, end-point in the radioactive decay series. Finally the dodecahedron fills up, and the twinned structure "hinges" open, creating the instability which leads to the fissioning of uranium.*

*The Moon model is thus consistent with much of the same experimental data which underlies the periodic table of the elements, and explains additional features not explained by the modern, electron-configuration presentation of the periodic table: the completed "shells" of the Moon model, correspond to the elements whose stability is attested by their abundance in the Earth's crust: Oxygen, Silicon, and Iron.... Palladium, which is an anomaly in the modern electron-configuration conception of the periodic table - because it has a closed electron shell, but occurs in the middle of a period - is not anomalous in the Moon model.*

*However, the Moon model seems to be inconsistent with the evidence from spectroscopy (upon which the electron-configuration conception rests) which suggests the periods of 2, 8, 18, and 32; it is also not consistent with the older "law of octaves," which was developed to explain the phenomena of chemical bonding, and was subsumed in Mendeleyev's conception."*

(All quotes from the article by Laurence Hecht:  
*Advances in Developing the moon Model.*  
 21st Century Science & Technology. Fall 2000.)

# SG203.3.10.3 Atomic Nucleus - Moon Model (3)

Note: this page points to advances made on the Moon model presented by Laurence Hecht, between the publication of the original article (1988) and 2009. We are only able to give some pointers here: for detailed technical information, go to the link below. Quotes & diagrams by L. Hecht.

## Octaves in the Moon model:

*"When the stable isotopes are grouped according to the four Platonic solid shells, as described by the Moon model of the nucleus, the number of neutrons is found to fall in octaves.*

*Thus, for the first 135 stable isotopes:*

*$^1\text{H}$  to  $^{15}\text{N}$ , number of neutrons = 0 to 8  
 $^{16}\text{O}$  to  $^{30}\text{Si}$ , number of neutrons = 8 to 16  
 $^{31}\text{P}$  to  $^{58}\text{Fe}$ , number of neutrons = 16 to 32  
 $^{59}\text{Co}$  to  $^{110}\text{Pd}$ , number of neutrons = 32 to 64.*

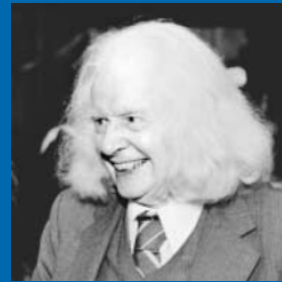
*The Moon structure of the nucleus thus defines a periodicity in the gross ordering of the stable isotopes, somewhat analogous to the periods of the Mendeleev table of the elements, but based on an implicit musical harmony."*

## The Ratio 1:137

*"Noting first that the presence of an impedance in so-called free space implied the existence of some sort of structure, Moon considered the fact that the ratio of the maximum Hall resistance (25,812 ohms) found in super-cooled thin-layer semiconductors, to the impedance of free space (376 ohms), was almost precisely 137/2.*

*Moon supposed that the configuration of the electrons in free space was related to the configuration of the nucleus. In the Moon model of the nucleus, the vertices provided by a nesting of cube-octahedron-icosahedron-dodecahedron are the resting place of 46 protons. Two dodecahedra then combine to form the structure for the 92 naturally occurring elements of the periodic table. In Moon's conception for the electrons in free space, three of the nested dodecahedra come together, providing 137 positions (138 minus one at the point of joining) for the electrons."*

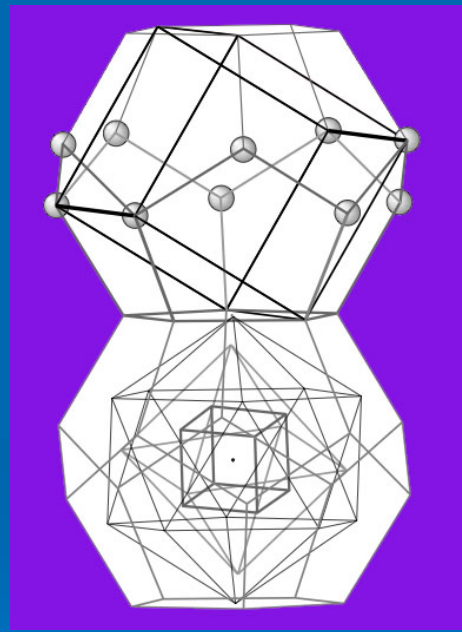
Go to this page for all the documents pertaining to the ongoing development of the Moon model:  
<http://www.21stcenturysciencetech.com/moonsubpg.html>



↑ Robert Moon. 1986.

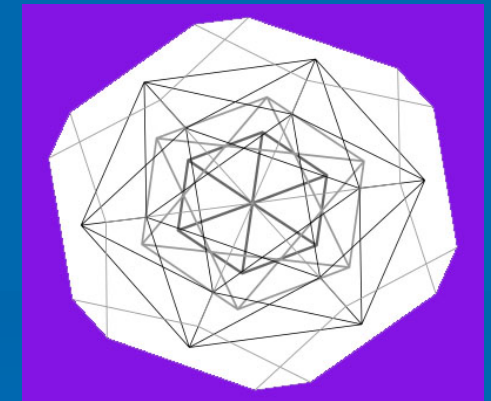


↑ Laurence Hecht delivering a lecture on the Moon model.



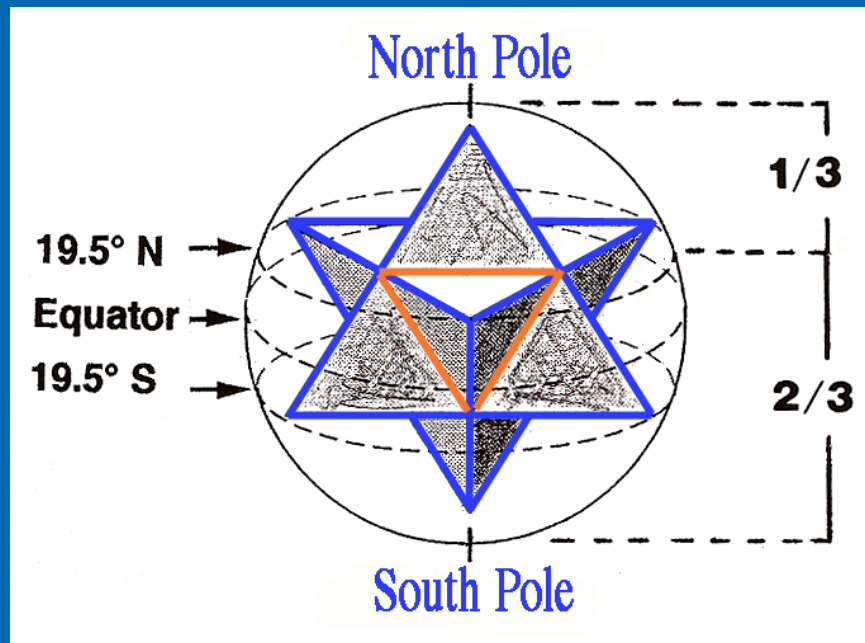
## ↑ The 'Salad Bowl' model

"The stability of the salad bowl is created by the cube implicit within the dodecahedron. Note the two bold edges of the cube, which also connect pairs of vertices of the salad bowl. Current elements moving along the dodecahedral diagonals between these vertices will experience a zero-force in one direction. The same occurs for other pairs on the salad bowl, creating a stable structure."



