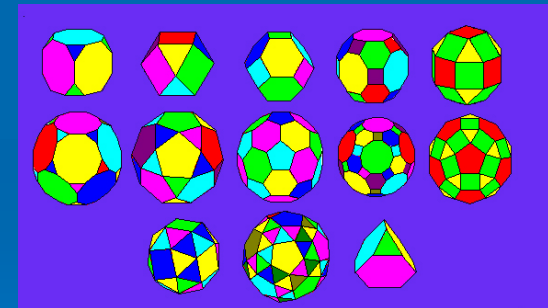
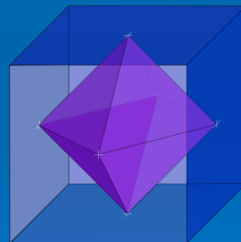
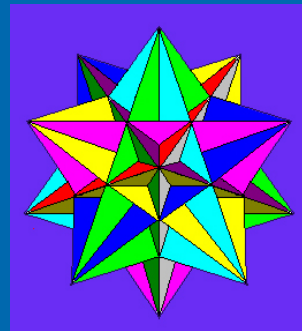
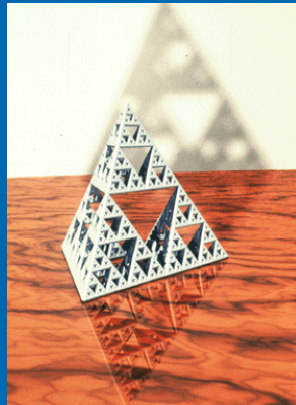
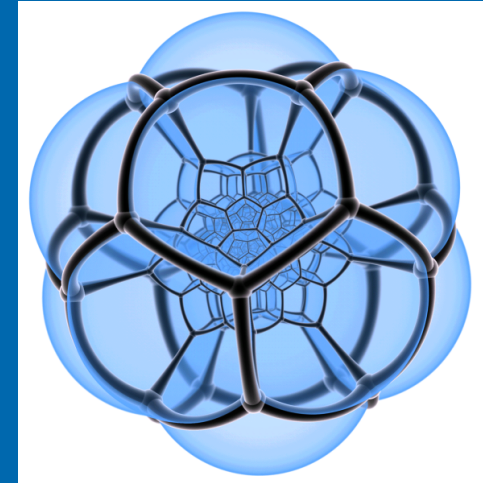
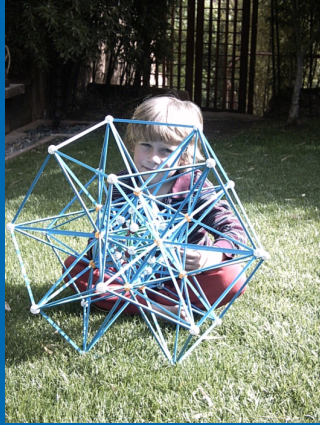


SG107.1a The 5 Platonic & 13 Archimedean Solids

Online Module SG107 Intro VII



Presented by AYA
& the School of Sacred Geometry
P. O. Box 3714
Sedona, AZ 86340

<http://www.SchoolOfSacredGeometry.org>
phi@schoolofsacredgeometry.org
www.starwheels.com

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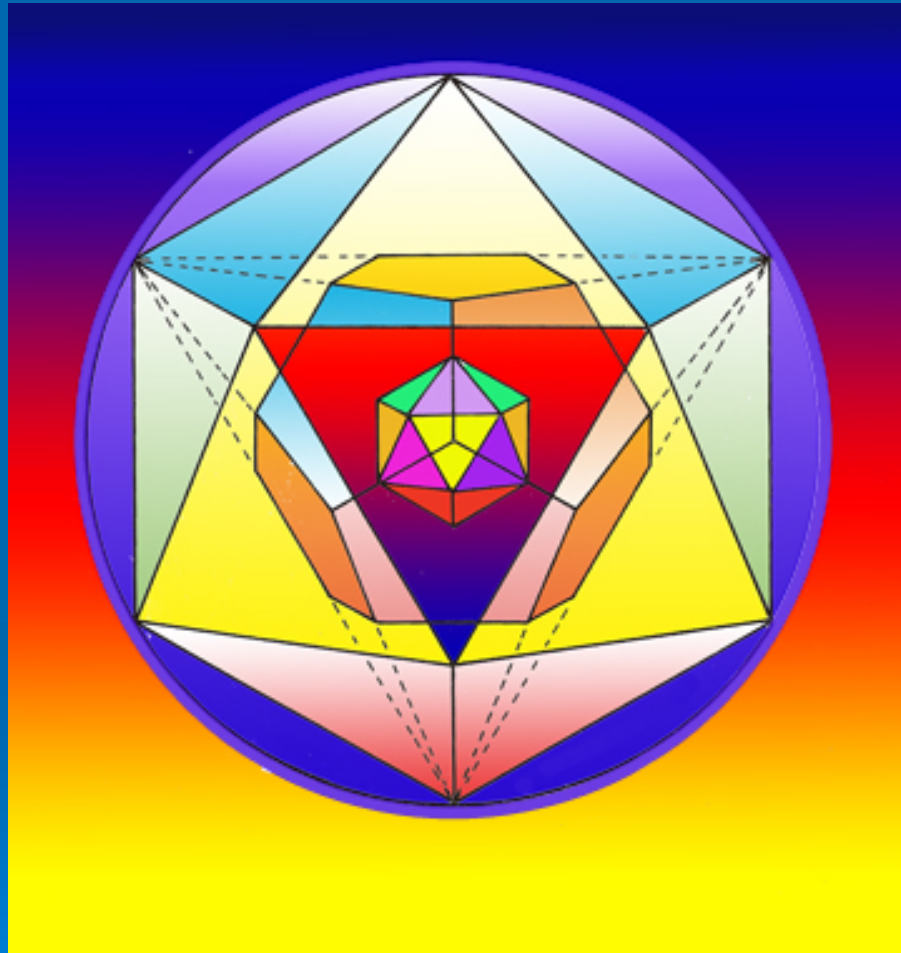
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SG107.1c Platonic & Archimedean Solids



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Kindly contact us
for omissions or corrections.*

phi@schoolofsacredgeometry.org

SG107.1d Platonic & Archimedean Solids - Intro

Welcome to the fun world of 3D Sacred Geometry!

The step up from 2D to 3D is an ascension in consciousness frequency. It is stepping out of Flatland and starting to fly and dance, as is the essence and destiny of consciousness.

As we become accustomed to perceive, see and visualize in 3D, we move from static to dynamic vision. We learn to move around these 3D shapes or enter them by modifying the angle and size of our perception point.

Much like “*avatars*” in virtual/digital reality scenes, we are starting to become part of the Play of Life.

And we realize, as quantum science discovered, that there is no such thing as a fixed “*object*”, whether it is our body or even a thought: nothing ever stops or freezes - except maybe where we focus our beliefs of limitation.

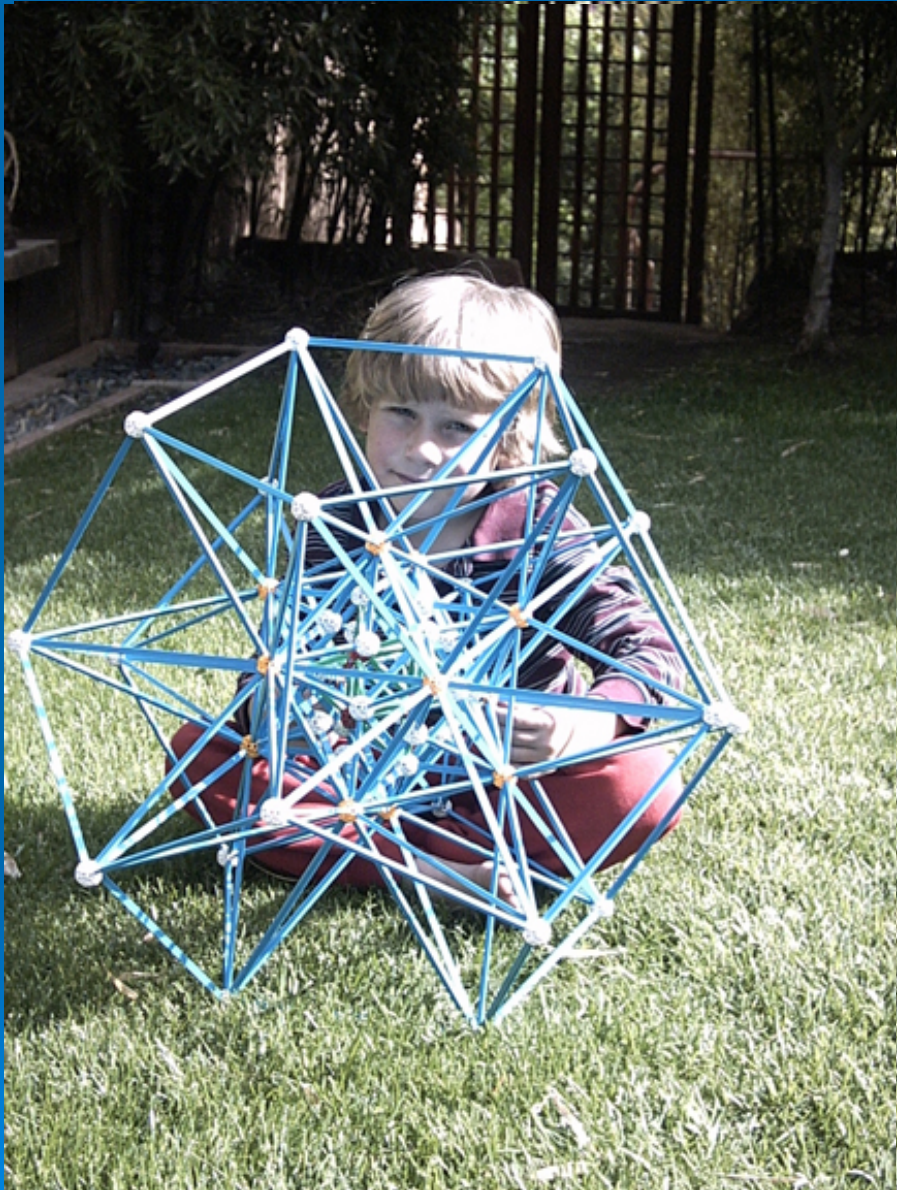
Platonic & Archimedean Solids are the alphabet of 3D consciousness, the building blocks of higher dimensional realities. By taking the time & effort to get familiar with them, we are learning an archetypal language of creation that can speak across continents, across space/time and across dimensions.

We are, in essence, upgrading and re-grooving our brains...

Watch out for and welcome a deep transformation at the end of this module!



SG107.1 Chapter 1.
The 5 Platonic
Solids



← Ben playing with a “*Star Mother*”
(a nested model of the 5 Platonic Solids)

SG107.1.1 Why Studying the 5 Platonic Solids?

“The chief reason for studying regular polyhedra is still the same as in the times of the Pythagoreans, namely, that their symmetrical shapes appeal to one’s artistic sense”.

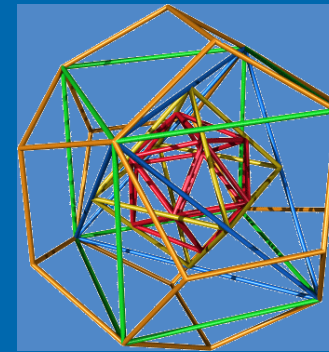
H.S.M. Coxeter. Regular Polytopes.
(Coxeter has been called the “Geometer of the 20th century”)

The 5 Platonic Solids are a cosmic primer of Sacred Geometry as they are embedded (nested) into each other, starting with the seed of PHI - the guiding principle to construct the Icosahedron.

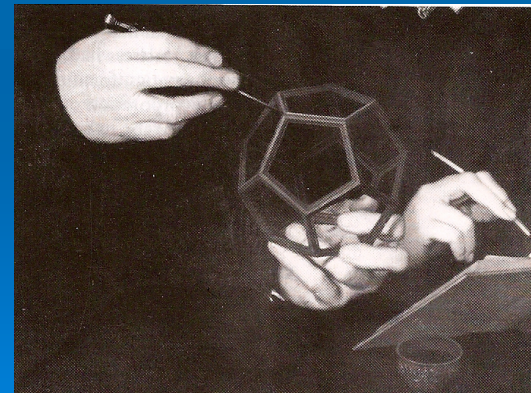
The 3D model of the nested Platonic Solids is called the StarMother. It is a beautiful pattern in space, replete with mathematical/geometrical symmetries, aesthetic elegance and philosophical insights.

Pioneers scientists, starting with Johannes Kepler, are using this model to describe complex systems like the solar system or the atomic nucleus.

The 5 Platonic Solids have been hailed by Tradition to be the “building blocks of the universe”. These forms were considered to be the “*crystallizations of the creative thoughts of God*” (Lawlor).



← Nested
Platonics.



↑ Nicolaus Neufchatel. 1561.

SG107.1.2.1 The 5 Platonic Solids (1)

The 5 “Platonic” Solids are almost mythical beings in the collective consciousness of humanity.

They are archetypal patterns of 3D space. And occur throughout nature on all scales: from microscopic radiolaria, to molecules, viruses, water clusters, crystals... to new models of sub-atomic structure, nested self-similarity systems and even the overall shape of the universe. They also show up extensively throughout history and world cultures in ancient artifacts, art, architecture, wisdom systems and current technology.

The Platonic Solids seem to be “*built in*” the blue-print of life itself. In fact, they have been traditionally regarded as the “*building blocks of the universe*”.

“Today, an intelligent child who plays with regular polygon shapes can hardly fail to rediscover the Platonic Solids”.

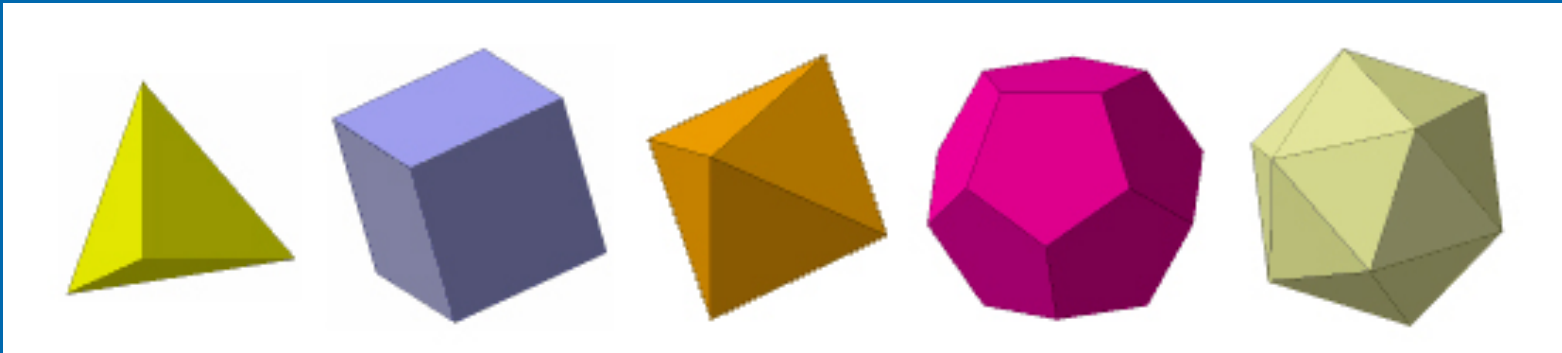
(H.S.M. Coxeter. Regular Polytopes. 1963)

In the next chapters, we will pay an extensive visit to the Platonic Solids and see many examples of their formative power in nature & culture.



SG107.1.2.2 The 5 Platonic Solids (2)

Meet the 5 Platonic Solids:



Tetra
(4)

Hexa
(6)

Octa
(8)

Dodeca
(12)

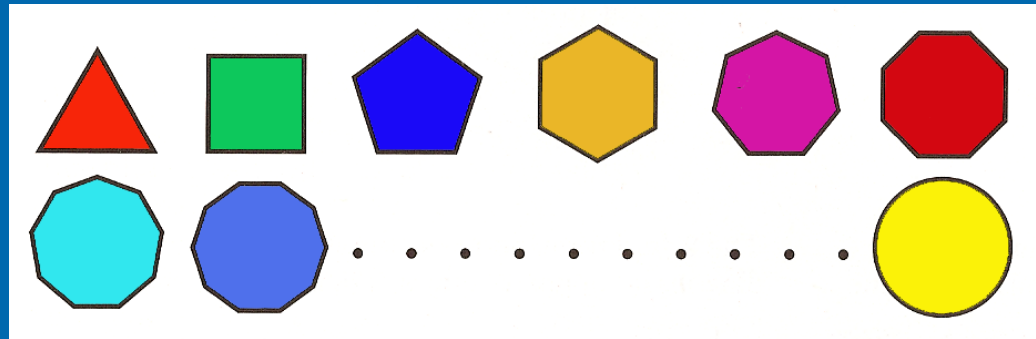
Icosa
(20)

Hedron = Face, in Greek language.



(Wire models)

SG107.1.2.3 The 5 Platonic Solids (3) - Definition



← There is an infinite number of regular polygons.

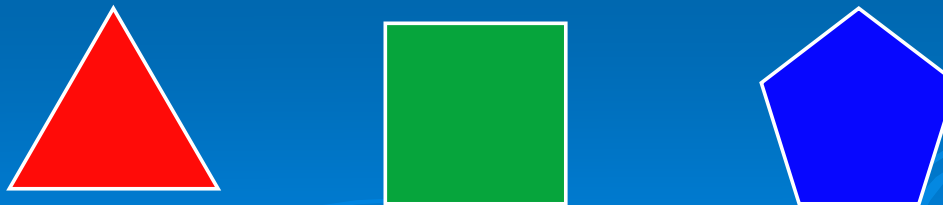
However there are only **5** regular polyhedra or “Platonic Solids”.

In a REGULAR POLYHEDRON:

All face are equal (they are “*congruent*” = they “coincide”)

All faces are regular polygons (equilateral triangle, square or pentagon)

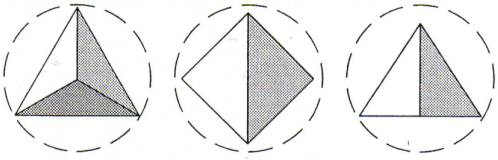
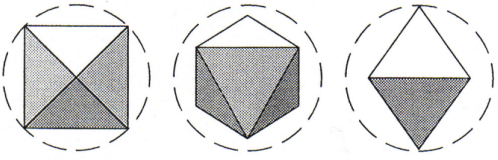
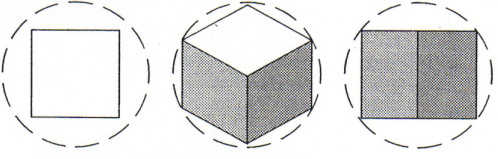
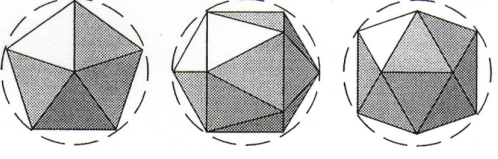
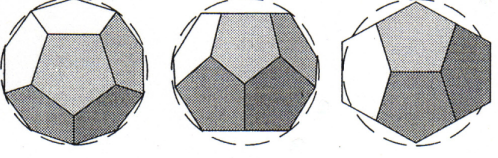
All vertices are alike (the number of edges at each vertex is the same)



↑ The 3 components of the Platonic Solids

The 5 Platonic Solids

Diagram from Richard Dannelley. Sedona: Beyond the Vortex. 1995)

	Edges	Faces	Vertices	Length	A	B
Tetrahedron - Fire 	6	4	4	$\sqrt{2}$	$60^\circ \times 3 = 180^\circ$	$180^\circ \times 4 = 720^\circ$
Octahedron - Air 	12	8	6	$\frac{1}{\sqrt{2}}$	$60^\circ \times 4 = 240^\circ$	$240^\circ \times 6 = 1440^\circ$
Cube - Earth 	12	6	8	1	$90^\circ \times 3 = 270^\circ$	$270^\circ \times 8 = 2160^\circ$
Icosahedron - Water - Biological Life Force 	30	20	12	Φ	$60^\circ \times 5 = 300^\circ$	$300^\circ \times 12 = 3600^\circ$
Dodecahedron - Ether - Universal Life Force 	30	12	20	$1/\Phi$	$108^\circ \times 3 = 324^\circ$	$324^\circ \times 20 = 6480^\circ$

Total Column B = 14,400

SG107.1.2.4 The 5 Platonic Solids (4) Table

Column A = Sum of angles around each vertex

Column B = Sum of angles x number of vertices.

Note 1: all the digits sums of A and B are 3, 6 or 9.

Note 2: Total of Column B = 14,400.