↑ "The 'Axis of the Universe' passes through the center of this figure. It is a diagonal axis for the dodecahedron and cube, and a face-centered axis of the icosahedron and octahedron. The unique pair of parallel faces of the icosahedron and octahedron is seen head on. (Note the hexagonal quality of the four solids in this view)."

## SG203.3.11.1 Tetrahedral Physics (1) - The Sphere



- The Tetrahedron is the only 3D volume whose corners (apices) are at the same distance from each other. There is no other volume with less than 4 corners.
- The Tetra has a special relationship with the Sphere: the sphere packs the most volume in the least surface area, while the Tetra packs the least volume with the greatest surface area.
- When the diameter of the circle is 12, the ratio of the area of the sphere to the tetra is 2 to 1. If a larger Tetrahedron has an inscribed sphere touching the centers of its 4 faces, then the characteristics of this superscribed tetra are 3 times those of the tetra inscribed in the sphere. Its height is twice the diameter of the sphere.
- When standing with one apex at the South pole (or the North pole), the Tetra divides the Sphere into 1/3 and 2/3.
- When the Tetra is placed within a rotating sphere with one apex at the North or South pole, the other 3 apices will lie at 19°.47, i.e. about 19.5° from the equator.
- The dance between the Tetra and the Sphere is basic to Buckminster Fuller's work (the "*isotropic vector matrix*"), as a mathematical blueprint for the universe.
- The 7 symmetry spin axes of the Tetra, as a primary field of form, have been correlated by Dan Winter to the 7 layers of the heart muscle [[◆SG204](#)] and the 7 regions of the rainbow donut.



## SG203.3.11.2 Tetrahedral Physics (2) - Solar System

The Tetra-Sphere geometry has been found in locations as diverse as Teotihuacan, Mexico and the Cydonia site on Mars. It is an integral part of Tetrahedral Physics and what is now called Hyperdimensional Physics. This geometry ( $19.5^\circ$ ) is also appearing throughout the solar system as a primary planetary grid structure. Consider the following:

Energy upwelling at tetrahedral latitudes ( $19.5^\circ$ ) in the solar system:

SUN: sunspot activity and region of peak temperature limited to  $19.5^\circ$  North and South.

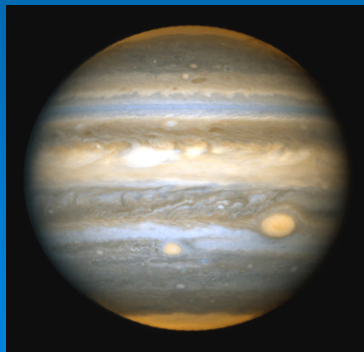
EARTH: largest cone volcano at  $19.5^\circ$  (Mauna Kea, HI).

VENUS: volcanic complexes Alpha and Beta Regio.

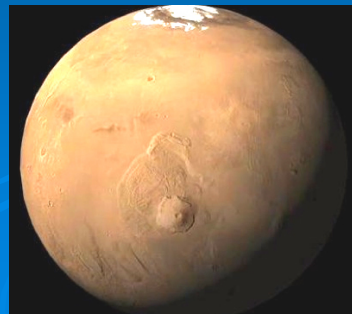
MARS: Olympus Mons cone volcano at  $19.5^\circ$  North.

JUPITER: "Red Spot" vortex.

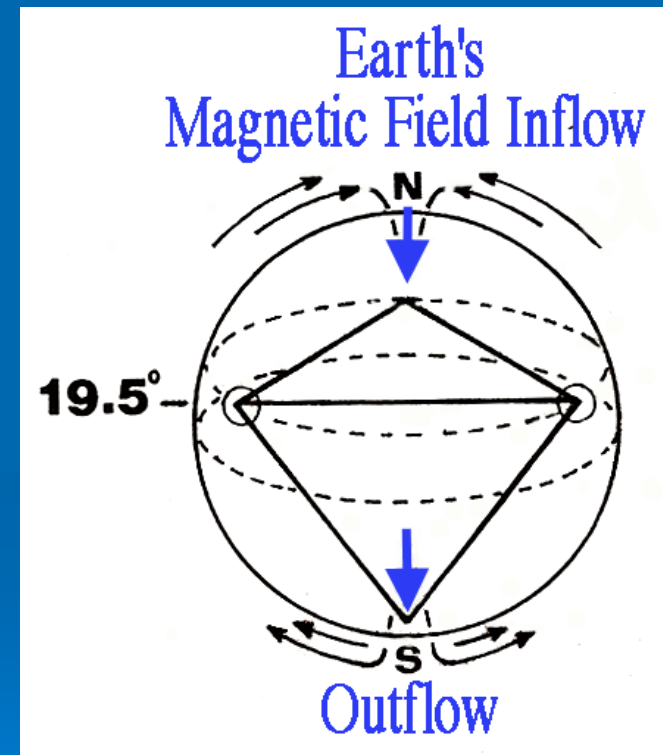
NEPTUNE: Voyager II discovered a vortex point at  $19.5^\circ$ .



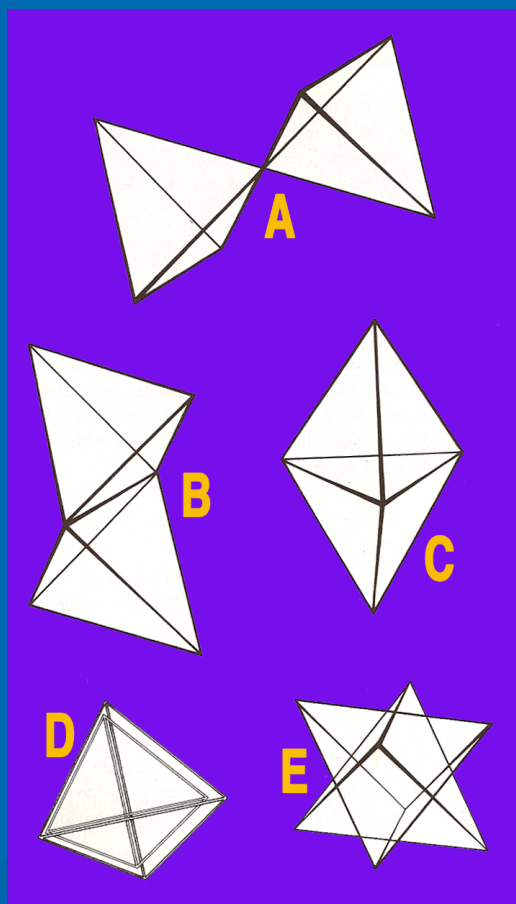
← Jupiter's Red Spot



← Olympus Mons on Mars

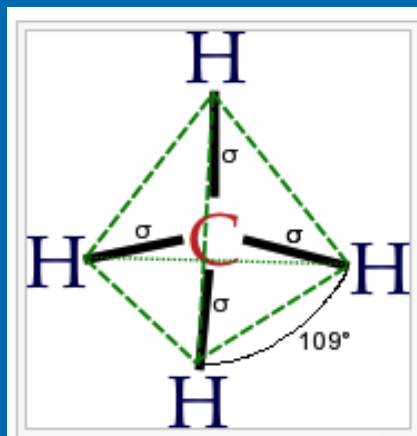


## SG203.3.12.1 Platonic Chemistry (1) Tetrahedral



### ↑ Tetrahedral Chemical Bonding

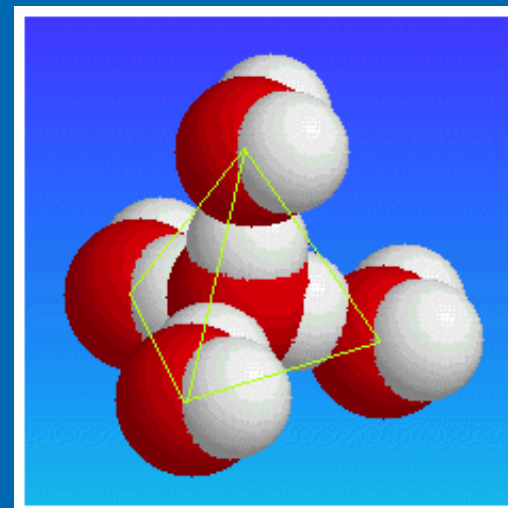
- A. Single-bonded tetra (gases)
- B. Double-bonded tetra (liquids)
- C. Triple-bonded tetra (crystals)
- D, E Quadri-bond / mid-edge tetra systems demonstrate the super strength of diamond & metals.



↑ The tetrahedral molecule Methane ( $\text{CH}_4$ )

In a tetrahedral molecular geometry a central atom is surrounded by four substituents that are located at the corners of a tetrahedron. The bond angles are  $\approx 109.5^\circ$  when all four substituents are the same, as in  $\text{CH}_4$ .

This molecular geometry is common throughout the first half of the periodic table. Tetrahedral molecules can also be chiral (left or right-handed).



↑ Tetrahedral structure of water molecules

The hydrogens on each water molecule point toward the oxygens on neighboring water molecules, such that the whole structure is based around a tetrahedral shape. Although since this is a liquid, the molecules are continually moving and rotating, breaking and reforming hydrogen bonds, so the tetrahedral structure can be thought of as only a time averaged approximation.

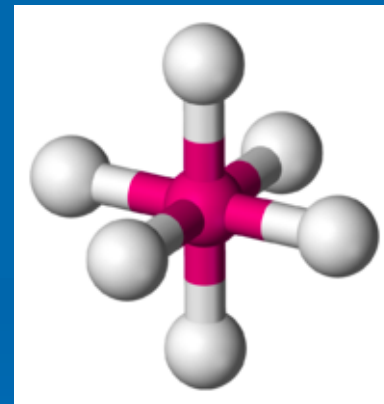
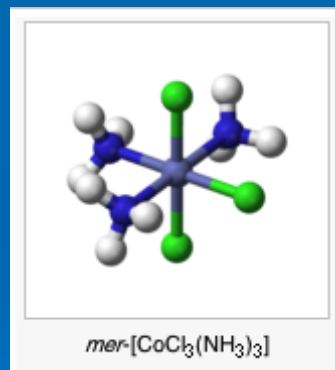
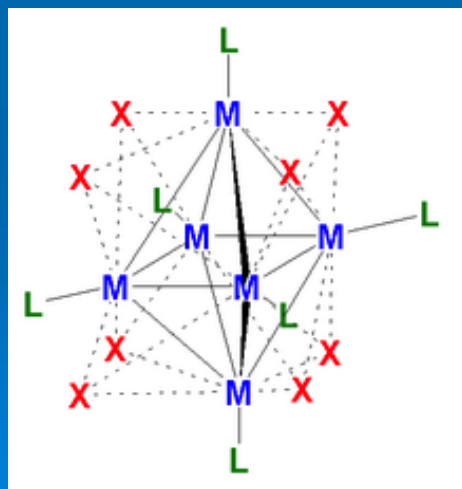
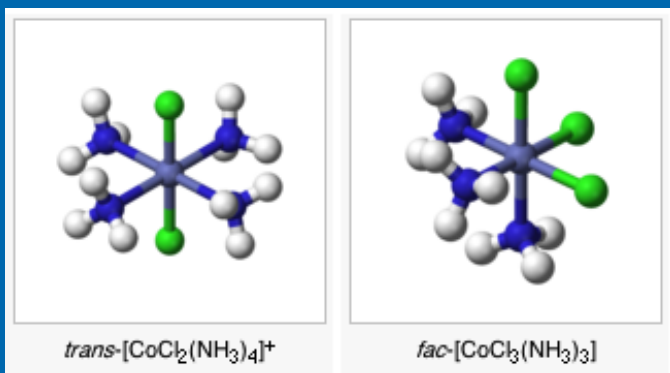
[www.ch.ic.ac.uk](http://www.ch.ic.ac.uk)

## SG203.3.12.2 Platonic Chemistry (2) Octahedral

In chemistry, octahedral molecular geometry describes the shape of compounds in six atoms (or groups of atoms) that are symmetrically arranged around a central atom, defining the vertices of an octahedron.

Examples of octahedral compounds are sulfur hexafluoride  $\text{SF}_6$  and molybdenum hexacarbonyl  $\text{Mo}(\text{CO})_6$ . A virtually uncountable variety of octahedral complexes exist with a wide variety of reactions.

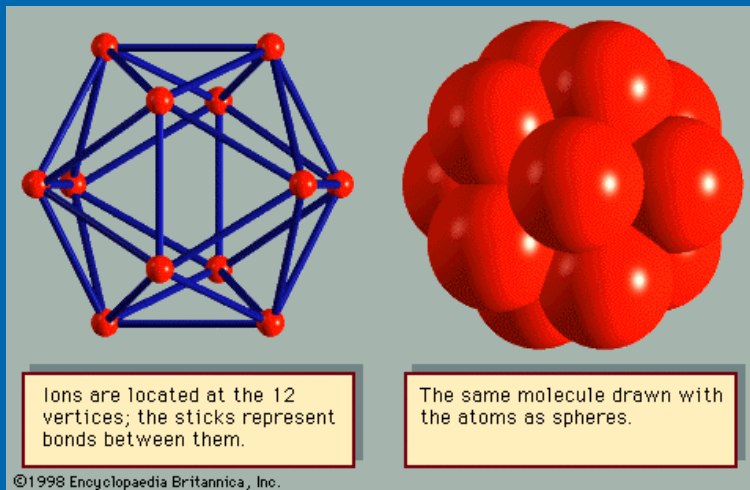
The concept of octahedral coordination geometry was developed by Alfred Werner to explain the *stoichiometries* (quantitative relationships that exist among the reactants and products) and *isomerism* (same molecular formula but different structural formulas) in coordination compounds. His insight allowed chemists to rationalize the number of isomers of coordination compounds. (Wikipedia)



↑ Idealized structure of a compound with octahedral coordination geometry.