Euler's formula:

$$F + V - E = 2$$

(F = faces

V = vertices

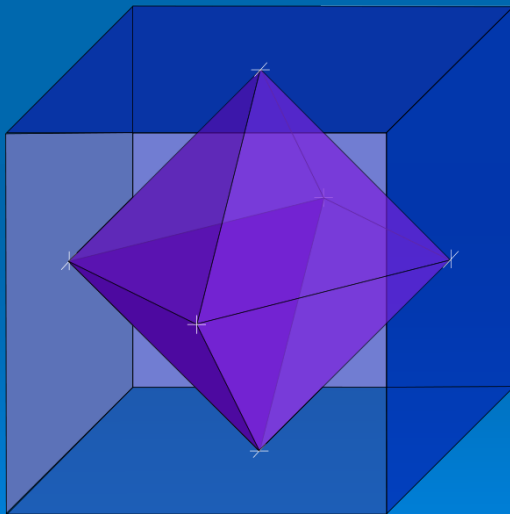
E = edges)

SG107.1.3 Two Duals & One Self-Dual

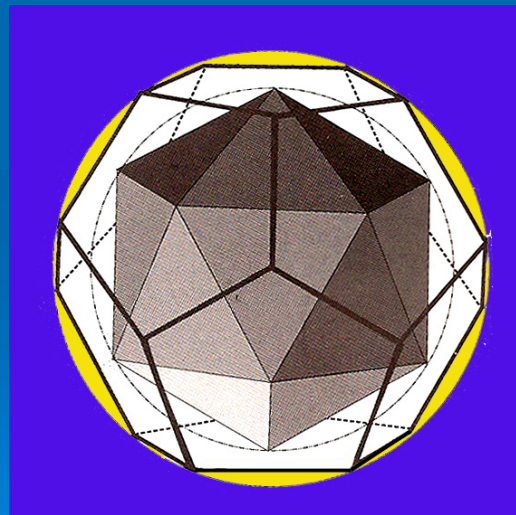
I like to say that the tribe of the 5 Platonic Solids comprises two couples and one *androgyn*. In mathematics, the Tetrahedron, as an *androgyn*, is called a “*self-dual*” i.e. it is self-sufficient and self-reflective: when the centers of its faces are joined, this forms another tetrahedron and the resulting figure is called a Star-Tetrahedron.

The first couple is formed by the symmetry relationship between the Cube and the Octahedron: the cube has 8 faces + 6 vertices whereas the Octa has 6 faces + 8 vertices - therefore the centers of one become the vertices of the other.

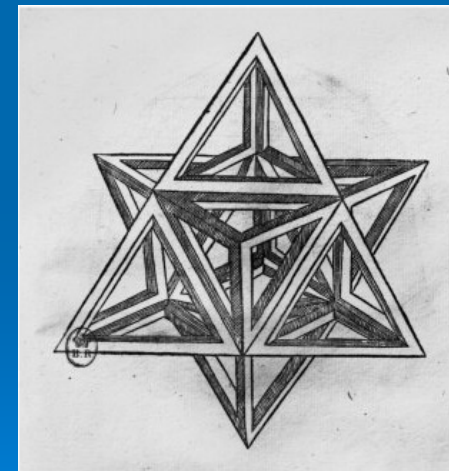
The second couple is formed by the Icosa and the Dodeca. The icosahedron has 20 faces + 12 vertices whereas the dodecahedron has 12 faces + 20 vertices. They morph perfectly into each other. (See the blue & orange arrows in preceding page).



↑ The couple Cube-Octa

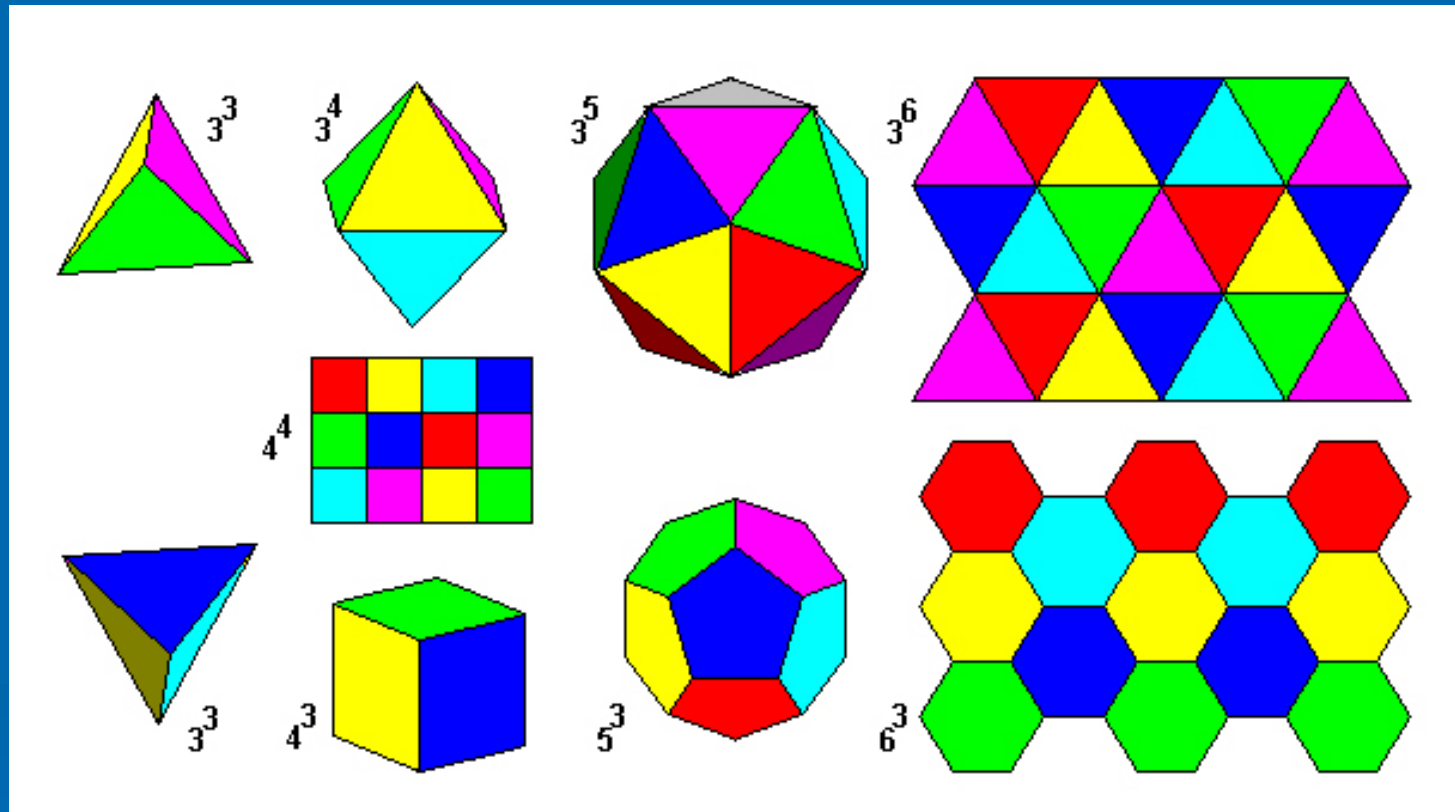


↑ The couple Icosa-Dodeca



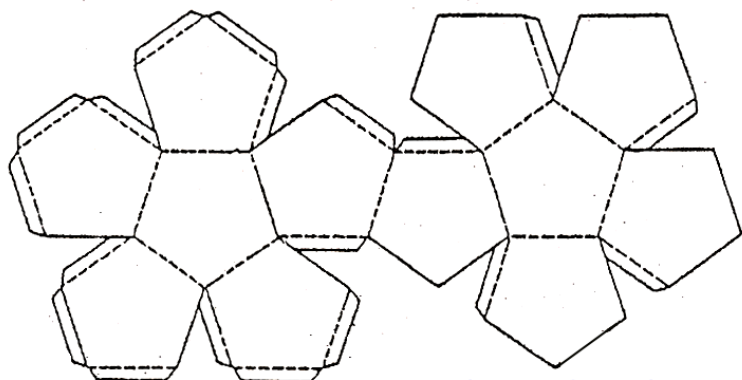
↑ The self-dual Tetra
(here seen as a double Tetra
or “Star-Tetra”)

SG107.1.4 Platonics & Tiling



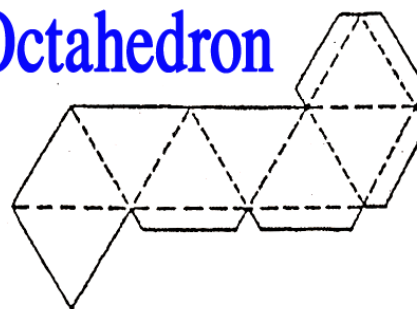
Platonic solids and *uniform tilings* are closely related. Starting from the tetrahedron we have polyhedra with three, four and five triangles at each vertex, then the plane tiling with six triangles (top row). Also starting from the tetrahedron we have polyhedra with three triangles, squares and pentagons at each vertex, then the plane tiling with three hexagons (bottom row). The figures in the top and bottom rows are duals of each other. The tetrahedron is its own dual.

SG107.1.5 Fold-Out Nets

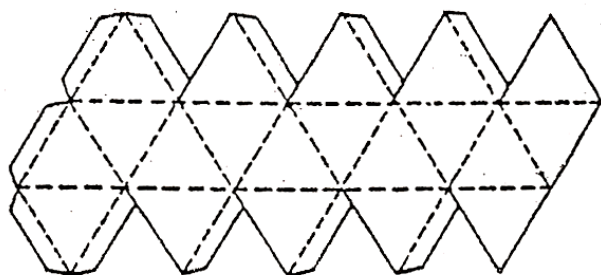


Dodecahedron

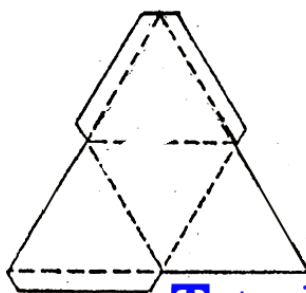
Octahedron



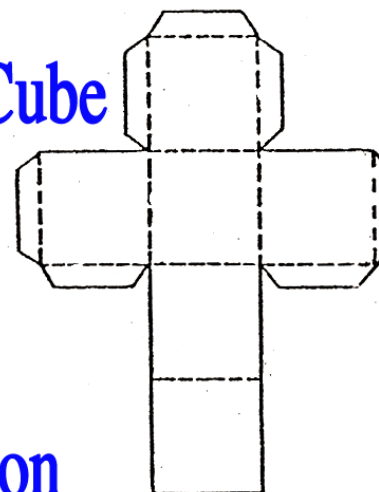
Cube



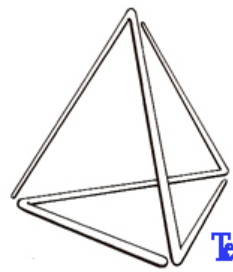
Icosahedron



Tetrahedron



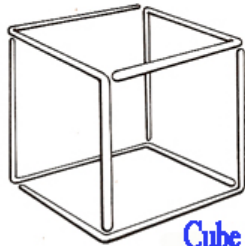
Fold-out patterns (or “nets”) are a great way to understand hands-on the “personality” of polyhedra. In 2D, these patterns give insights into the component shapes, the symmetries and the dynamics of the polyhedra. Here, we meet the 5 Platonic Solids “flat on”. What do these shapes suggest in terms of the “character” of each Platonic Solid?



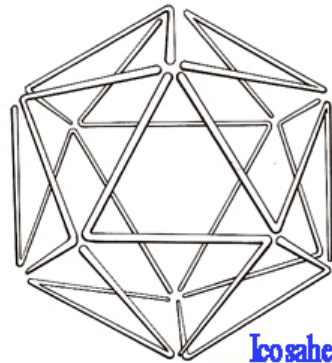
Tetrahedron



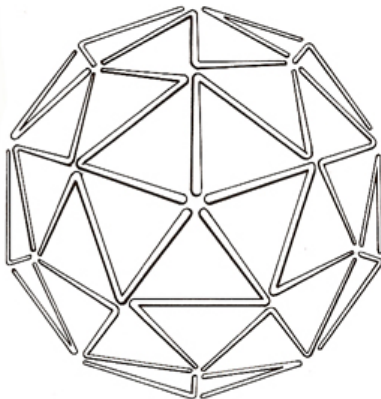
Octahedron



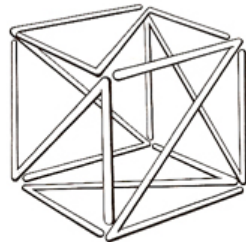
Cube



Icosahedron



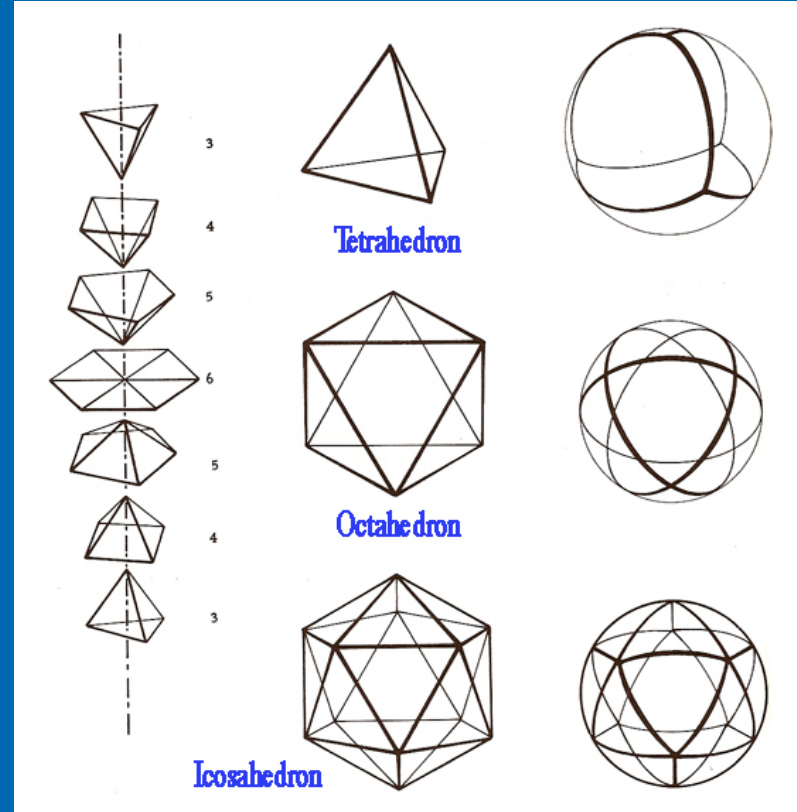
2-frequency Geodesic Sphere



Vector Diagonal Cube

↑ At their most fundamental level, as Buckminster Fuller noted, polyhedra are made of “spiral triangular events”: action, reaction and resultant.

SG107.1.6.1 Platonics & Symmetry (1)



↑ The 3 basic structural systems in nature:

“There are three types of omni-triangular, symmetrical structural systems. We can have around each vertex:

3 triangles - a tetrahedron

4 triangles - an octahedron

5 triangles - an icosahedron

The Tetrahedron, Octahedron and Icosahedron are made up, respectively, of one, two and five pairs of positively and negatively functioning open triangles.”

(B. Fuller)

SG107.1.6.2 Symmetry (2)

The 5 Platonic Solids and other Polyhedra are treasure troves of SYMMETRY.

The concept of symmetry (Greek *sun* = together, equal + *metria* = measurement) has been called “*a unifying concept*” (Hargittai) as it provides a bridge between apparently separate fields of study or scientific disciplines. Moreover, it links strict geometrical descriptions with the more aesthetic/artistic notions of harmony and proportion.

“The bridging ability of the symmetry concept is a powerful tool - it provides a perspective from which we can see our world as an integrated whole”.

Istvan & Magdolna Hargittai. Symmetry. 1994.

During a full revolution, the object is reproduced:

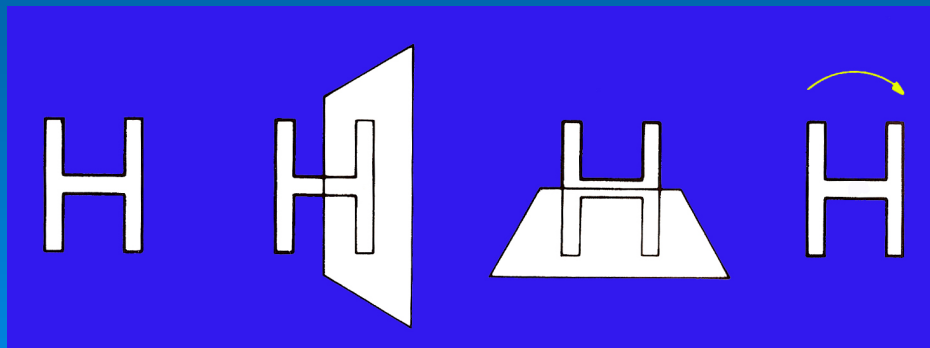
2 times = 2-fold symmetry

3 times = 3-fold symmetry

4 times = 4-fold symmetry

5 times = 5-fold symmetry

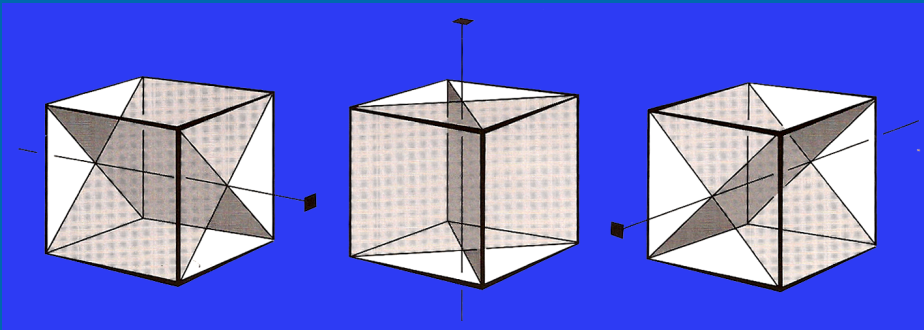
There are two main types of symmetry: reflection & Rotation.
REFLECTION SYMMETRY: Reflecting one half of an object reconstructs the image of the whole object.
ROTATION SYMMETRY: When an object is rotated around its axis, it appears in the same position two or more times.



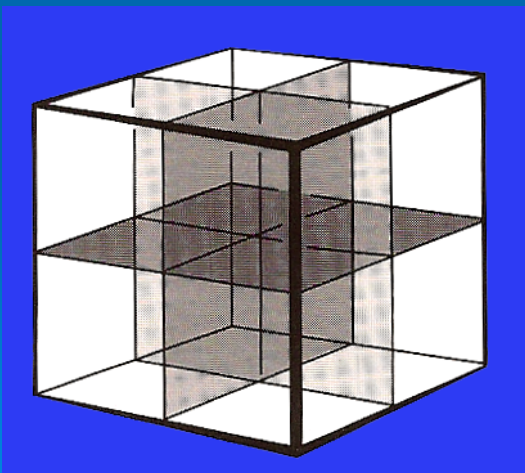
← The letter “H” has two reflection planes + one 2-fold rotation axis. (After Hargittai)

SG107.1.6.3 Symmetries in the Platonics (3)

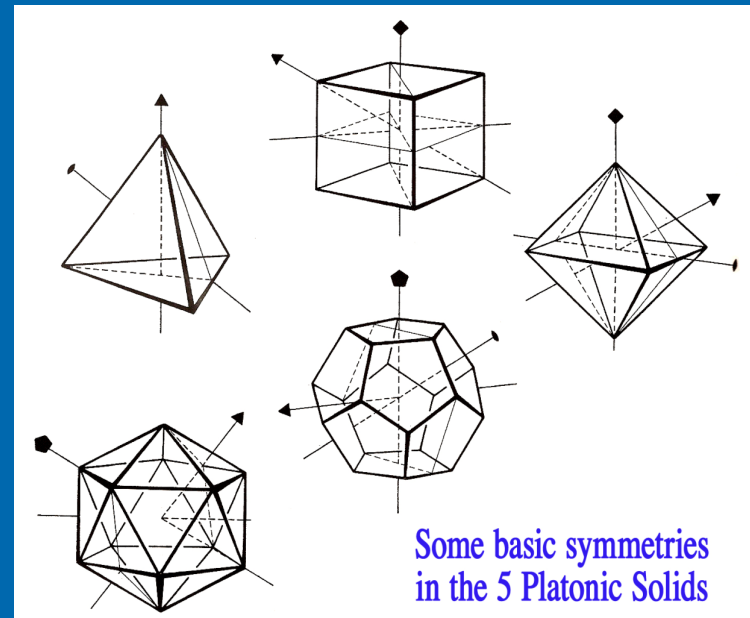
The Platonic Solids, being regular shapes, display many symmetries. Below is the example of the cube.