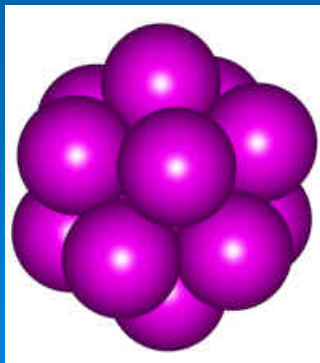
← Octahedral clusters are inorganic or organometallic cluster compounds composed of six metals in an octahedral array. Important classes of octahedral clusters are *chalcogenide* and *molybdenum* clusters.

# SG203.3.12.3 Platonic Chemistry (3) Icosahedral (1)

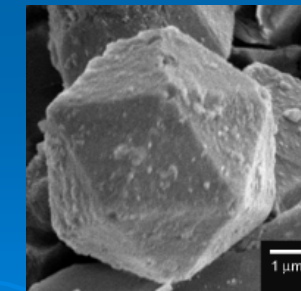


↑ Icosahedral arrangement of the boron molecule

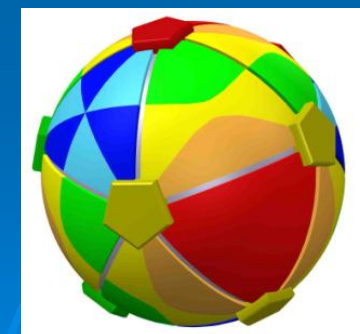
[♦SG203A chapter 2 for the icosahedral water clusters]



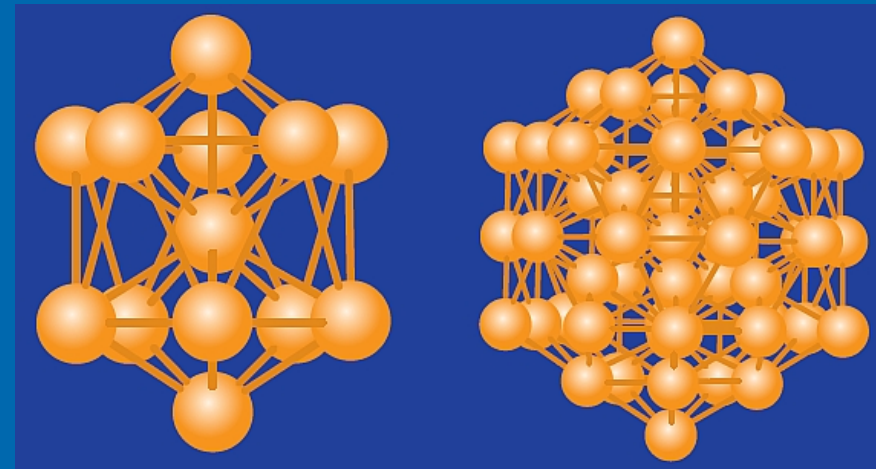
← An icosahedral cluster of thirteen identical spherical atoms as found in liquid argon, krypton, xenon and molten metals. Such five-fold symmetry is optimal for short-range close packing. Its preferred formation has been shown to prevent crystallization in liquid metal melts and be the cause of their extensive super cooling.



↑ Mesoporous silica crystal



↑ Icosa-Dodeca model mapping the 64 DNA codons.  
[//www.codefun.com](http://www.codefun.com)



↑ Icosahedral gold clusters with 13 (left) and 55 atoms (right)  
Note: 13 & 55 are Fibonacci numbers. [//www.scidacreview.org](http://www.scidacreview.org)

Interestingly the Golden ratio also appears in aqueous chemistry as the ratio between atomic and ionic diameters. Thus the diameter of an anion ( $A^-$ ) is twice its atomic diameter divided by Phi and the diameter of a cation ( $A^+$ ) is twice its atomic diameter divided by Phi<sup>2</sup>; with the diameter of  $A^-$  being the golden ratio times the diameter of  $A^+$ , and simple functions of F also relating ion-water distances to covalent radii. [♦SG203B - Research of R. Heyrovska]

Plato certainly was on the right track when connecting the WATER element or liquid structure in general to icosahedra: spherical atoms and molecules (for example, the larger noble gases) in the liquid phase prefer icosahedral clustering which has a lower energy than crystal structures.

# SG203.3.12.4 Platonic Chemistry

## (4) Icosahedral Viruses

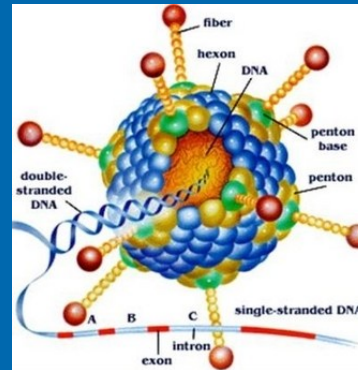
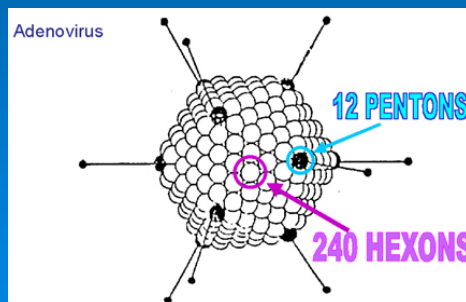
Since the initial discovery of the *tobacco mosaic virus* by Martinus Beijerinck (1898), about 5,000 viruses have been described in detail, although there are millions of different types. Viruses are found in almost every ecosystem on Earth and are the most abundant type of biological entity and one of the largest genetic pools on Earth. They also are a natural mean of transferring genes between different species, thus increasing genetic diversity. It is thought that viruses played a central role in the early evolution, before the diversification of bacteria, archaea and eucaryotes and at the time of the *last universal common ancestor* of life on Earth.

An intriguing fact about viruses is that the large majority of them display FULL ICOSAHEDRAL SYMMETRY, the most esthetically-pleasing symmetry shown in nature. Why should it be so? Beauty hiding lethality? Or a more complex story...

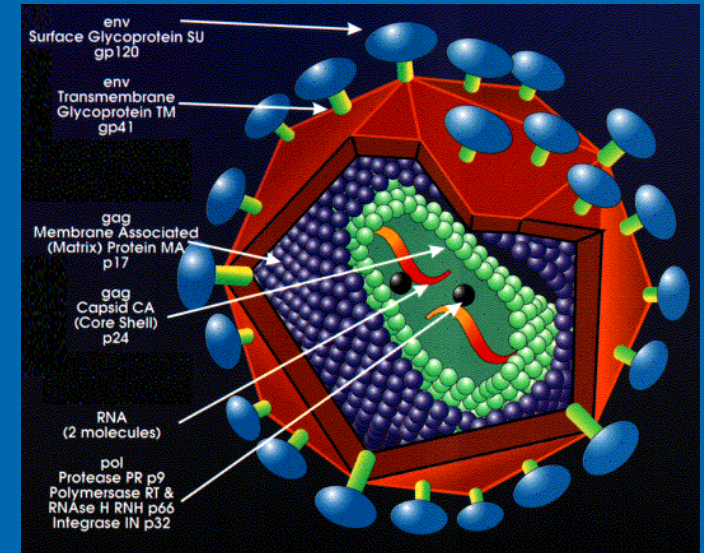
The elements of icosahedral symmetry involve 6 five-fold rotation axes, 10 three-fold, and 15 two-fold.

Viruses. "Most animal viruses are icosahedral or near-spherical with icosahedral symmetry. A regular icosahedron is the optimum way of forming a closed shell from identical sub-units. The minimum number of identical capsomers required is twelve, each composed of five identical sub-units. Many viruses, such as rotavirus, have more than twelve capsomers and appear spherical but they retain this symmetry. Capsomers at the apices are surrounded by five other capsomers and are called pentons. Capsomers on the triangular faces are surrounded by six others and are called hexons." (Wikipedia)

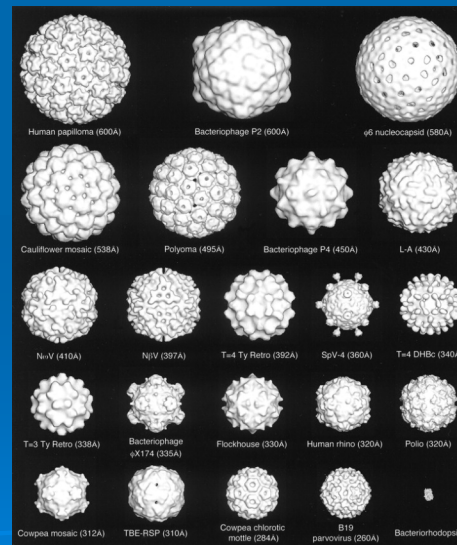
[//pathmicro.med.sc.edu](http://pathmicro.med.sc.edu)



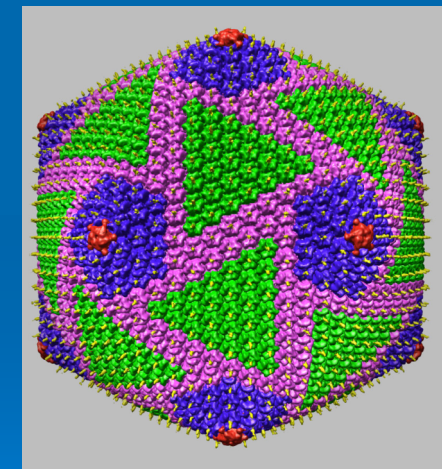
↑ Virus with 12 antenna-like projections.



↑ Structure of a retrovirus [//pathmicro.med.sc.edu](http://pathmicro.med.sc.edu)



↑ Icosahedral shapes of common viruses [//virus.chem.ucla.edu/](http://virus.chem.ucla.edu/)



↑ UCSF image of the CIV capsid color-coded to show the 12 pentasymmetrons (blue and red) and 20 trisymmetrons (pink and green) in the icosahedral surface lattice. [//www.cgl.ucsf.edu](http://www.cgl.ucsf.edu)

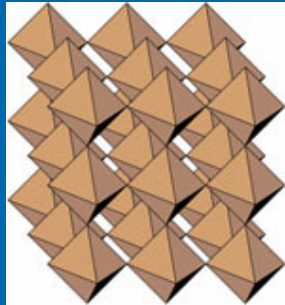
## SG203.3.13.1 Cosmic Harmonics (1) Galactic Octahedra

In the universe, large scale structures are characterized by a noticeable regularity and periodicity suggesting a network of filaments... In 1997, when taking magnetic fields into account, Battaner & Florido have come to the following conclusion :

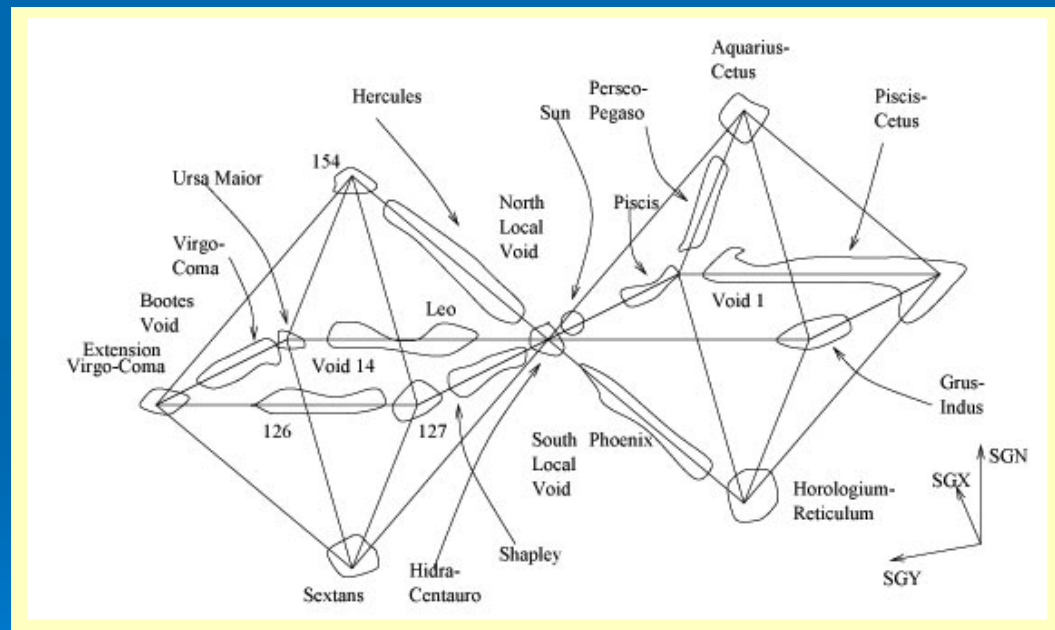
*The filament network, if magnetic in origin, must be subject to some magnetic restrictions. The simplest lattice matching these restrictions is an "egg-carton" network, formed by octahedra joining at their vertexes. This "egg-carton" universe would have larger amounts of matter along the edges of the octahedra, which would be the sites of the super-clusters. Outside the filaments there would be large voids..."*

These theoretical speculations are compatible with present observations of the large scale structure as delineated by the distribution of superclusters. It is easy to identify at least four of these giant octahedra in real data, which comprise observational support for the egg-carton universe. Two of them, those which are closest and therefore most unambiguously identified, are seen below (right).