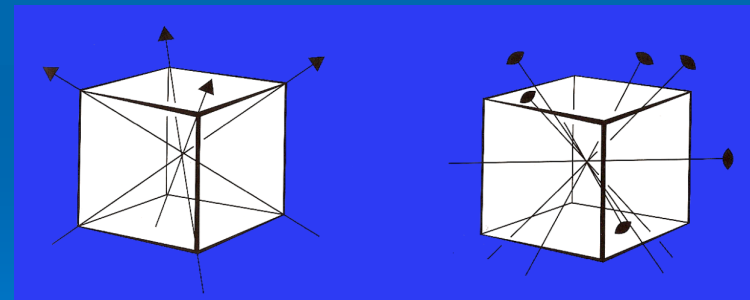
↑ The cube has 6 diagonal mirror plane symmetries. The crossings of these diagonal planes are also 4-fold symmetry axes.



← The cube has 3 parallel mirror plane symmetries.



Some basic symmetries in the 5 Platonic Solids

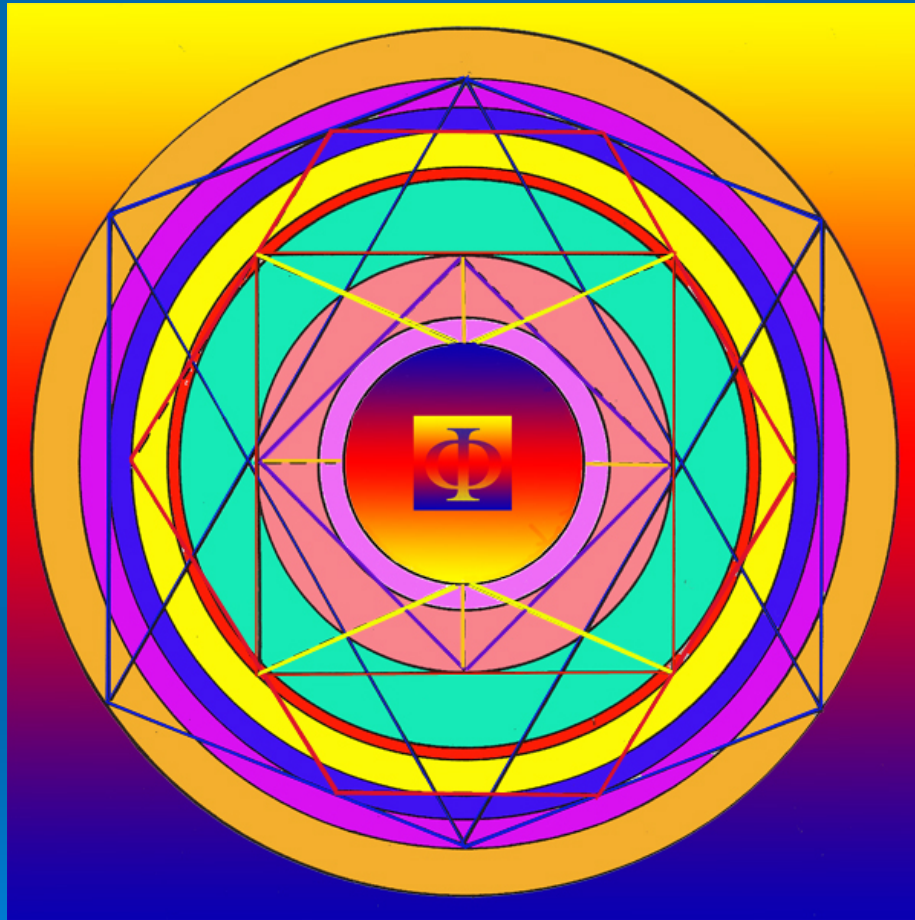


↑ The cube has four 3-fold rotation axes (left) and six 2-fold rotation axes (right).

(From Hargittai. Symmetry, 1994)

SG107.1.6.4 Platonic Nesting 2D (4)

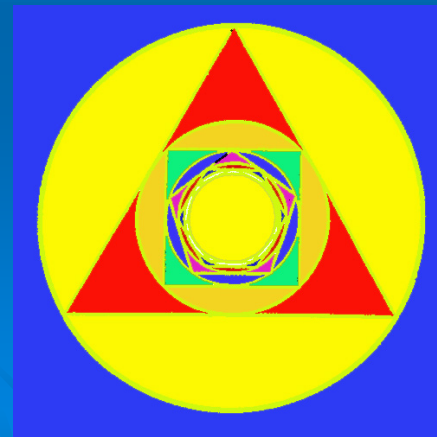
The 9 Circles



The “9 Circles of Creation”
Harmonic nesting within the 5 Platonic Solids

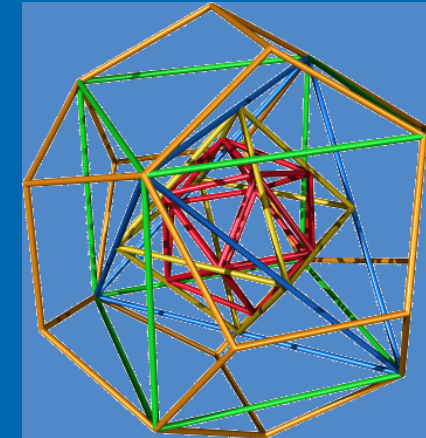
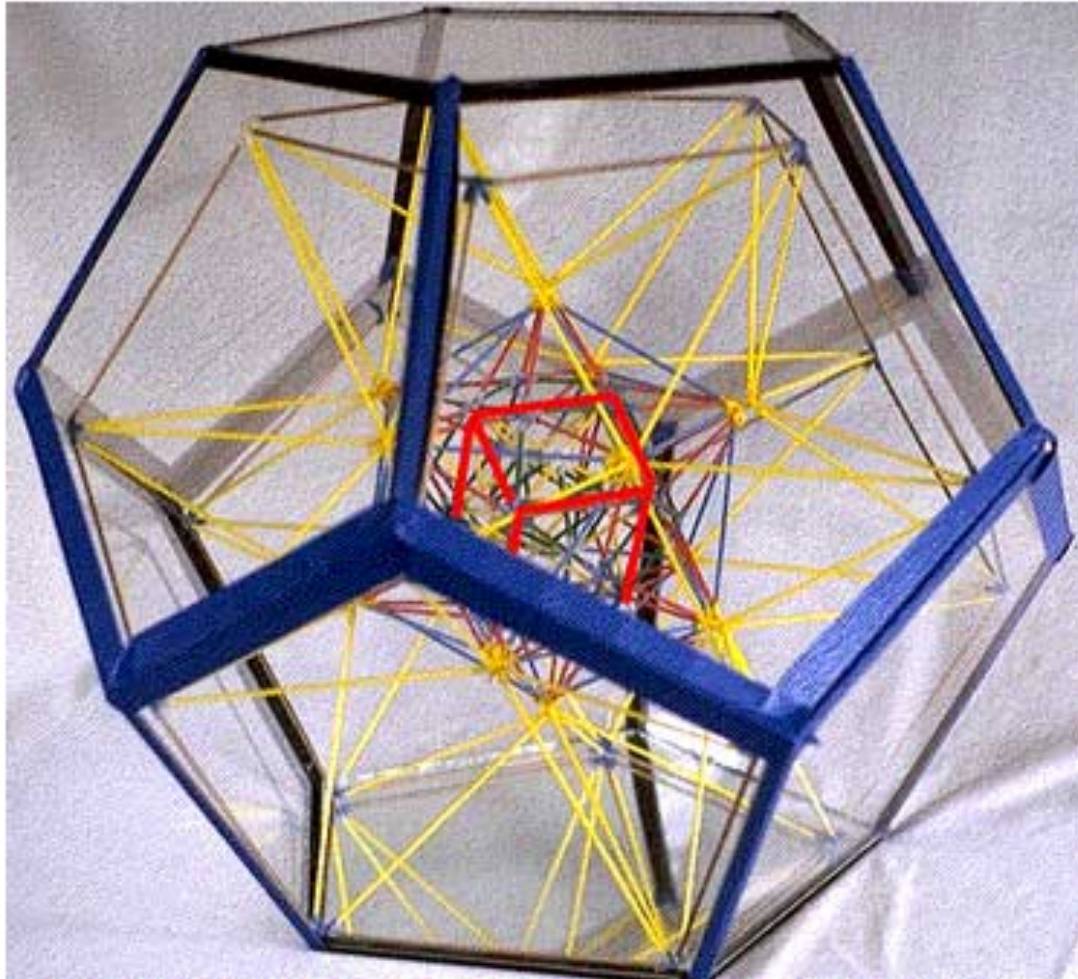
According to Robert Lawlor in his seminal Sacred Geometry, the 5 regular polyhedra were classically drawn within 9 concentric circles, each solid touching the sphere circumscribing the next solid within it. These 9 circles yield all the data necessary for the building of the 3D forms.

“Each volume is in simple harmonic relationship to the others, and it is the same transcendental functions, $\sqrt{2}$, $\sqrt{\Phi}$ and Φ that make up these patterns of relationships... But if one of the concentric circles is removed, then the pattern cannot yield the remaining volumes. This is an image of the great Buddhist idea of the co-dependent origination of the archetypal principles of creation.”



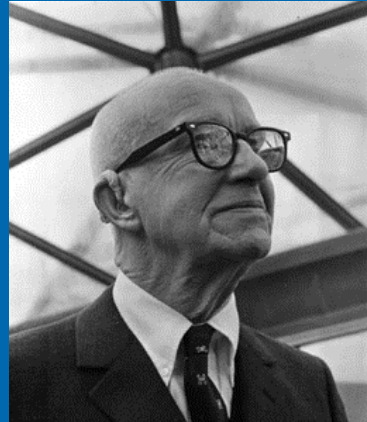
← Another 2D version of the nested Platonics

SG107.1.6.5 3D Platonic Nesting 3D (5)



The 5 Platonic Solids,
due to their common
PHI base,
can be nested
in various ways.

← On the left, the original model
built by Dan Winter and called the
“*Star Mother*”.



Buckminster Fuller (1895-1983)

*“The Leonardo Vinci of our times”
“The Planet’s Friendly Genius”*

Author, scientist, artist, inventor, architect, engineer, philosopher, patent holder, mathematician, metaphysician, cartographer, visionary, social historian... The creator of *dymaxion* homes & cars, father of *geodesic domes & tensegrity* architecture. The geometer who discovered *Synergy & Synergetics*. The man who, in his 27th year, dedicated himself to the service of humanity. The globalist who coined the term '*Spaceship Earth*' and organized the *World Game* and *World Resources* collective vision.

SG107.1.7.1 The Jitterbug (1)

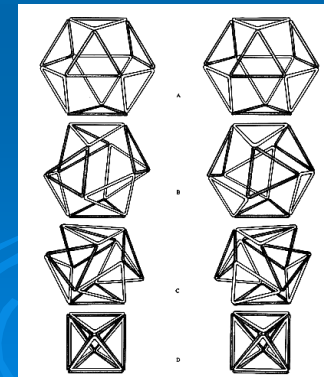
The “*Jitterbug*” is a wonderful Platonic Solid toy to demonstrate polyhedral progression from Icosa to Octa to Tetrahedron.

Due to flexible joints between the 8 triangles & 6 squares of the starting shape of the “*Vector Equilibrium*” (or Cuboctahedron - an Archimedean Solid), the Jitterbug can collapse symmetrically and morph into 3 different - yet harmonically resonant - Platonic Solids.

“In the Jitterbug, we have a sizeless, nuclear, omnidirectionally pulsing model. The Jitterbug is a conceptual system independent of size, ergo cosmically generalizable.”

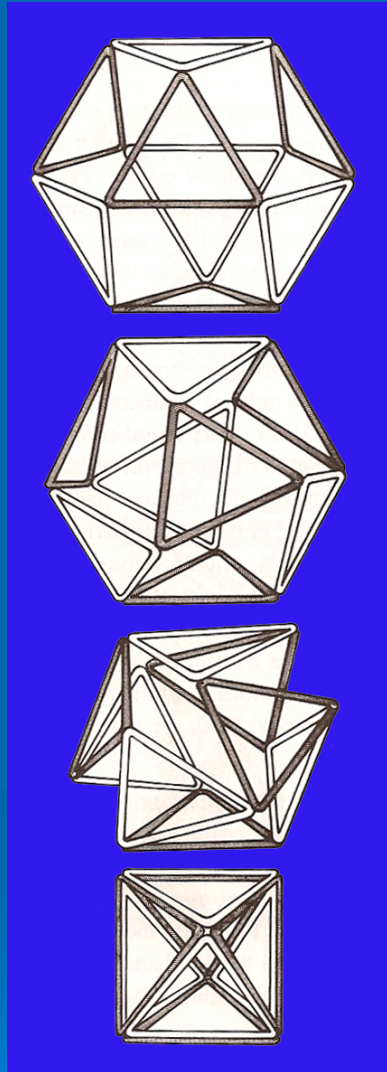
Fuller, *Synergetics*, 460.08

A dogged and globally loved “*planetary citizen*” who has received 47 honorary doctorate degrees from around the world and whose genius has been felt for over half a century. The first modern cosmic futurist.

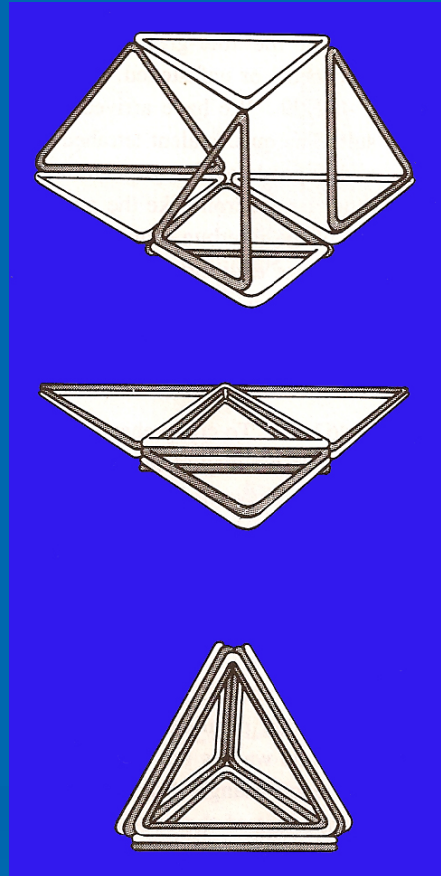


SG107.1.7.2 The Jitterbug (2)

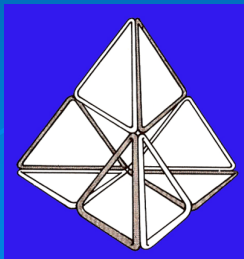
In the Jitterbug, the icosahedron contracts to a radius less than the radii of the cuboctahedron from which it derives.



Harmonic progression
from cuboctahedron
to icosahedron
to octahedron



Final progression
to tetrahedron



www.bfi.org

SG107.1.7.3 The Jitterbug (3)



↑ Dan Winter's slide about the "Jitterbug".

Quicktime Links for Gerald de Jong Nucleated Jitterbug:

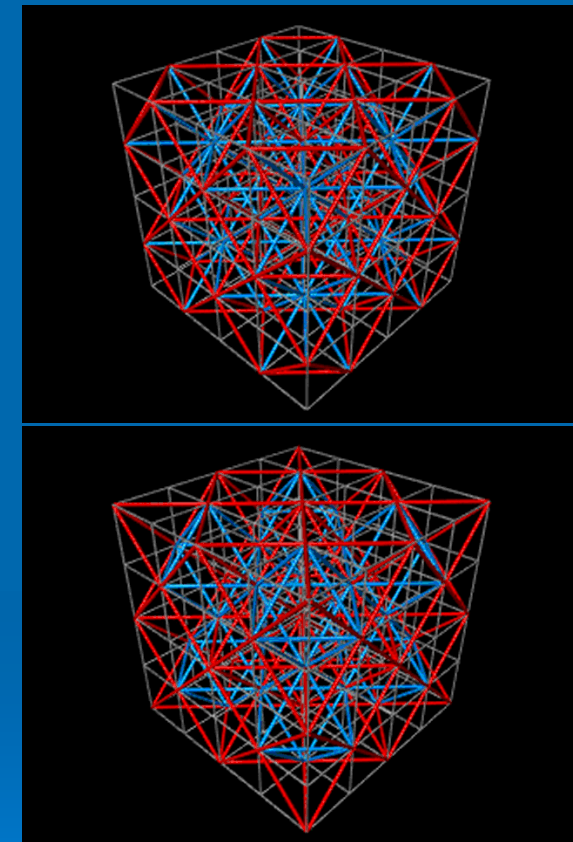
http://www.newciv.org/Synergetic_Geometry/charhawk/lrt.htm

http://www.newciv.org/Synergetic_Geometry/charhawk/universe.htm

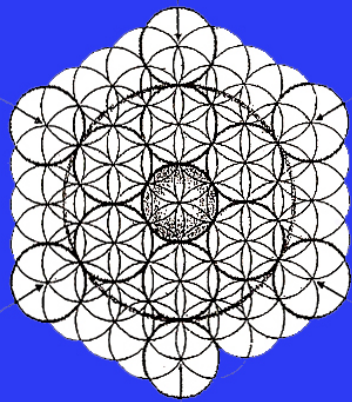
http://www.newciv.org/Synergetic_Geometry/64cell.htm

http://www.newciv.org/Synergetic_Geometry/tnsjtr.mov

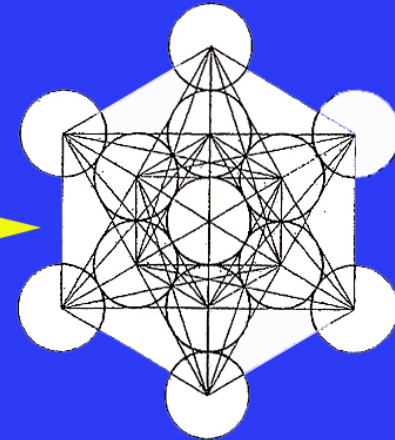
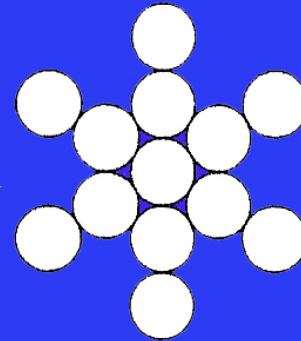
http://www.newciv.org/Synergetic_Geometry/64cell.htm



↑ Two views of a 64-cell
Jitterbug array
by Gerald de Jong

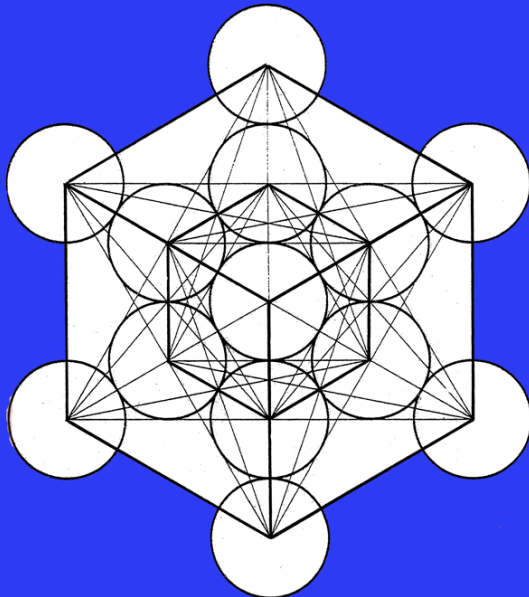


13 Circles
(5th rotation of the
Flower of Life)



The Fruit of Life

Metatron's Cube



Drunvalo Melchizedek. *The Ancient Secret of the Flower of Life*. 1998.



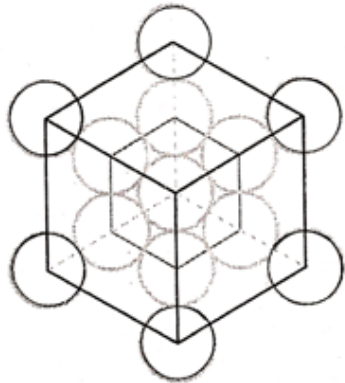
www.floweroflife.org

SG107.1.8.1 Metatron's Cube (1)

← By connecting the inner lines of the *Fruit of Life* design (originating with the Vesica Piscis ♦SG108), one can obtain the 3D outlines of Hexa-stars and also of a large and smaller cube. Hence the name *Metatron's Cube*.

Metatron's Cube happens to be a template for the Platonic Solids. All 5 Platonics are precisely embedded in the matrix of Metatron's Cube, as the next page illustrates.

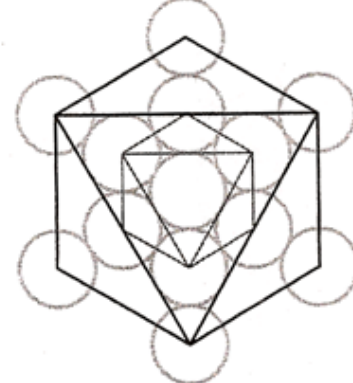
SG107.1.8.2 Metatron's Cube & Platonics (2)



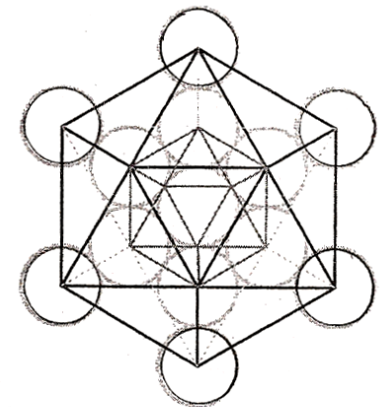
Hexahedron
or Cube



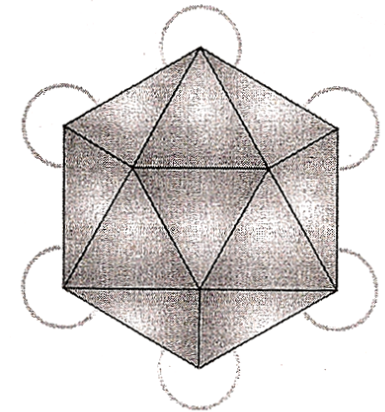
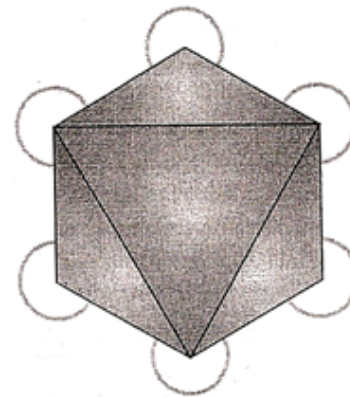
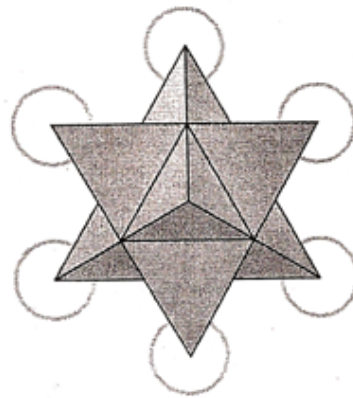
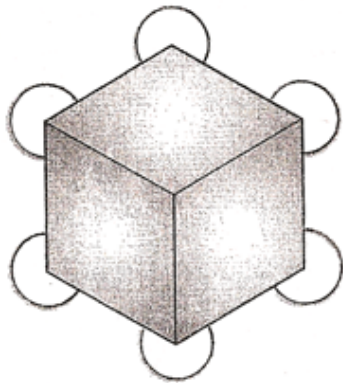
Tetrahedron and
Star-Tetrahedron



Octahedron

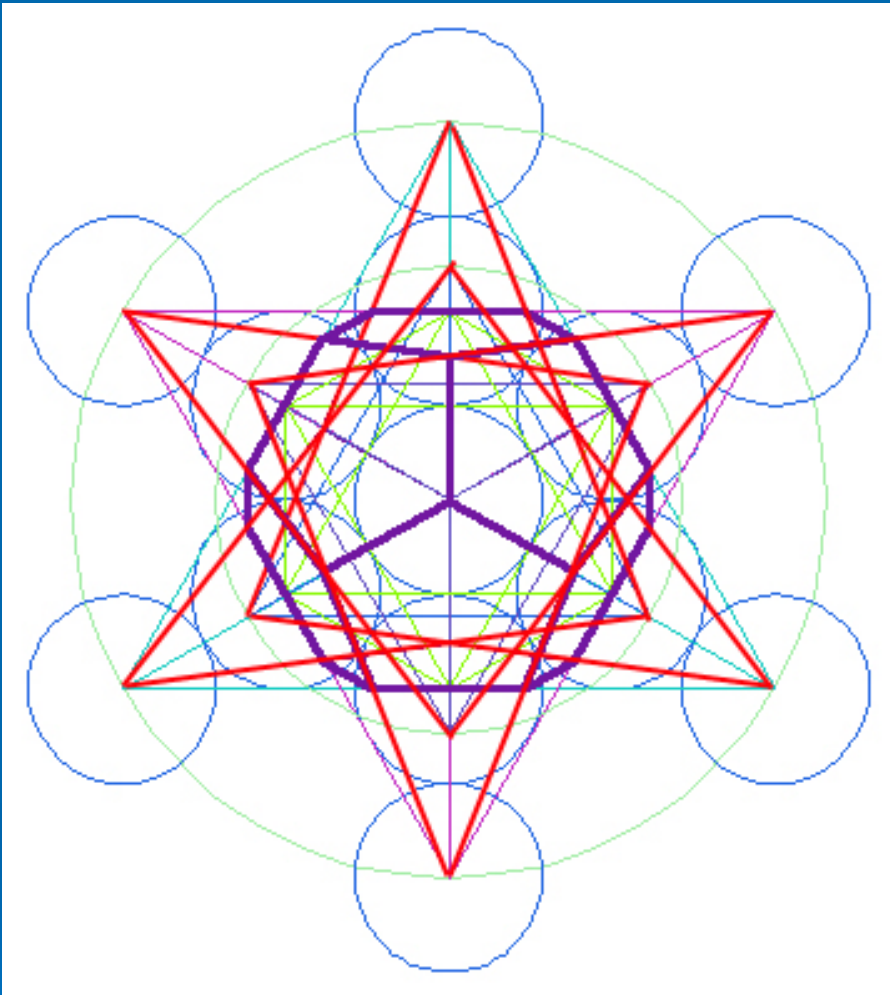


Icosahedron

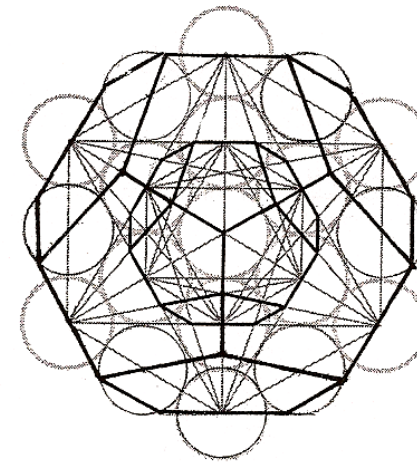


The 5 Platonic Solids can be directly derived from Metatron's Cube:
Tetra, Cube, Hexa & Icosa. (See next page for the dodeca).

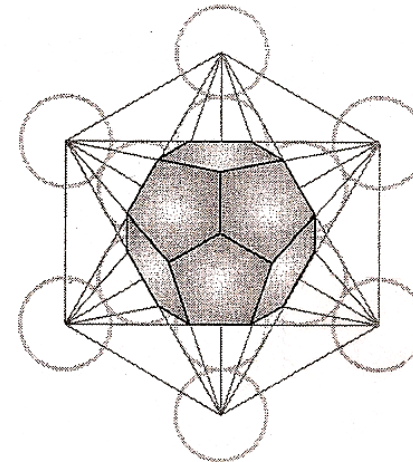
SG107.1.8.3 Metatron's Cube & Dodeca (3)



Whereas to draw the large size dodecahedron within Metatron's cube requires additional circles, the small dodeca can be precisely drawn by connecting the 6 apices (summits) of the large Star Tetra to the alternate 6 apices of the small Star Tetra.

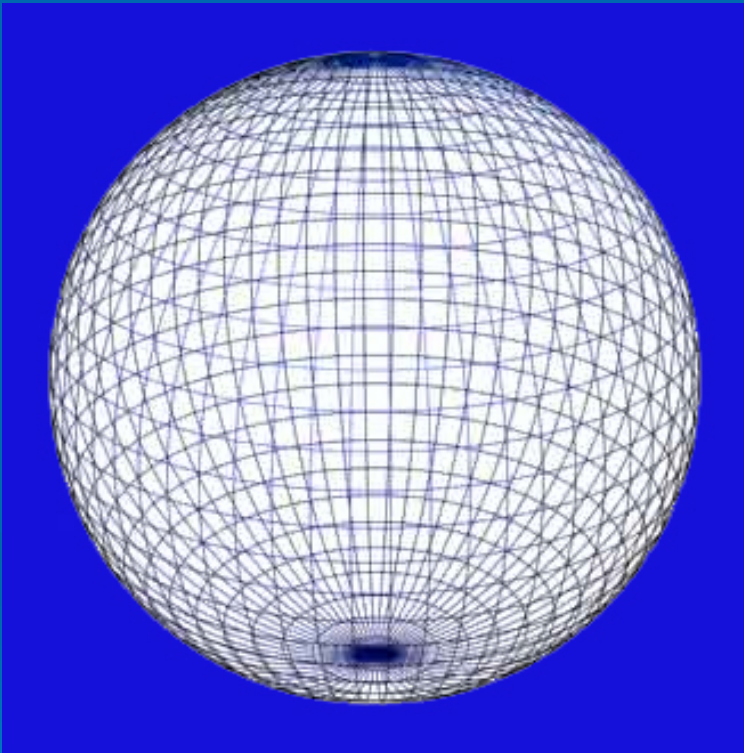


Dodecahedron



SG107.1.9 The Sphere

The most perfect - and unpublicized - Platonic Solid is the Sphere.
It is the 6th regular Solid, source and destination of them all.



The SPHERE is a symbol and model of perfection and primordial Origin:

*"... Coming from the Sphere
and returning to the Sphere,
Coming from Source and returning to Source ..."*

The 5 Platonic Solids are aspects of the archetypal Sphere: they are all inscribed within the Sphere i.e. their apices (summits) all touch the circumscribed sphere.

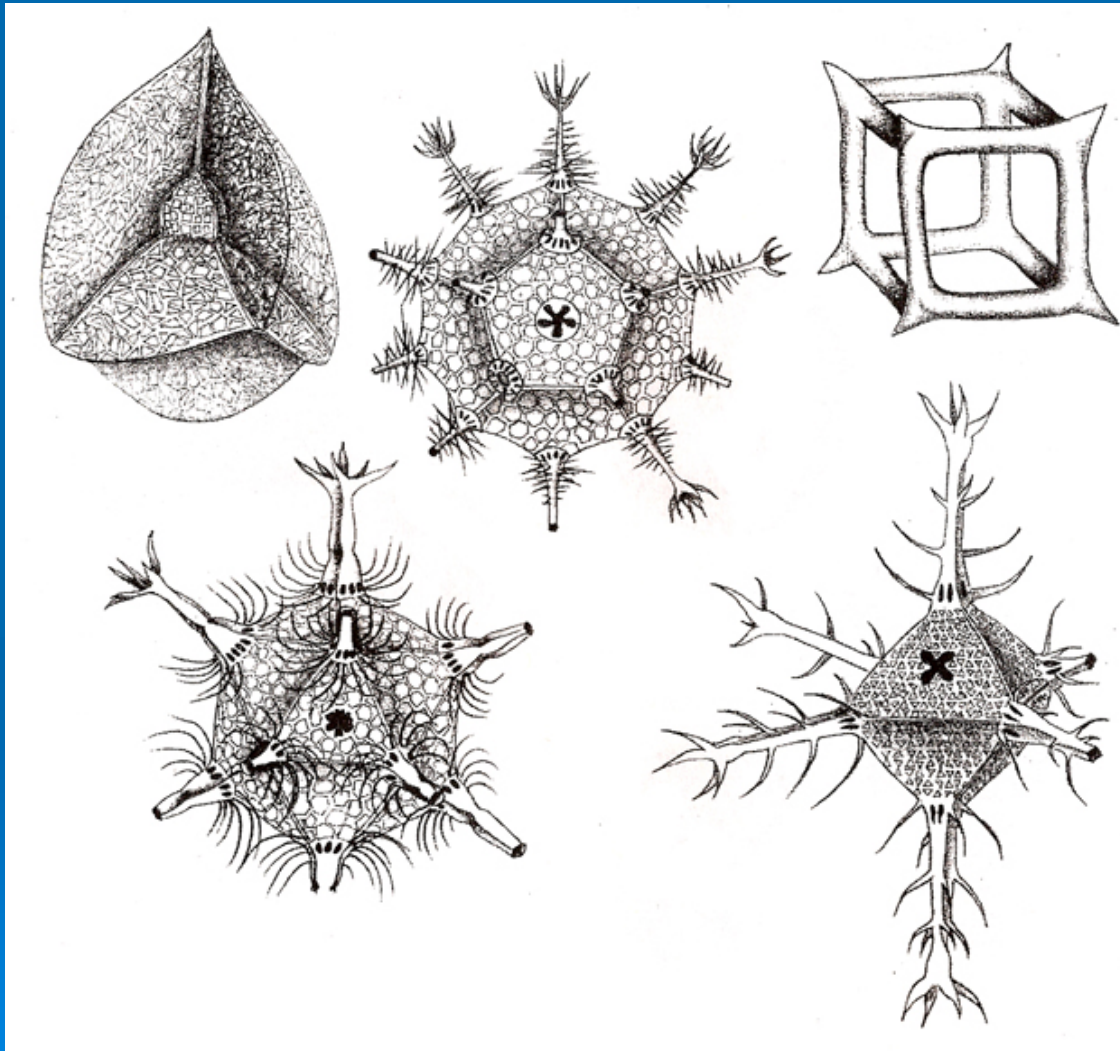
The Circle and the Sphere have an infinite number of planes of symmetry and an infinite-fold axis of rotation.

The Sphere is also described as having an infinite cylindrical or radial symmetry.

*"God id a Sphere whose Center is everywhere
and circumference nowhere".*

(Quote attributed to Hermes Trismegistus and re-quoted through all ages).

SG107.1.10 Platonics in Nature



Radiolarians, the “*Living Jewels of the Sea*”, are tiny marine protozoans. Size: 1-2 micrometer (1-2 millionth of a meter).

The fossil remains of radiolarians exhibit precise geometric shapes.

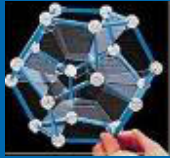
← Here, we have perfect examples of the 5 Platonic solids in the radiolarian world. The scientific names themselves reflect the geometries:

Circogonia Icosahedra

Circorhagma Dodecahedra

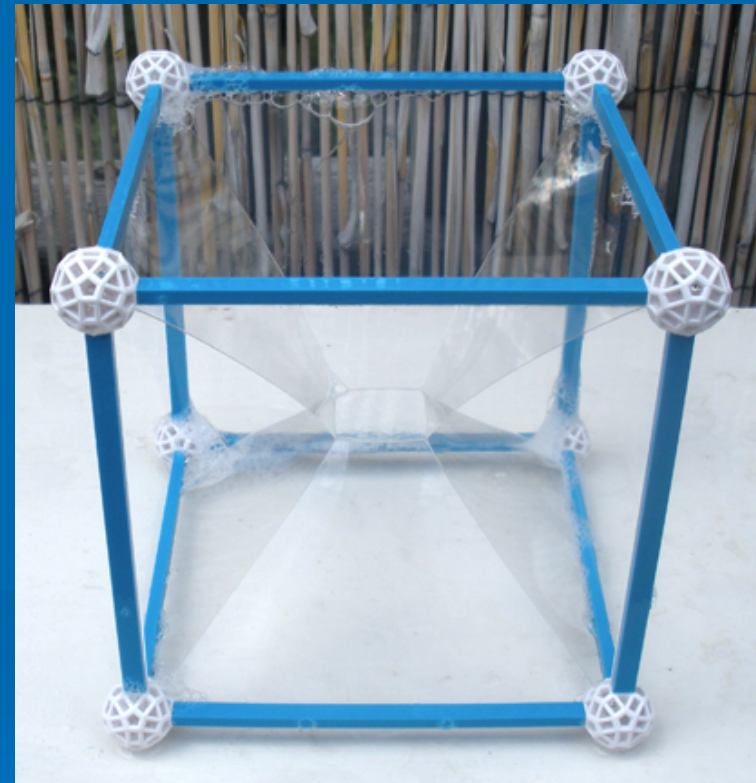
C. Octahedrus etc...

(From L. Blair. *Rhythms of Vision*. 1975. The original drawings are by Ernst Haeckel, 1904).



SG107.1.10 Platonic Bubbles

Immersing Platonic Solid structures in soap-bubble water is a fascinating educational game: it reveals the lines of inner geometry.



www.youtube.com/watch?v=ipf1XE21rMo

A number of different bubbles shapes are produced by making a soap bubble inside a cluster of soap bubbles and adding more bubbles one by one.

The shapes are: Tetrahedron, Prism, Bubble Cube, Pent...

SG107.1.11.1 Ancient Cultures (1) The 5 Elements



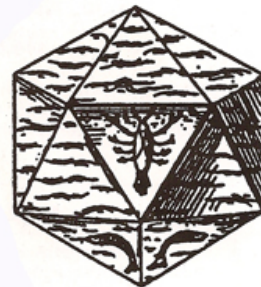
**Tetrahedron
FIRE**



**Octahedron
AIR**



**Cube
EARTH**



**Icosahedron
WATER**



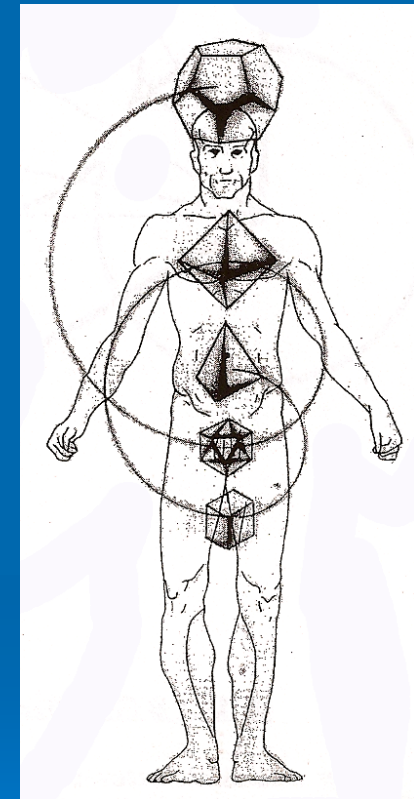
**Dodecahedron
AETHER**

From ancient Greece all the way to Kepler, the 5 Platonic Solids were associated with the 5 elements as follows:

**FIRE - Tetrahedron
AIR - Octahedron
EARTH - Hexahedron
WATER - Icosahedron
AETHER - Dodecahedron**

These, in turn, can be related to the 5 main chakras from the Hindu Yogi Tradition:

**Muladhara/Base:
Cube - Earth
Svadhithana/2nd:
Icosa - Water
Manipura/3rd: Tetra - Fire
Anahata/Heart: Octa - Air
Sahasrara/Crown:
Dodeca - Prana**