(Data & images from: [nedwww.ipac.caltech.edu](http://nedwww.ipac.caltech.edu))



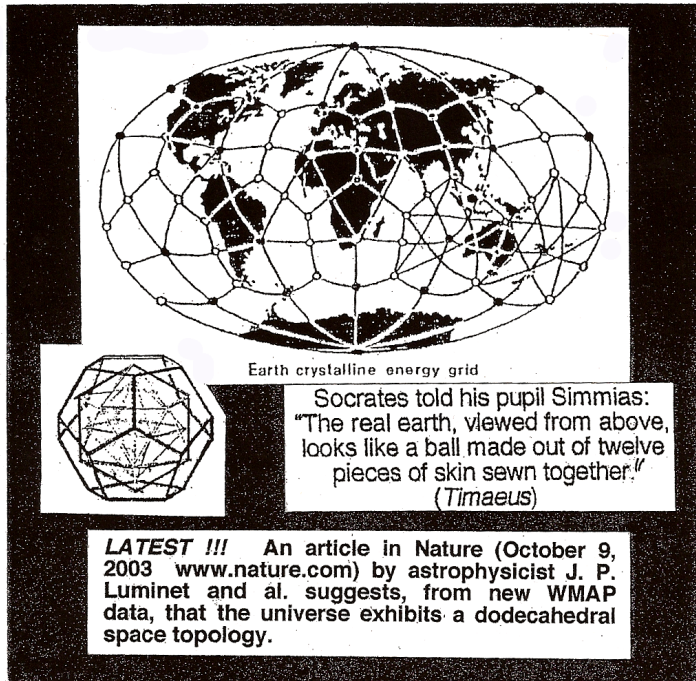
↑ Ideal scheme of the "egg-carton universe" formed with octohedra only contacting at their vertexes. (Battaner and Florido)



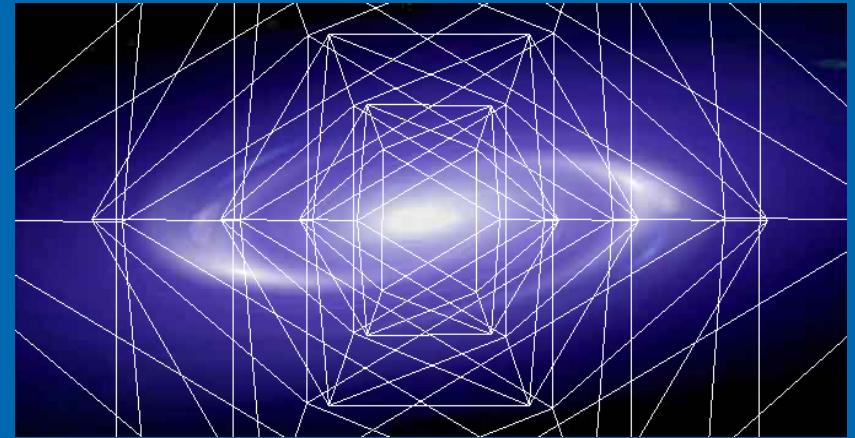
↑ The two closest octahedra of our local super-cluster, seen from our sun's position near their meeting point. The super-clusters, which may each contain billions of galaxies, are found along the edges and at the corners of the shapes, and relative voids are indeed found within the shapes. (From Battaner & Florido)

*"A fractal nature could be compatible with the octahedron web, in agreement with the identification of fractals by Lindner et al. (1996) from the observational point of view. There could be sub-octahedra within octahedra, at least in a limited range of length scales."*

## The Geometry of the Universe



## SG203.3.13.2 Cosmic Harmonics (2) Universal Dodeca?



The question of the *'shape and structure of the universe'* is an age-old quest. Explanations have come and gone, from descriptions of shamanic journeys to string theory...

In October 2003, new data about the cosmic background radiation brought by NASA's Wilkinson Microwave Anisotropy Probe (WMAP) may hint at a possible answer along the line of ancient Sacred Geometry: the universe is finite and resembles a dodecahedron. A team of cosmologists (Luminet, Weeks and al.) came to that model after careful measurements of the WMAP data. The density fluctuations of the cosmic background radiation can tell a lot about the physical shape & structure of space. The conclusion was that the mathematics add up nicely to support a finite dodecahedral topology. The title of the article submitted to *Nature* (425, 2003, 593) is: *Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background*.

Mathematician George Ellis wrote: "*Can this theory be confirmed? Yes, indeed*", explaining that WMAP's successor will provide even more precise key data on the cosmic background radiation that will confirm or disprove Weeks & colleagues' theory.

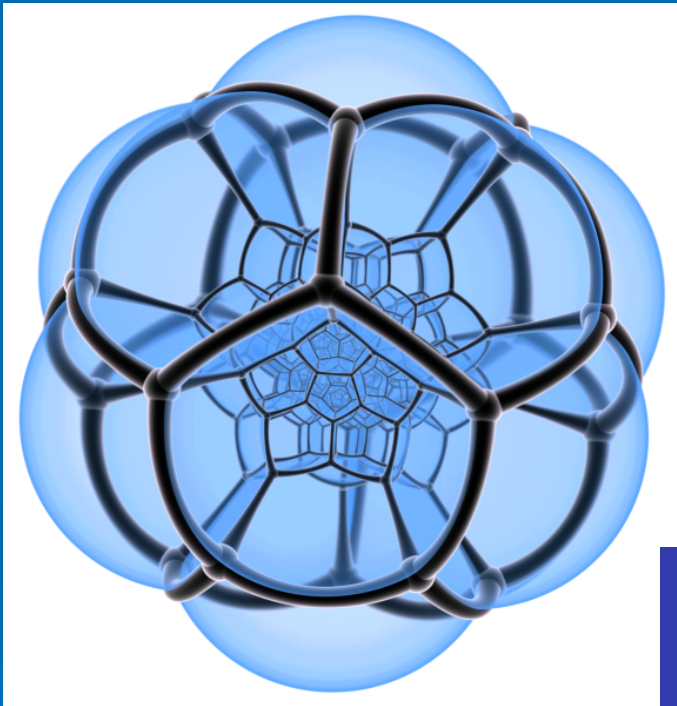
The question has been asked: is the entire Cosmos in evolutionary flux and possibly shifting from simpler Platonic grid-structures to more complex ones: from an octahedron model to a hyper-dodecahedron matrix? How would that affect us?

## SG203.3.14 Hyper Platonics

In mathematics, a convex regular 4-polytope (or *polychoron*) is 4-dimensional polytope which is both regular and convex. These are the 4D analogs of the Platonic solids (in 3D) and the regular polygons (in 2D).

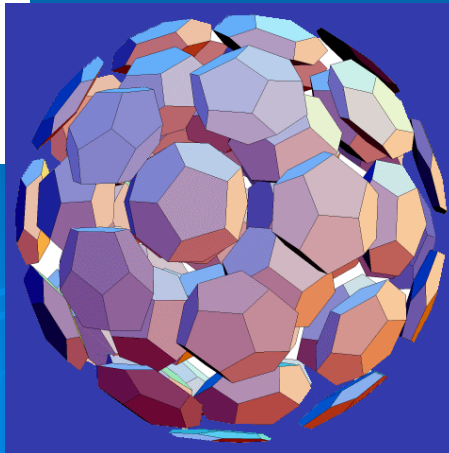
These polytopes were first described by the Swiss mathematician Ludwig Schläfli in the mid-19th century. Schläfli discovered that there are precisely six such figures. Five of these may be thought of as higher dimensional analogs of the Platonic solids. There is one additional figure (the 24-cell) which has no 3D equivalent.

Each convex regular 4-polytope is bounded by a set of 3-dimensional *cells* which are all Platonic solids of the same type and size. These are fitted together along their respective faces in a regular fashion. [♦SG107 chapter 7]. Below the example of the 120-cell. These higher-dimensional Platonics will shape many scientific models and soon become household items.