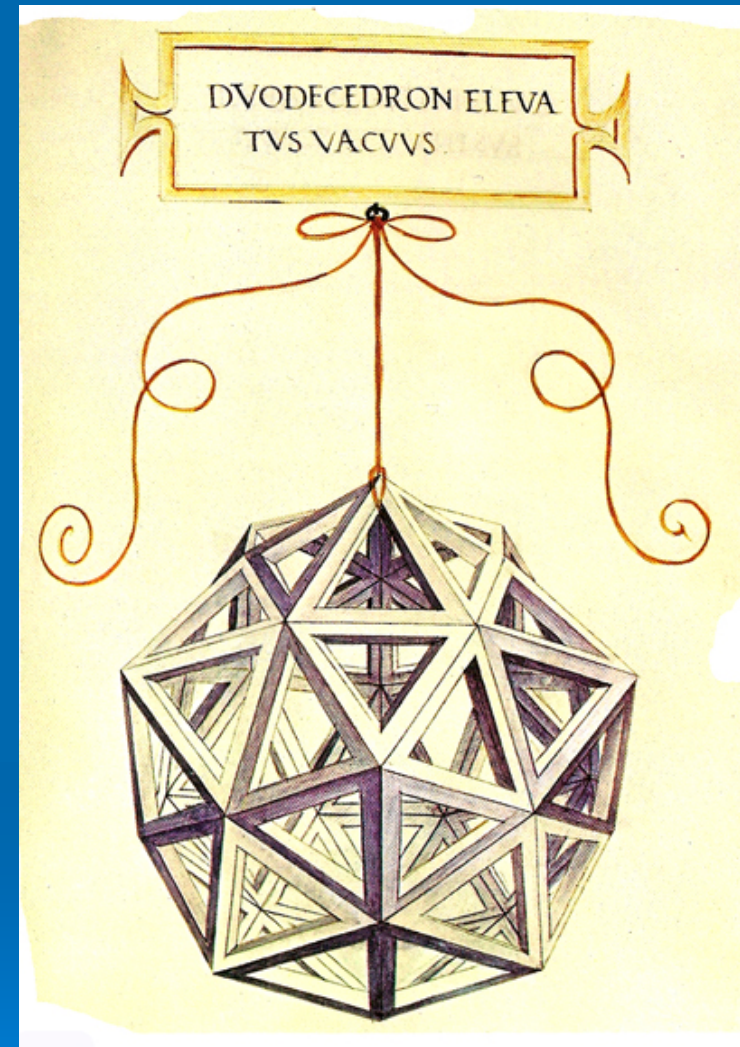
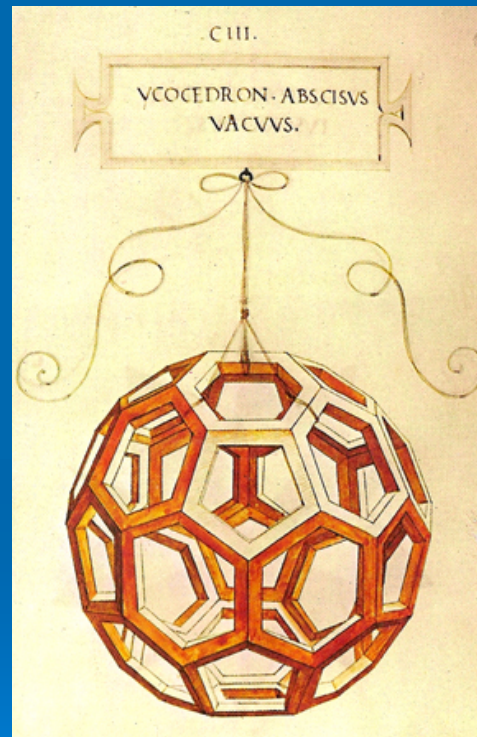
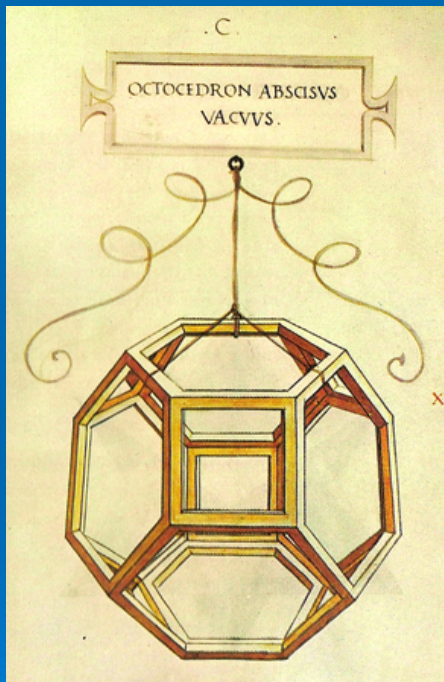


**The 5 Platonic Solids
and the 5 main chakras**

SG107.1.11.2 Cultures (2) Da Vinci's Platonics

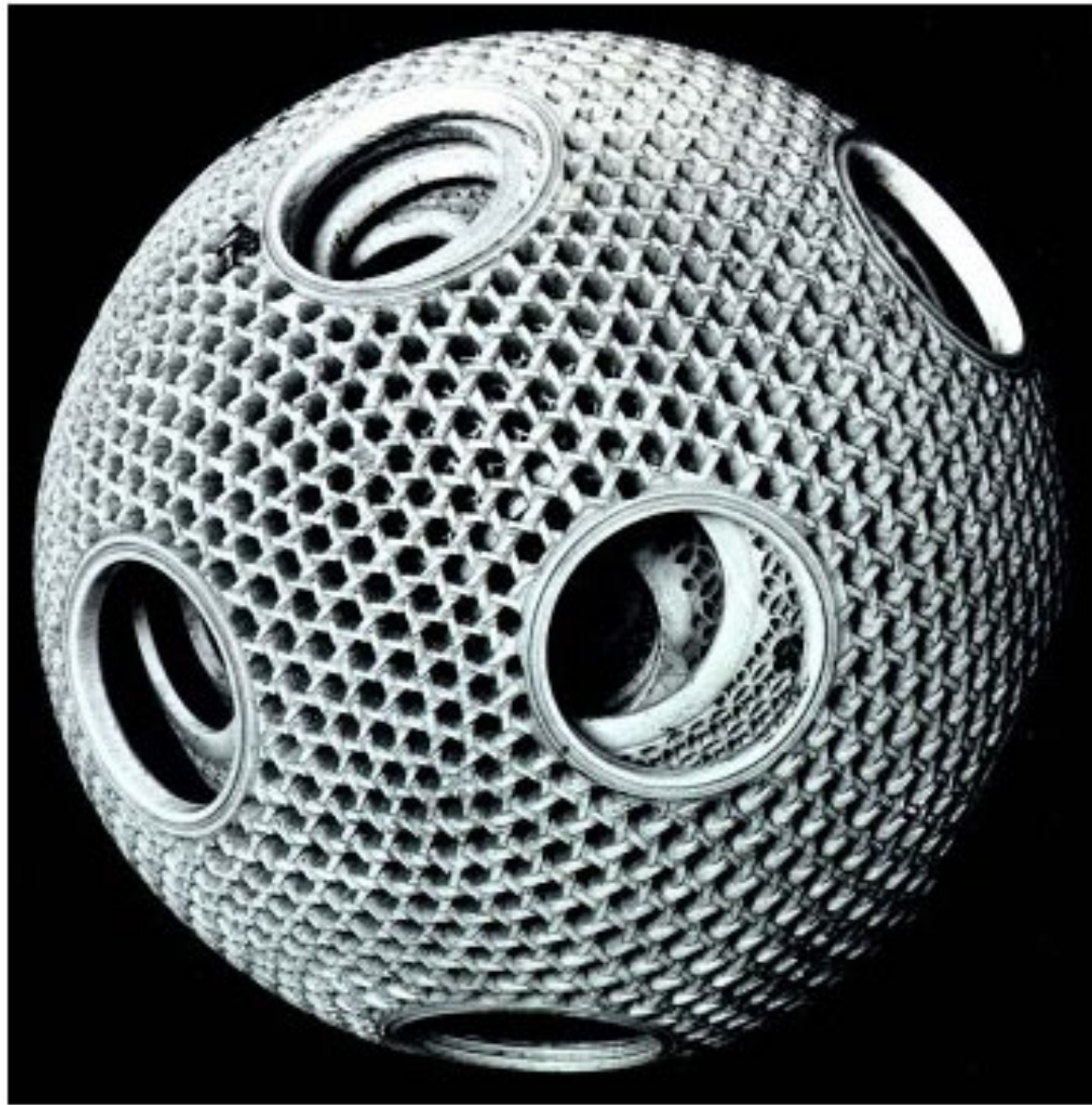
Leonardo's illustrations for Luca Pacioli's *De Divina Proportione* (1509) were complex 3D designs in solid geometry.

Pacioli extolled them as “*extraordinary and most beautiful figures*”.



Please see [◆SG102](#) for historical pages on Leonardo Da Vinci and the Golden Ratio.

SG107.1.11.4
Cultures (4)
**Post-
Renaissance**



← A beautifully carved
dodecahedron.

Post-Renaissance.
Artist unknown.

SG107.1.11.5 Cultures (5) Contemporary

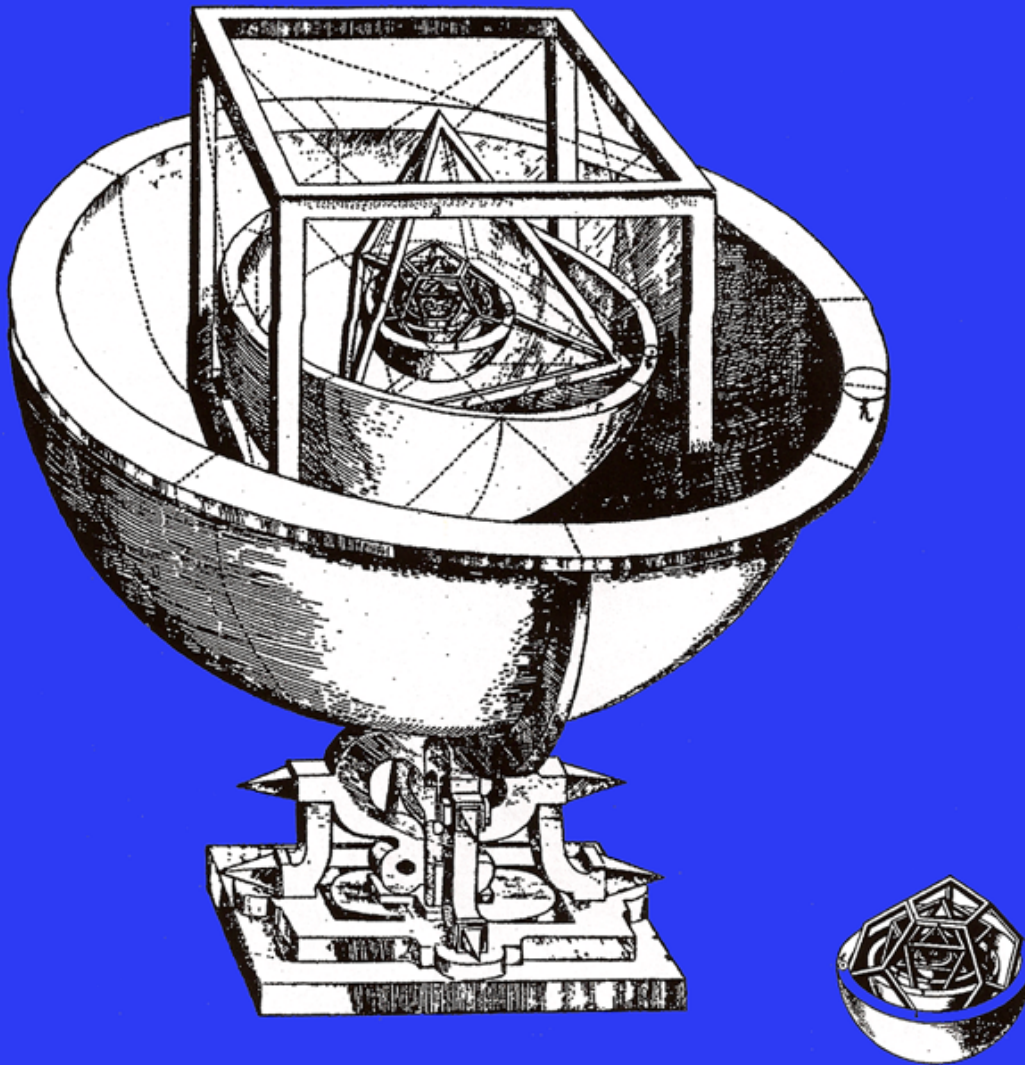


SW#71 "Grail Sutra"
www.starwheels.com



↑ "Christuskind" (The Christ Child)
By Sulamith Wülfig. www.artscraftsy.com

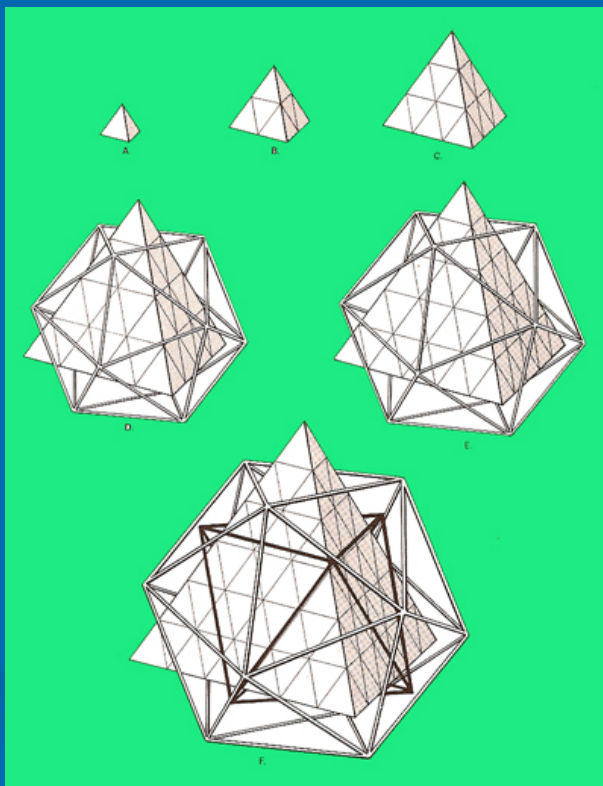
SG107.12.1 Platonic Models (1)



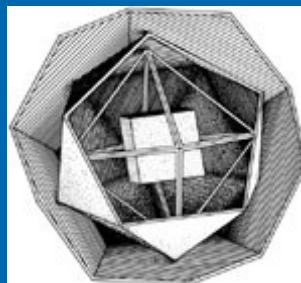
In his *“Mysterium Cosmographicum”* (1597), Johannes Kepler (1571-1630) represented the orbits of the six then-known planets as spheres. He nested the 5 Platonic Solids within these spheres, so that the inner planet, when at its greatest distance from the sun, lies on the inscribed sphere of a solid, whereas the outer planet, when at its least distance, lies on the circumscribed sphere.

While the model does not exactly fit, Kepler’s overall insight (explaining the number & properties of the planets by the symmetries of the Platonic Solids) opened the way to understand the harmonic geometry of the solar system as proven by contemporary data. In more scientific terms, Kepler understood that the rotational energy of the sun is distributed throughout the solar system in a quantized way, according to the Golden Ratio principle.

SG107.12.2 Platonic Models (2)



↑ Buckminster Fuller’s concept of a vector model for the atomic nuclei, based on the Tetrahedron, Octahedron and Icosahedron. This Platonic Nesting model is proposed to account for the “*Magic Numbers*”: in the structure of atomic nuclei there are certain numbers of neutrons & protons which correspond to states of increased stability.



← The Moon’s Model of the atomic nucleus

The nesting of the 4 Platonic Solids whose 46 vertices form half of the Moon model.

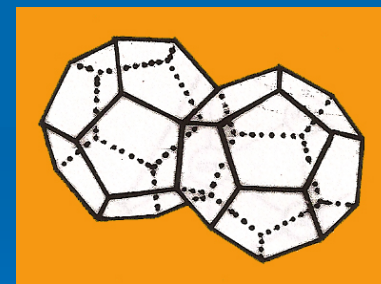
Around 1985, Dr. Robert James Moon (1911-1989), professor emeritus at the U of Chicago, developed an atomic nucleus model based on the nesting structure of 4 of the 5 Platonic solids.

This “Moon Model of the nucleus” considers protons to be located at the vertices of the solids and thus lays a geometric foundation for the periodicity occurring in the Table of the 92 naturally existing elements.

The Moon Model follows on the Sacred Geometry tradition pioneered by Plato and insightfully applied by Kepler to the solar system: the 5 Platonic Solids explain the harmonic structure of the universe.

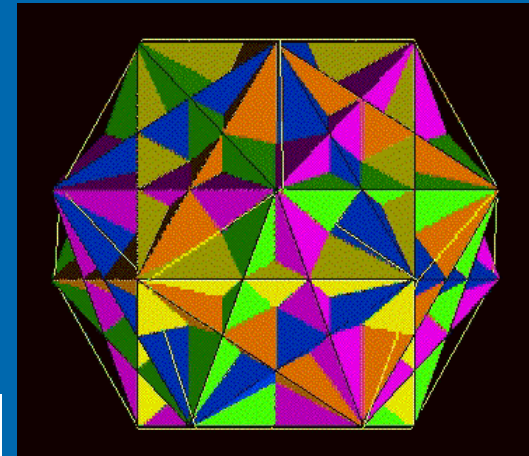
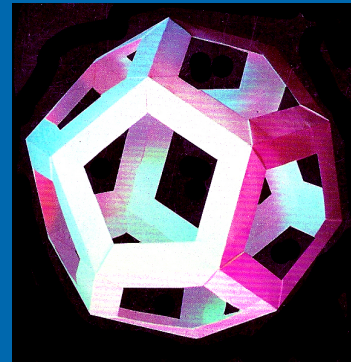
Since the original Moon Model, further developments have been achieved: the addition of the Tetrahedron at the center of the nucleus, a distribution of the neutrons at the edge-midpoints of the solids and an ordering relationship between the nucleus and the electron shells.

[♦SG203B]

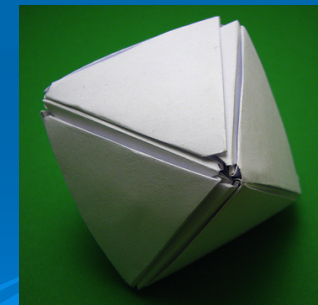
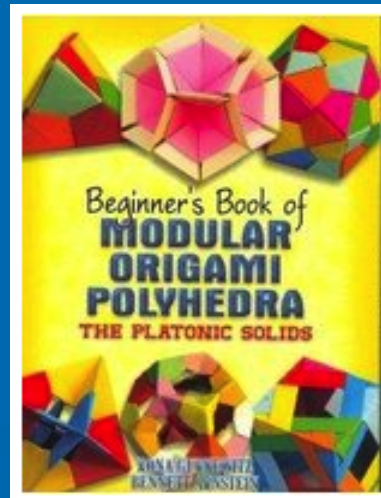


↑ Beyond Palladium (46), a twin dodeca frames the nucleus of radon (86). Beyond radon, the two dodecas open up to create the first unstable element (Uranium - 92).

SG107.1.13.1 Platonic Origami (1)



↑ "Digital Origami"
 A Dodeca is the 4th axis
 of spin of a cube.
 The cube edge
 = $.618$ or $1 / \Phi$
 of the Dodeca.



↑ Models of the Platonic Duals. Wenninger.

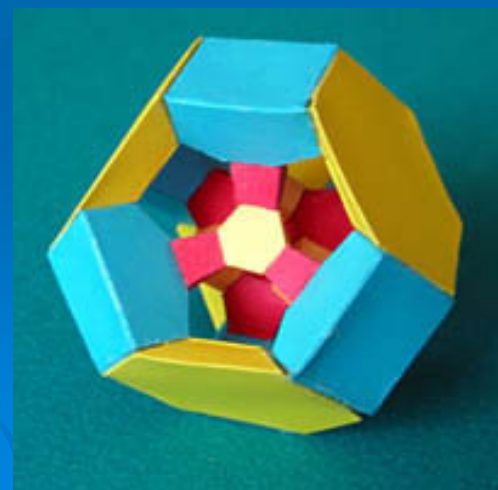
SG107.1.13.2 Platonic Origami (2) Magnus Wenninger (1)

Father Magnus Wenninger OSB is an ordained catholic priest who is also a mathematician and an outstanding pioneer of polyhedral art models.

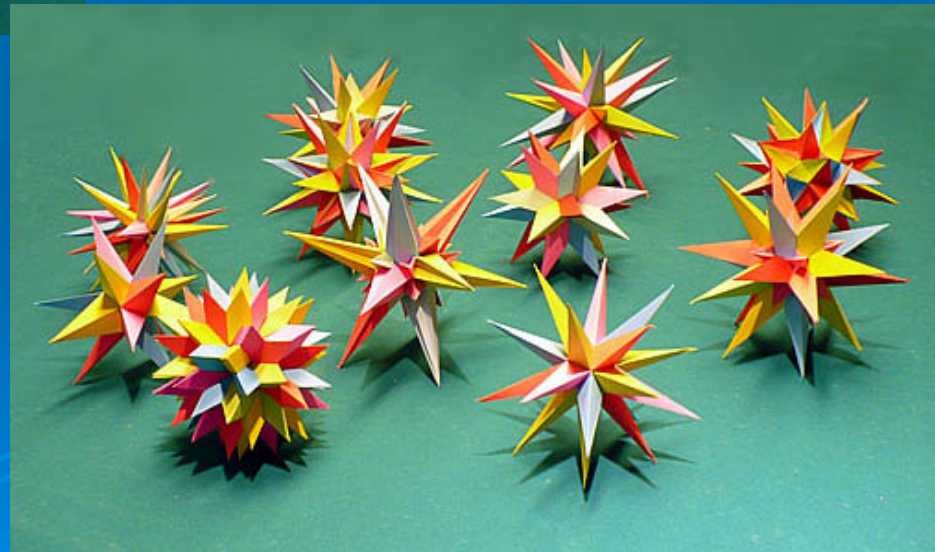
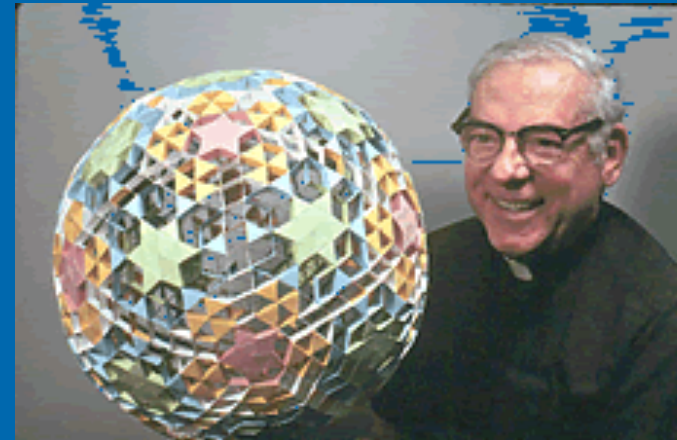
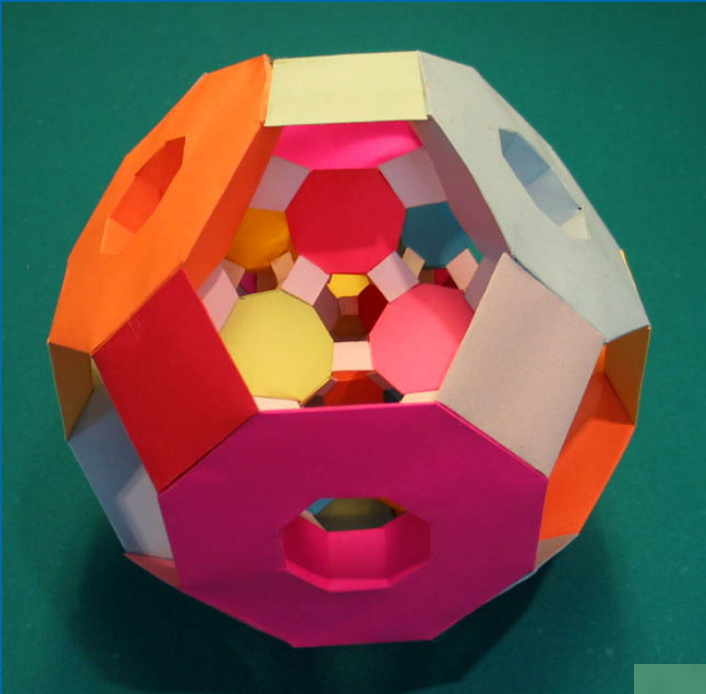
His many books and models “*have inspired a new generation of mathematical artists*”, according to the American Mathematical Society.



www.saintjohnsabbey.org/wenninger/



SG107.1.13.3 Platonic Origami (3) Magnus Wenninger (2)

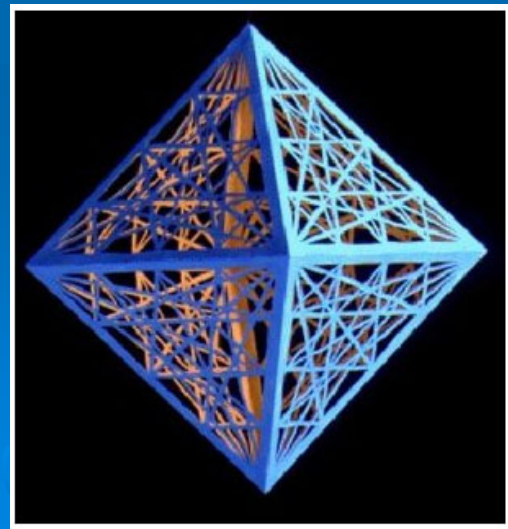
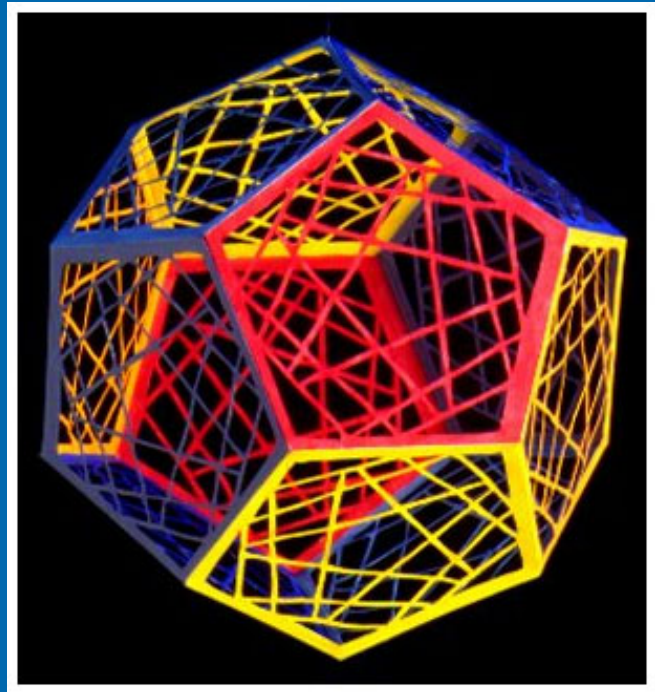
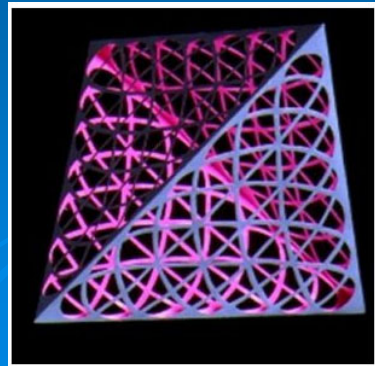
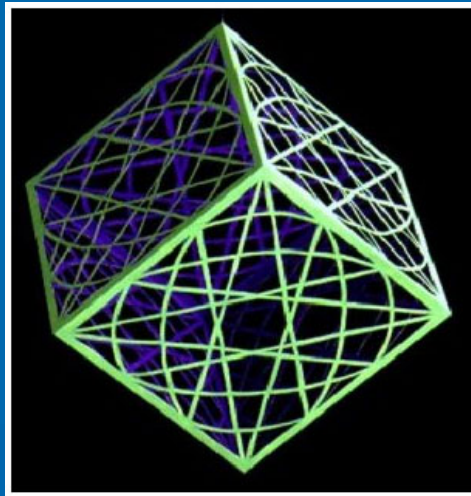
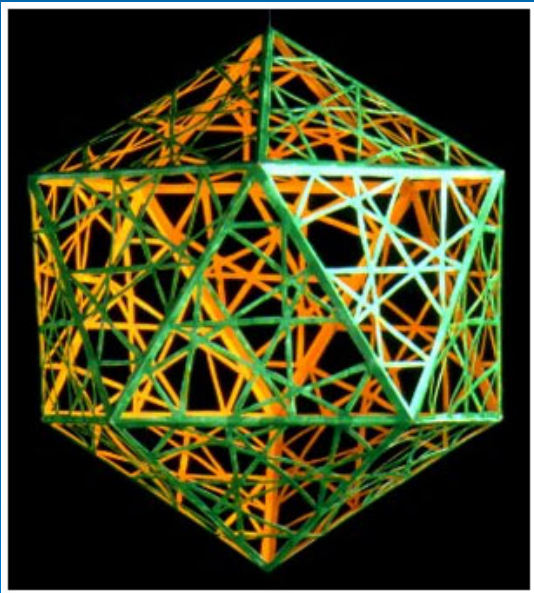


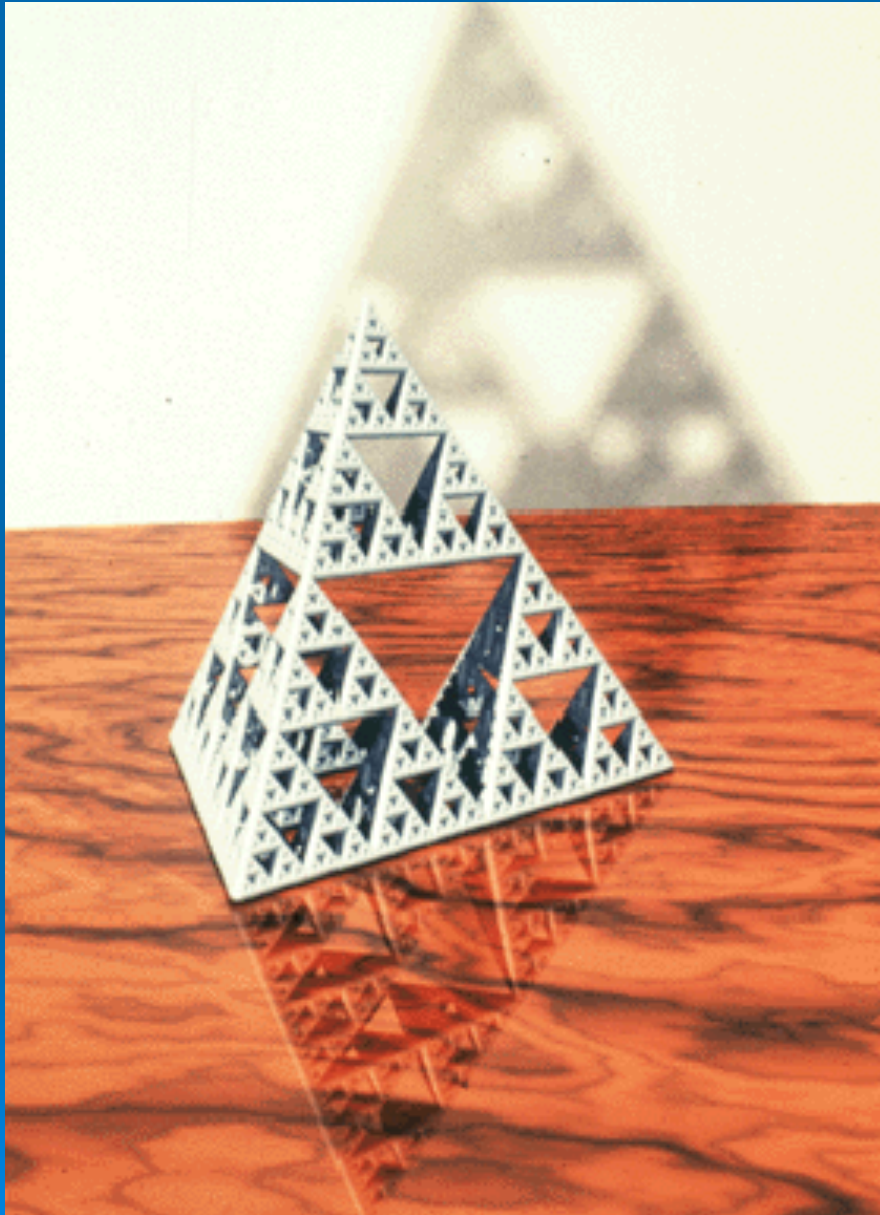
SG107.1.13.4 Platonic Origami (4) U.Mikloweit

German Polyhedra artist Ulrich Mikloweit has a website he describes as his *Garden of Polyhedra*. Please visit this extraordinary collection of exquisite paper models. Below are the 5 Platonic Solids created in paper by Ulrich Mikloweit.



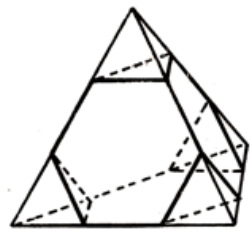
www.polyedergarten.de



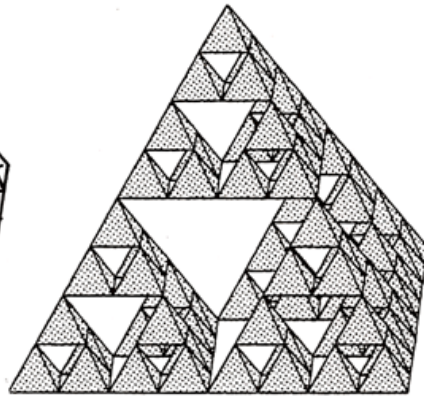


SG107.2 Chapter 2. The Tetrahedron

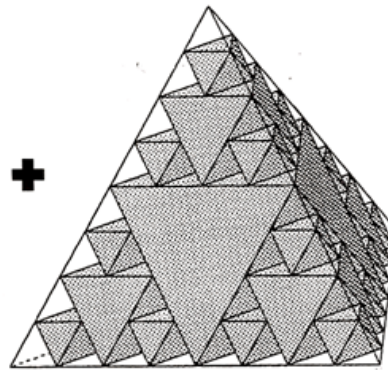
SG107.2.1 The Primordial Tetrahedron



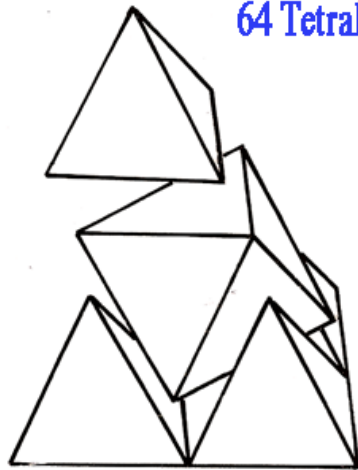
Truncating
a tetra



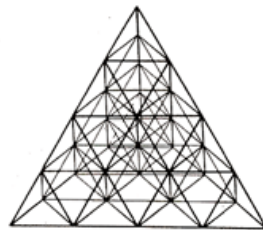
+



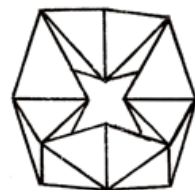
64 Tetrahedra + 21 Octahedra = one big solid Tetrahedron



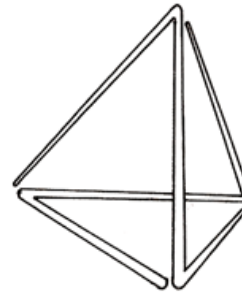
The Octahedron
within the Tetra



Isotropic Vector
Matrix (B. Fuller)



Tetrahedra
linked
face-to-face



"Triangular spiral
event in a Tetra"
(B. Fuller)

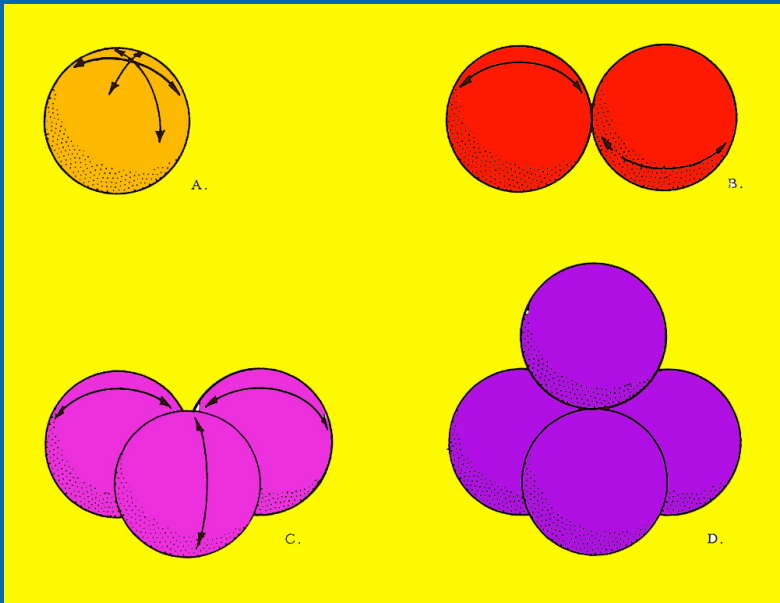
In his epoch-making
Synergetics (1975),
Buckminster Fuller
writes of the
tetrahedron:

"The tetrahedron is the first and simplest subdivision of Universe because it could not have an inside-ness and an outside-ness unless it had four vertexes and six edges... Between the four stars that form the vertexes of the tetrahedron there are six edges that constitute all the possible relationships between these four stars..."

By tetrahedron, we mean the minimum thinkable set that would subdivide Universe and have interconnectedness where it comes back upon itself... There are two kinds of number systems involved: four being prime number two and six being prime number three. So there are two very important kinds of oscillating quantities numberwise..."

(*Synergetics*. Section 620.00)

SG107.2.2 Tetrahedral Packing



↑ Four Spheres lock as a tetrahedron:

- A. A single sphere is free to rotate in any direction.
- B. Two tangent spheres must rotate cooperatively.
- C. Three spheres can rotate cooperatively only about axes parallel to the edges of the equilateral triangle defined by joining the spheres centers, i.e. each sphere rotates toward the center of the triangle.
- D. Four spheres lock together. No rotation is possible, making the minimum stable closest-packed-sphere system: the tetrahedron.



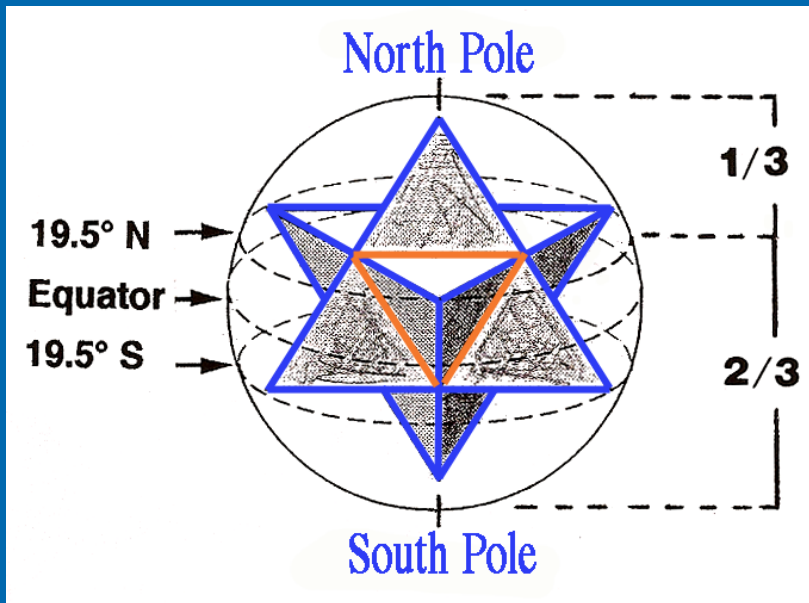
↑ Tetrahedral closest-packing of spheres:

Left: In any number of successive planar layers of tetrahedrally organized sphere packings, every third triangular layer has a sphere at its centroid (a nucleus). The 36-spheres tetrahedron with five spheres on an edge (4-frequency tetra) is the lowest frequency tetrahedron system which has a central sphere or nucleus.

Right: The 3-frequency tetrahedron is the highest frequency without a nucleus sphere.

[Buckminster Fuller. *Synergetics*. 1975]

SG107.2.3 The Tetrahedron & the Sphere



- The Tetrahedron is the only 3D volume whose corners (apices) are at the same distance from each other. There is no other volume with less than 4 corners.
 - The Tetra has a special relationship with the Sphere: the sphere packs the most volume in the least surface area, while the Tetra packs the least volume with the greatest surface area.
 - When the diameter of the circle is 12, the ratio of the area of the sphere to the tetra is 2 to 1. If a larger Tetrahedron has an inscribed sphere touching the centers of its 4 faces, then the characteristics of this superscribed tetra are 3 times those of the tetra inscribed in the sphere. Its height is twice the diameter of the sphere.
-
- When standing with one apex at the South pole (or the North pole), the Tetra divides the Sphere into 1/3 and 2/3.
 - When the Tetra is placed within a rotating sphere with one apex at the North or South pole, the other 3 apices will lie at 19°.47, i.e. about 19.5° from the equator.
 - The dance between the Tetra and the Sphere is basic to Buckminster Fuller's work (the "*isotropic vector matrix*"), as a mathematical blueprint for the universe.
 - The 7 symmetry spin axes of the Tetra, as a primary field of form, have been correlated by Dan Winter to the 7 layers of the heart muscle and the 7 regions of the rainbow donut.

SG107.2.4 Tetrahedral Physics

The Tetra-Sphere geometry has been found in locations as diverse as Teotihuacan, Mexico and the Cydonia site on Mars. It is an integral part of Tetrahedral Physics and what is now called Hyperdimensional Physics. This geometry (19.5°) is also appearing throughout the solar system as a primary planetary grid structure. Consider the following:

Energy upwelling at tetrahedral latitudes (19.5°) in the solar system:

SUN: sunspot activity and region of peak temperature limited to 19.5° North and South.

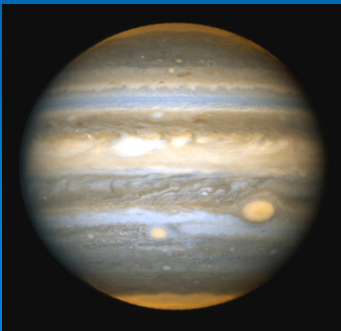
EARTH: largest cone volcano at 19.5° (Mauna Kea, HI).