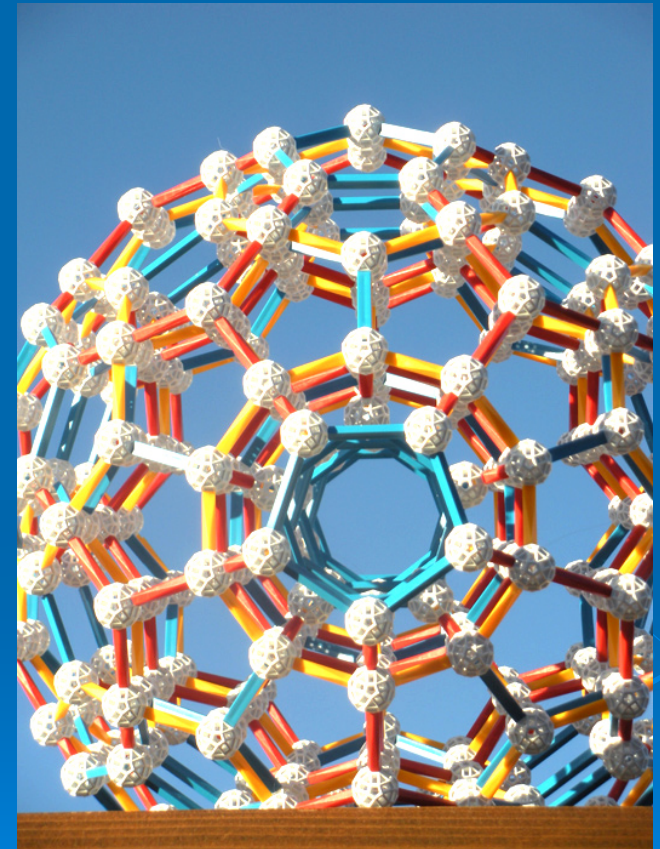


↑ Stereographic projection of the 120-cell or *hyper-dodecahedron*.  
(Wikipedia)

The "120-cell"  
is the 4D  
analog  
of the 3D  
Dodecahedron



→ 120-cell  
"exploded" view  
[//weimholt.com](http://weimholt.com)

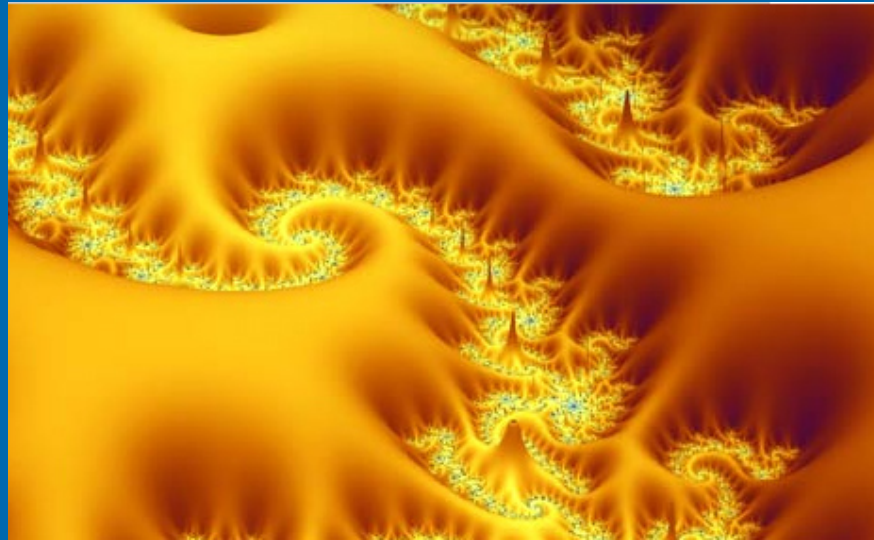


↑ 120-cell model built  
with Zometools.



## SG203A.Ca Conclusion to Part A

In the first 3 chapters of SG203, we have been exploring fractals & self-similarity, looking at newly discovered Phi-based penta-symmetries, and pointing to the re-emergence of the Platonic solids in many research areas.



↑ 3D Mandelbrot fractal.  
[www.miqel.com](http://www.miqel.com)

The story of this New Paradigm Science, as rediscovering & extending ancient knowledge,  
is continued in Part B of the SG203 module:

Chapter 4: Fibonacci/Lucas & PHI  
Chapter 5: Vortex Science  
Chapter 6: A Golden Matrix Universe

SG203A.Cb StarWheel Blessing



SW#69. "Pent-Atman" . [www.starwheels.com](http://www.starwheels.com)

# **SG203A. Cc Online SG School Curriculum: Intro & Intermediate**

## **Sacred Geometry Introductory Level: 8 Modules**

<b>SG 101</b>	<b>Intro I</b>	<b>Sacred Geometry: Universal Order &amp; Beauty</b>
<b>SG 102</b>	<b>Intro II</b>	<b>History &amp; Traditions of Sacred Geometry</b>
<b>SG 103</b>	<b>Intro III</b>	<b>Sacred Geometry: A Grand Tour</b>
<b>SG 104</b>	<b>Intro IV</b>	<b>PHI: the Golden Ratio &amp; the Fibonacci Series</b>
<b>SG 105</b>	<b>Intro V</b>	<b>The Golden Rectangle &amp; Golden Spiral</b>
<b>SG 106</b>	<b>Intro VI</b>	<b>Pentagons, Pentagrams &amp; the Penta-Modules</b>
<b>SG 107</b>	<b>Intro VII</b>	<b>The Five Platonic &amp; 13 Archimedean Solids</b>
<b>SG 108</b>	<b>Intro VIII</b>	<b>The Vesica Piscis: Cosmic Womb of Creation</b>

## **Sacred Geometry Intermediate Level: 8 modules**

<b>SG 201</b>	<b>Interm I</b>	<b>The Monochord, Music &amp; Cymatics</b>
<b>SG 202</b>	<b>Interm II</b>	<b>The Power of Archetypal Numbers</b>
<b>SG 203A</b>	<b>Interm IIIA</b>	<b>Sacred Geometry Resurgence in Science - Part 1</b>
<b>SG 203B</b>	<b>Interm IIIB</b>	<b>Sacred Geometry Resurgence in Science - Part 2</b>
<b>SG 204</b>	<b>Interm IV</b>	<b>PHI in the Human Body, Biology &amp; DNA</b>
<b>SG 205A</b>	<b>Interm VA</b>	<b>The SG of Nature - Part 1: Plants &amp; Phyllotaxis</b>
<b>SG 205B</b>	<b>Interm VB</b>	<b>The SG of Nature - Part 2: Animals &amp; Minerals</b>
<b>SG 207</b>	<b>Interm VII</b>	<b>SG in Architecture, Sacred Sites &amp; Green Design</b>

## SG203A. Cd Online SG School Curriculum: Advanced

### Sacred Geometry Advanced Level: 8 modules

SG 301	Adv I	Golden Cosmos: Planets, Stars & Cosmology
SG 302	Adv II	SG in Art, Culture & Creativity
SG 303	Adv III	Universal Symbols: Primordial Knowledge
SG 304	Adv IV	Labyrinths: a Mini-Pilgrimage to Self
SG 305	Adv V	Mandalas & Yantras: Sacred Vortices
SG 306	Adv VI	Languages & Gematrias: Sacred Communication
SG 307	Adv VII	Sacred Geometry in the Healing Arts
SG 308	Adv VIII	Harmony on Earth: Science & Consciousness of Harmony

*Upon completion of each level (Introductory, Intermediate & Advanced), a Certificate of Graduation from the Sedona School of Sacred Geometry will be presented to Certification Students.*

*Postgraduate seminars on current Sacred Geometry research, discoveries & updates will be organized in the harmonic future.*

*Questions:* [phi@schoolofsacredgeometry.org](mailto:phi@schoolofsacredgeometry.org)



## SG203A. Ce Contact Info

**Sedona School of Sacred Geometry**  
[www.schoolofsacredgeometry.org](http://www.schoolofsacredgeometry.org)  
[phi@schoolofsacredgeometry.org](mailto:phi@schoolofsacredgeometry.org)  
PO Box 3714, Sedona, AZ 86340

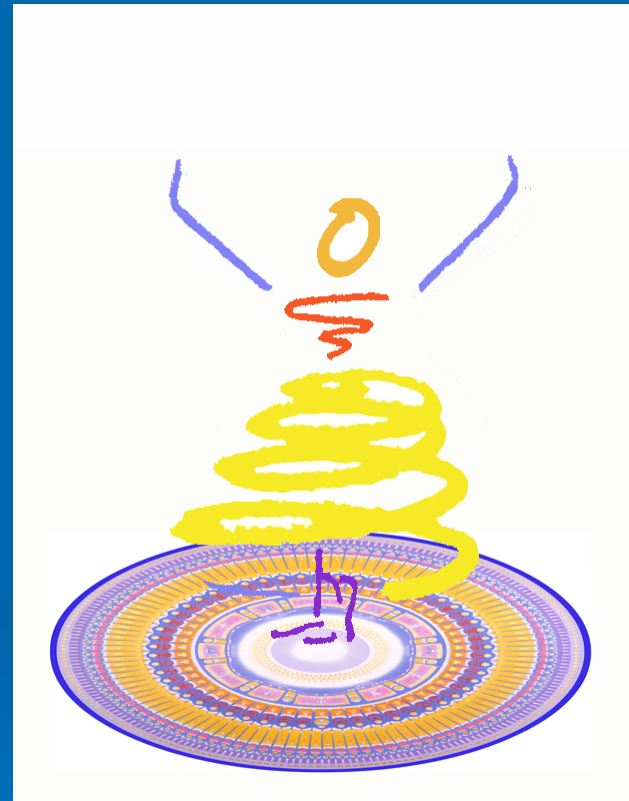
**StarWheel Mandalas by Aya**  
[www.starwheels.com](http://www.starwheels.com)  
[www.starwheels.com/infopage.php?pagename=starwheelgallery](http://www.starwheels.com/infopage.php?pagename=starwheelgallery)  
[aya@starwheels.com](mailto:aya@starwheels.com)

**Our non-profit: [www.starwheelfoundation.org](http://www.starwheelfoundation.org)**  
[www.starwheelfoundation.org/index.php?p=globalecocampus](http://www.starwheelfoundation.org/index.php?p=globalecocampus)  
[www.starwheelfoundation.org/index.php?p=acroyoga](http://www.starwheelfoundation.org/index.php?p=acroyoga)  
[www.starwheelfoundation.org/index.php?p=poona1hbooks](http://www.starwheelfoundation.org/index.php?p=poona1hbooks)  
[www.starwheelfoundation.org/index.php?p=treesponsorship](http://www.starwheelfoundation.org/index.php?p=treesponsorship)

**Our online store: [www.starwheelmandalas.com](http://www.starwheelmandalas.com)**  
[www.starwheelmandalas.com/index.php?p=originals](http://www.starwheelmandalas.com/index.php?p=originals)  
[www.starwheelmandalas.com/index.php?p=wisdomcards](http://www.starwheelmandalas.com/index.php?p=wisdomcards)  
[www.starwheelmandalas.com/index.php?p=deck1](http://www.starwheelmandalas.com/index.php?p=deck1)



**On Facebook: Aya Sheevaya**  
**FB Group: Sedona School of Sacred Geometry**



Φ celebration



## SG203A. Cf About Aya

A native of France, Aya is a visionary artist and celebration yogi who has dedicated his life to serve humanity and to develop sacred arts education. In his late 20's, Aya realized that his professional life in the French diplomatic service was not fulfilling his heart's desires; he quit everything to go on an extended vision quest. His path took him around the world to visit a variety of sacred sites & cultures and to receive inspiration from many teachers.

In 1985, in Santa Monica, CA, Aya was gifted with a spiritual vision prompting him to create a series of 108 airbrushed neo-mandala paintings: the "StarWheels". The StarWheels, a happy family of vibratory flowers for the Earth, are looking for sacred spaces to be graced with their presence... ([www.starwheels.com](http://www.starwheels.com) / [www.starwheelmandalas.com](http://www.starwheelmandalas.com))

Moving to Sedona, Arizona, in 1997, Aya has been involved with sacred arts classes & events, mandala creation, Sedona guided tours, labyrinth making and Sacred Geometry teaching. Aya has presented several StarWheel art exhibits, has sponsored community awareness events at the Sedona Library, has developed, in collaboration with Gardens for Humanity, the Peace Garden arboretum at the Sedona Creative Life Center, was a speaker at the Sacred Geometry Conference (Sedona, 2004), co-designed several labyrinth sites (The Lodge at Sedona, Mago's Ranch...), and was on the management team of the Raw Spirit Festival in 2006 - 2008.

Realizing that Sedona was progressively becoming a global spiritual university for many seekers from around the world, Aya founded in 2005 the Sedona School of Sacred Geometry. The school is offering online access to Sacred Geometry PDF modules, with 17 modules completed so far. In the school's website, Aya states: *"We are living at the extraordinary and exciting times of a global transformation to a higher order of human consciousness... Sacred Geometry is the expression and resurrection of our deep innate wisdom, now awakening from a long sleep: seeing again the all-encompassing, fractal-holographic unity of nature, life and spirit... The keyword is HARMONY."* ([www.schoolofsacredgeometry.org](http://www.schoolofsacredgeometry.org))

Aya's visionary dream, supported by his non-profit educational organization, the StarWheel Foundation, is the co-creation of an international eco-village "The School of Celebratory Arts" - a green, tropical environment encouraging young people of all nations to develop their creative consciousness and thus contribute to a new, spirited, life-respecting global civilization on Earth. ([www.starwheelfoundation.org](http://www.starwheelfoundation.org)).

Since 2012, Aya is dancing the body divine, after his re-discovery of Yoga, Partner Yoga and AcroYoga. Aya is currently the AcroYoga.org Jam coordinator for Sedona and a teacher of yoga swing asanas.

Blessings in Anjali!