VENUS: volcanic complexes Alpha and Beta Regio.

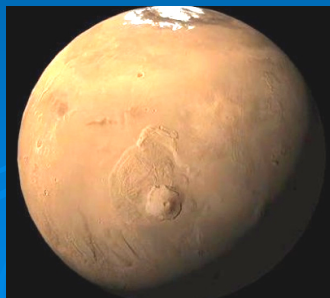
MARS: Olympus Mons cone volcano at 19.5° North.

JUPITER: "Red Spot" vortex.

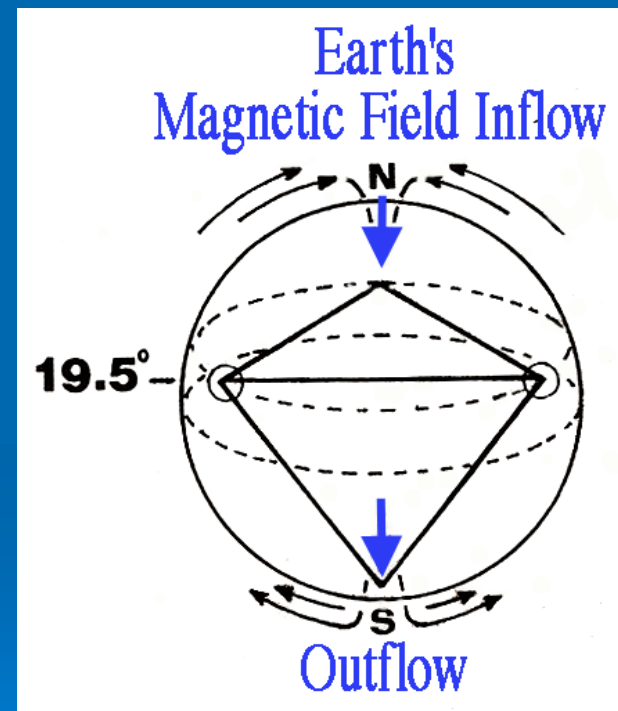
NEPTUNE: Voyager II discovered a vortex point at 19.5° .



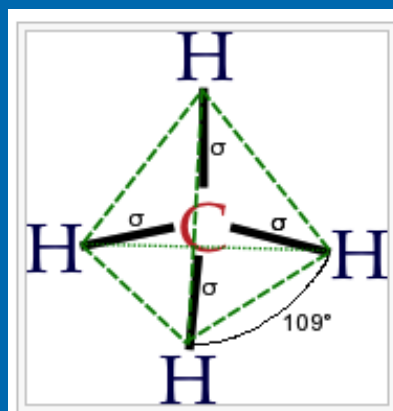
← Jupiter's Red Spot



← Olympus Mons on Mars



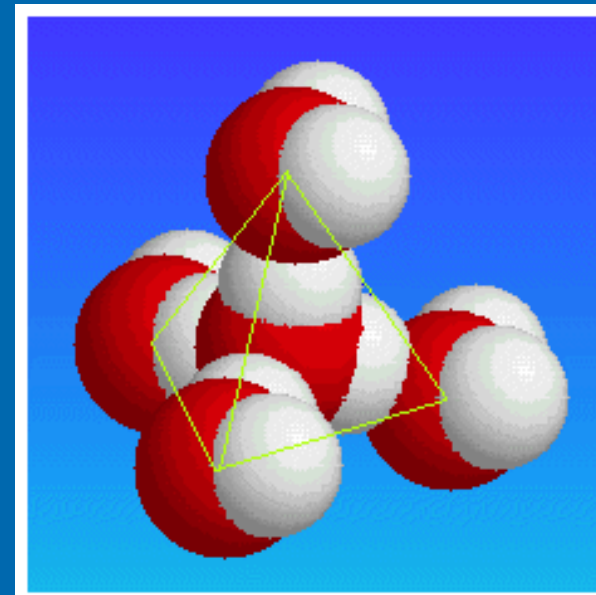
SG107.2.5 Tetrahedral Chemistry



↑ The tetrahedral molecule
Methane (CH₄)

In a tetrahedral molecular geometry a central atom is surrounded by four substituents that are located at the corners of a tetrahedron. The bond angles are $\approx 109.5^\circ$ when all four substituents are the same, as in CH₄.

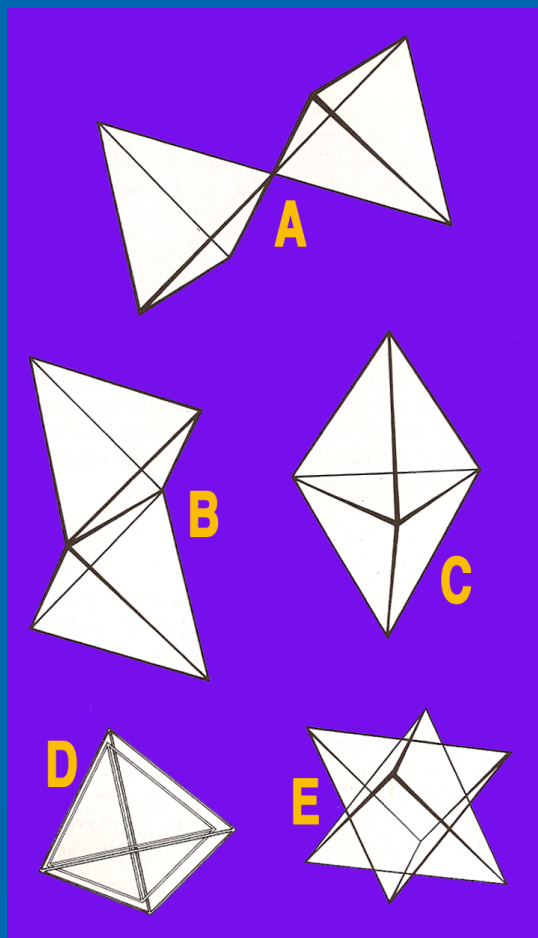
This molecular geometry is common throughout the first half of the periodic table. Tetrahedral molecules can also be chiral (left or right-handed).



↑ Tetrahedral structure
of water molecules

The hydrogens on each water molecule point toward the oxygens on neighboring water molecules, such that the whole structure is based around a tetrahedral shape. Although since this is a liquid, the molecules are continually moving and rotating, breaking and reforming hydrogen bonds, so the tetrahedral structure can be thought of as only a time averaged approximation.

[www.ch.ic.ac.uk]

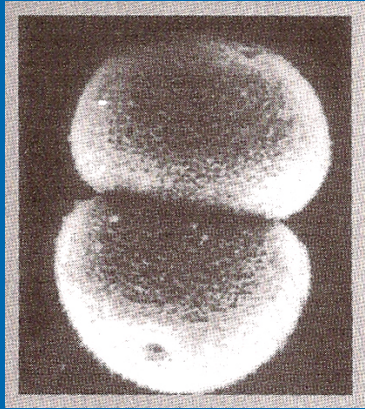


↑ Tetrahedral Chemical Bonding

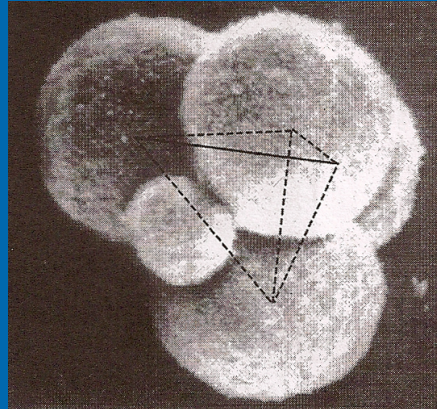
- A. Single-bonded tetra (gases)
- B. Double-bonded tetra (liquids)
- C. Triple-bonded tetra (crystals)
- D, E Quadri-bond / mid-edge tetra systems demonstrate the super strength of diamond & metals.

SG107.2.6 Tetrahedral Biology

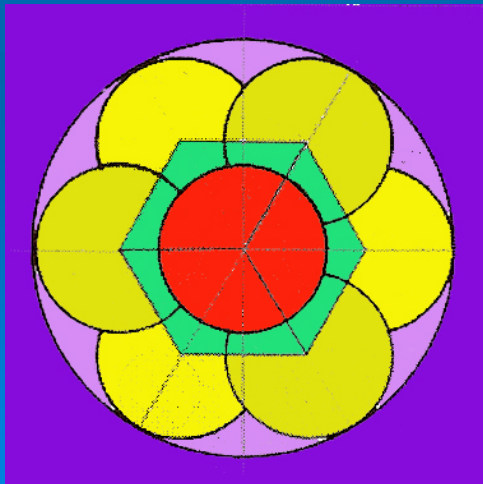
When the process of mitosis (cell division) progresses from 2 to 4, the first 4 cells from a tetrahedron. Going from 4 to 8 cells, an interlocked tetrahedron (Star-Tetra) is formed.



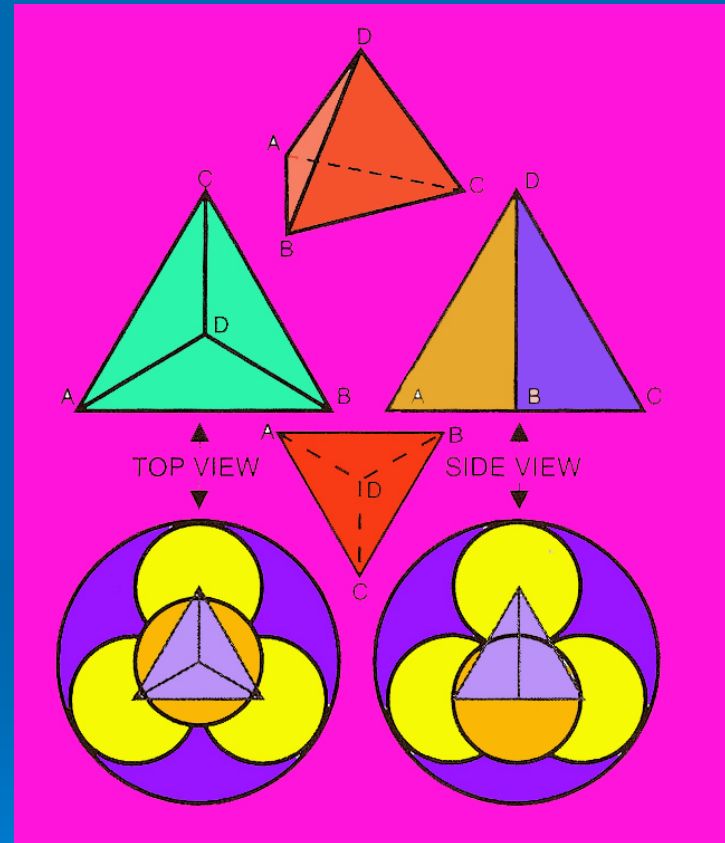
↑ First 2 cells in mouse egg



↑ First 4 cells in mouse egg



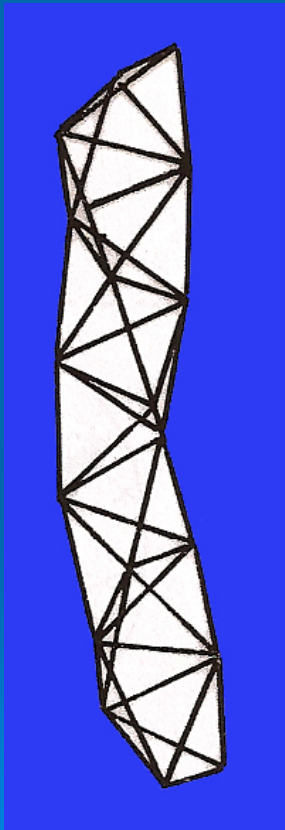
← Geometries of the first 8 cells: 2 interlocked tetrahedra inscribed in a cube (Star-Tetra)



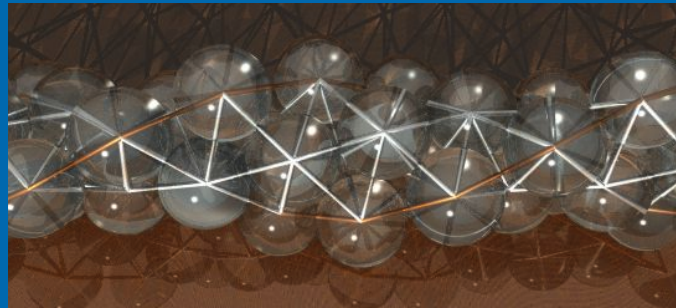
↑ Tetrahedral geometries of the first 4 cells

SG107.2.7.1 The Tetrahelix (1)

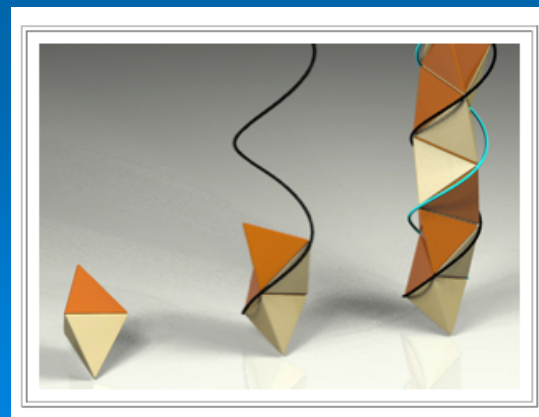
The tetrahelix of Fuller's Synergetics consists of face bond regular tetrahedra. The vertices of the regular tetrahedra of the tetrahelix all lay on the surface of a cylinder. The tetrahedral helix is also called the 'Bernal spiral' in the physics of liquids.



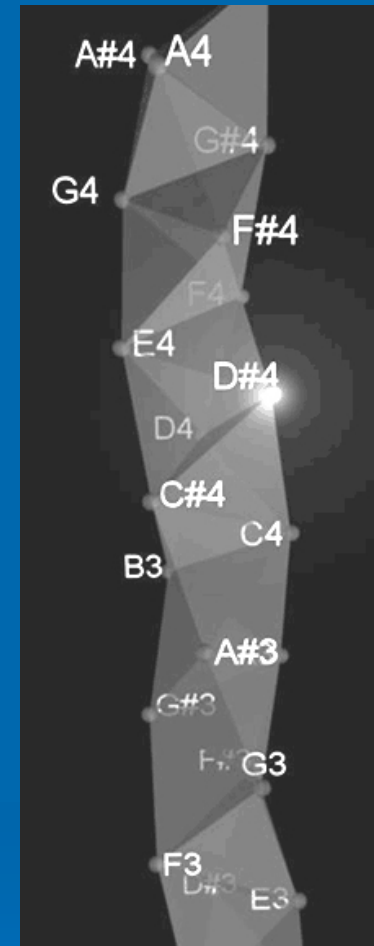
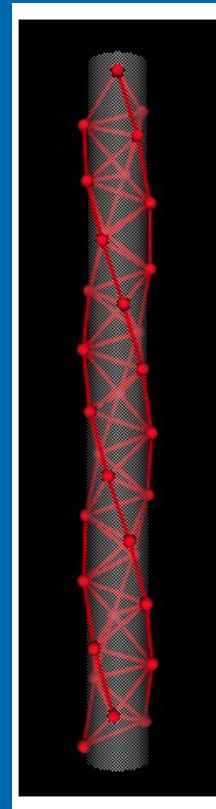
A Tetrahelix



↑ Rendering of a double tetrahelix showing both the struts & spheres versions, [//bobwb.tripod.com](http://bobwb.tripod.com)



← www.mi.sanu.ac.yu



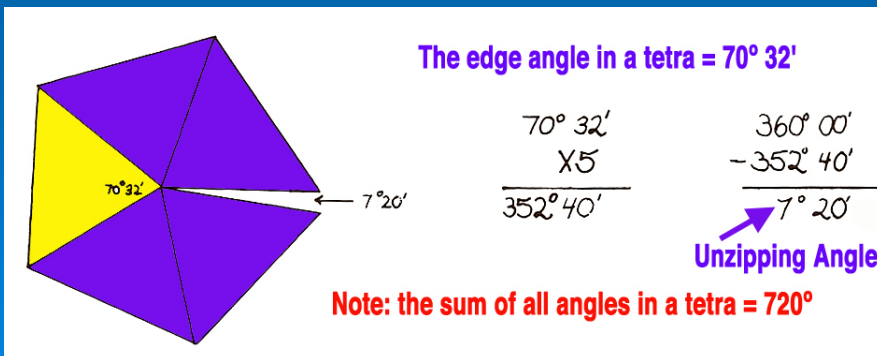
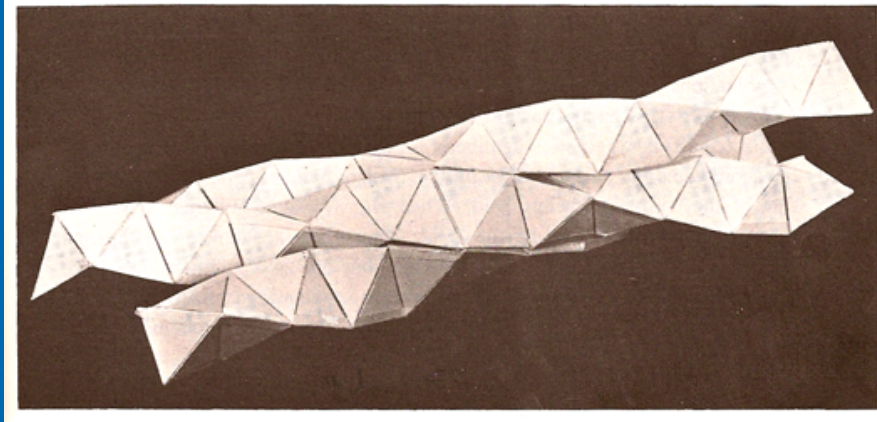
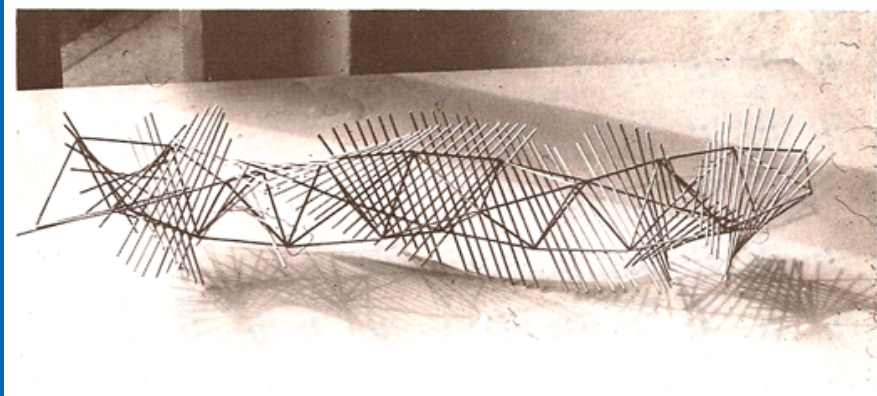
↑ A tetrahelix used in modeling Bach's vocal music. [//mto.societymusictheory.org](http://mto.societymusictheory.org)

SG107.2.7.2 The Tetrahelix (2)

← According to Buckminster Fuller, helical columns of tetrahedra (tetrahelices) explain the structuring of DNA.

“When nestled together, the tetrahedra are grouped in local clusters of 5 tetrahedra around a transverse axis. Because the dihedral angle of 5 tetra are 7°20' short of 360°, the 5 tetra do not close up to make 360°. But when they are brought together in a helix - due to the fact that a hinged helix is a coiled spring - the columns will twist enough to permit the progressive gaps to be closed.

The backed-up spring tries constantly to unzip one nesting tetra from the others of which it is a true replica. Unzipping occurs as the birth dichotomy and the new life breaks from the old pattern with a perfect imprint and repeats the other's growth pattern... When a column comes off, i.e. unzips, it is a replica of the original column.”



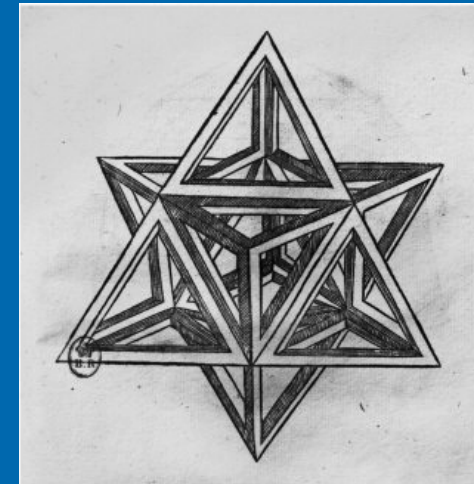
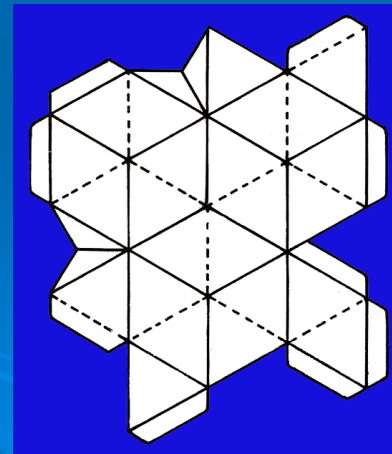
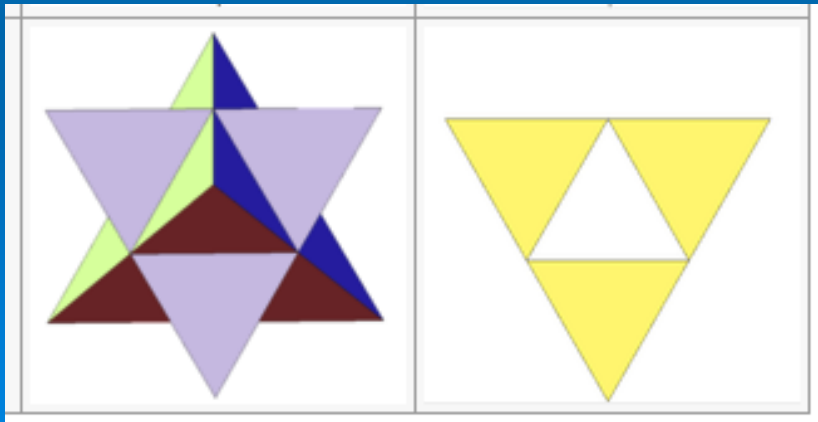
SG107.2.8.1 The Star-Tetrahedron (1)

The *Stella Octangula* was known to earlier geometers. It was first depicted in Pacioli's *De Divina Proportione*, 1509.

It is the simplest compound of the 5 Platonic Solids. It can be seen as either a polyhedron compound or a stellation.

As a compound, it is constructed as the union of two tetrahedra (a tetrahedron and its counter-tetrahedron). The vertex arrangement of the two tetrahedra is shared by a cube. The intersection of the two tetrahedra form an inner octahedron, which shares the same face-planes as the compound. Visualize it as an octahedron with tetrahedral pyramids on each face. It has the same topology as the convex Catalan solid, the *triakis octahedron*, which has much shorter pyramids.

As a stellation, it is the only stellated form of the octahedron. The stellation facets are very simple and shown below.



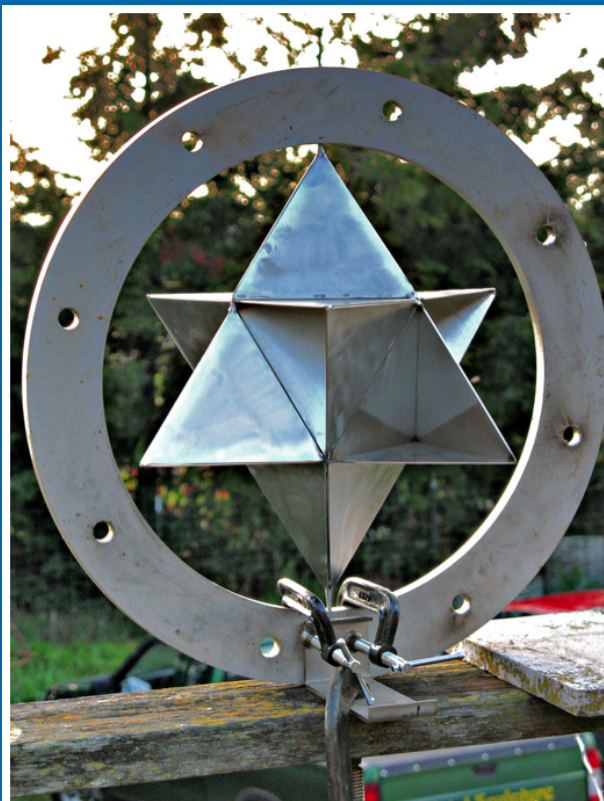
↑ Stella Octangula
by Da Vinci

3D names:
Stella Octangula
Stellated Octahedron
Star-Tetrahedron
Eight-pointed Star
Star Mer-Ka-ba

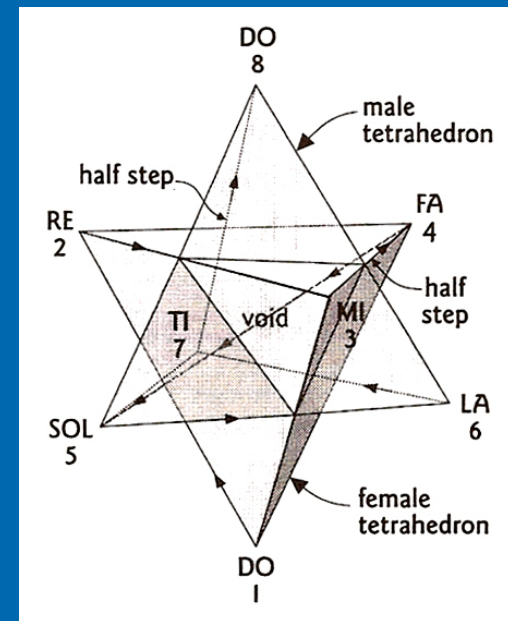
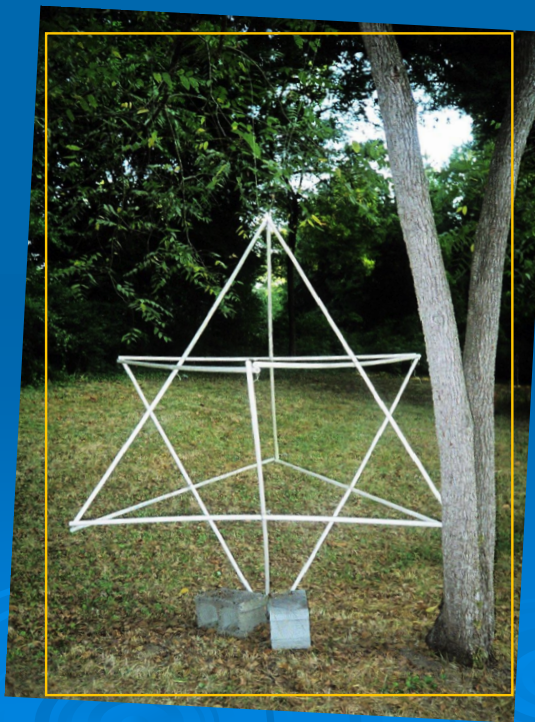
and in 2D:
Star of David
Solomon's Seal

← 2D net for the
Star-Tetra

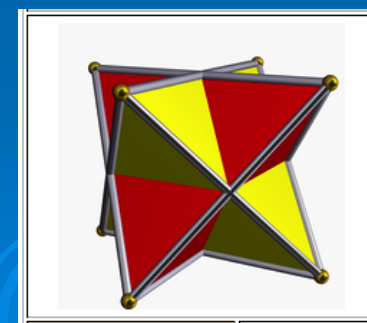
SG107.2.8.2 The Star-Tetrahedron (2) in Culture



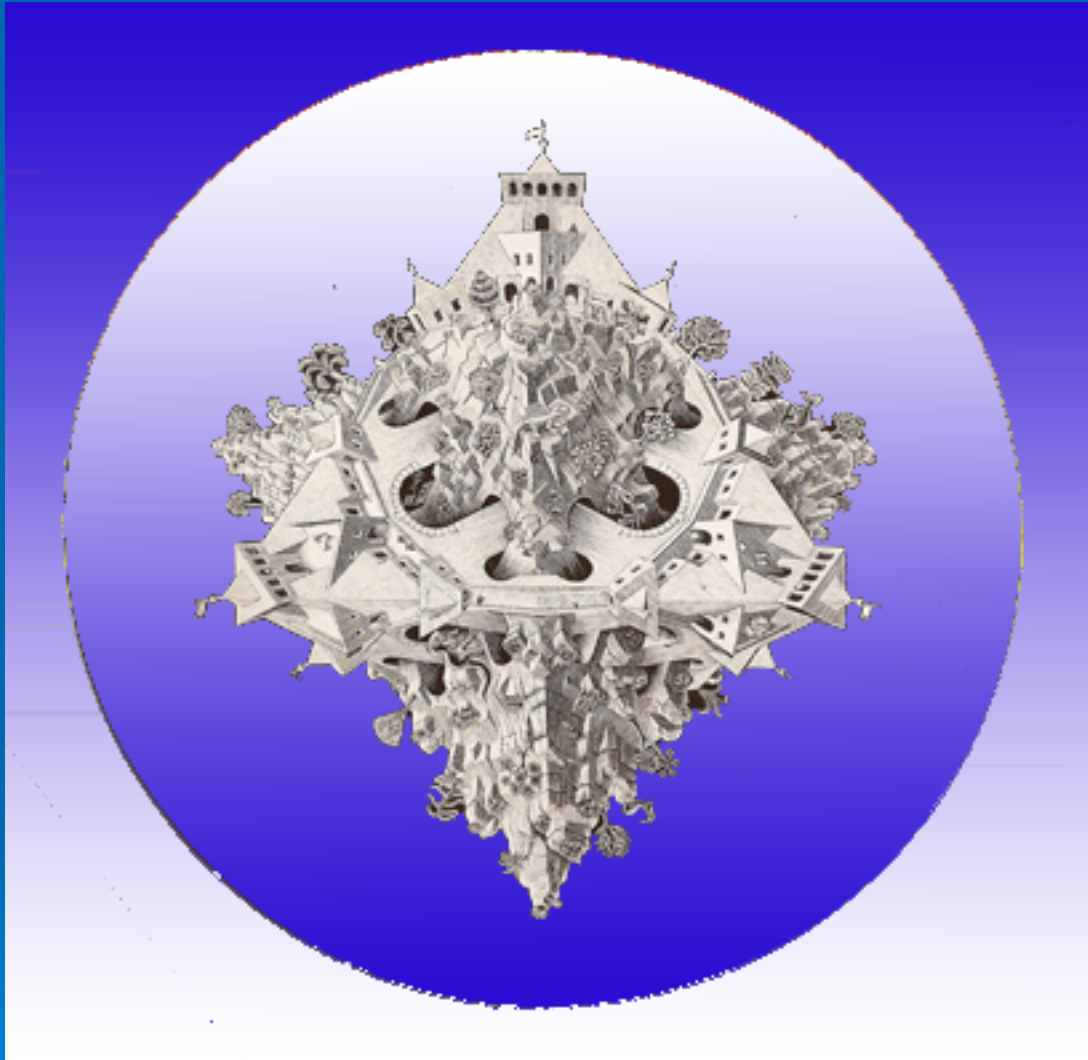
↑ Star-tetra Wind Spinner
by Stephen Fitzgerald
People.tribe.net



↑ Musical analogy of the energy
flows within a Star-Tetra.
(Drunvalo Melchizedek)



SG107.2.8.3 The Star- Tetrahedron (3)

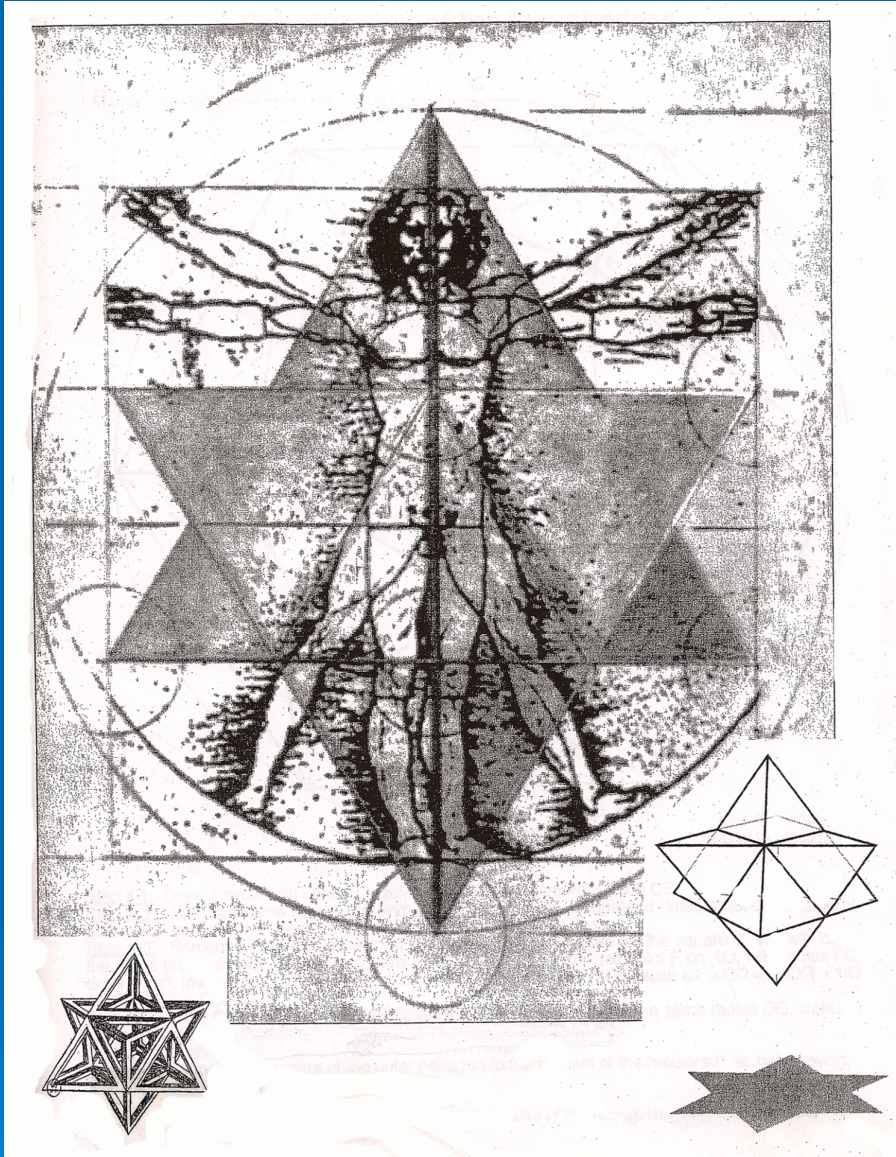


The “Double Planetoid”
wood engraving (1949)
by M. C. Escher features
a Star-Tetrahedron.

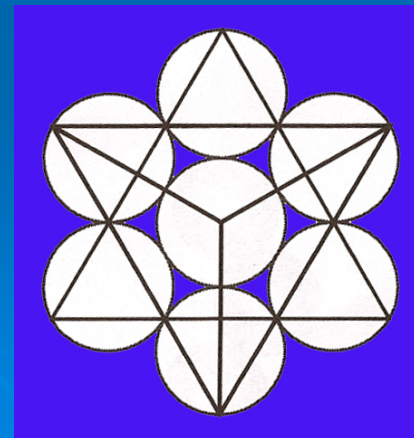
(Color background added).

SG107.2.9.1 Star-Tetra & Merkaba (1)

Drunvalo Melchizedek, in the seminal two volumes of *The Ancient Secret of the Flower of Life*, describes one aspect of the geometries of higher dimensional light infusing the physical human body-mind: the star-tetrahedral field of the “Mer-Ka-Ba” and its rotational spins. The Flower of Life workshops teach people the various steps necessary to activate this field through breath and meditation techniques. Originally, according to D. Melchizedek, the Star-tetra field is an extension of the original 8 cells called the “Egg of Life”. It then extends to a great distance around the body.

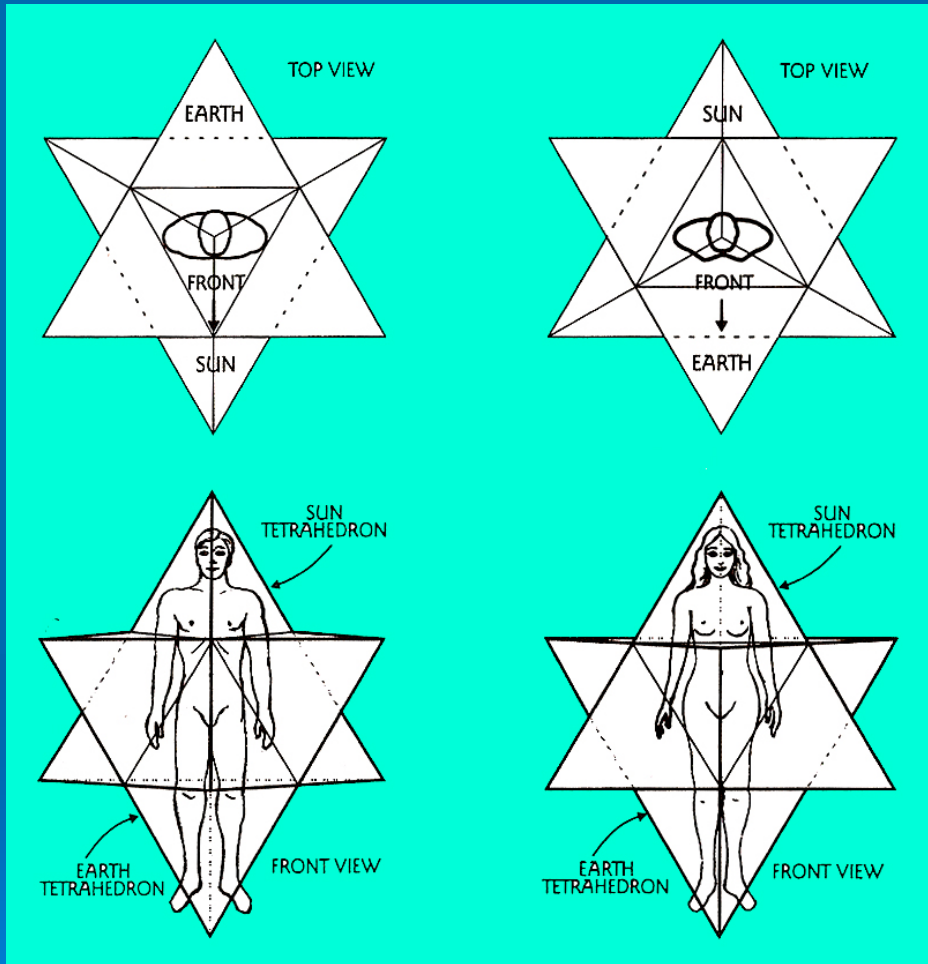


The “Da Vinci Man” with the star tetrahedron energy field (Mer-Ka-Ba) and Prana Tube.
Drunvalo Melchizedek. Flower of Life. 1990.

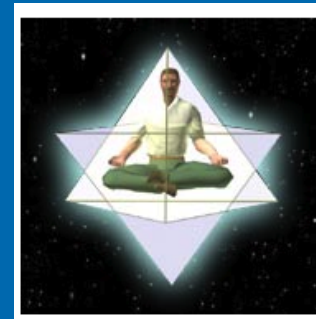


← Geometries of the Star-Tetra mapped on the Egg of Life.

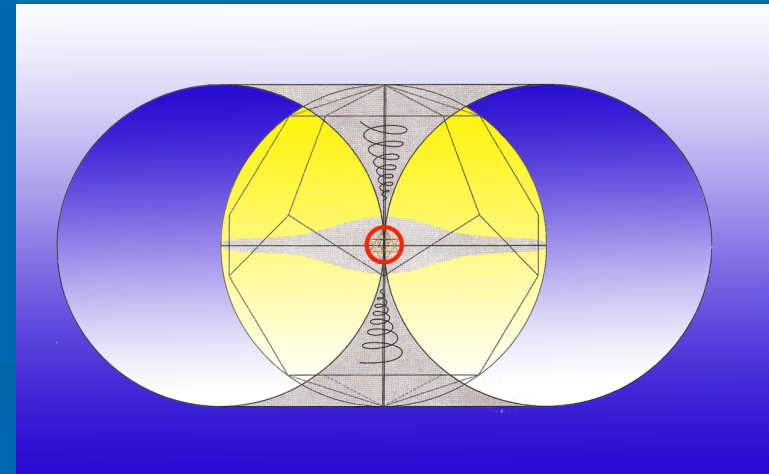
SG107.2.9.2 Star-Tetra & Merkaba (2)



↑ The male & female configuration of the Mer-Ka-Ba Star-Tetra fields.



Using a progression of 17 breaths practices, the Mer-Ka-Ba system creates a form of *spherical breathing* within the body and activates the counter-rotating fields of the Star-Tetra auric geometries.



The vortex-torus “Light Body” energy geometries surrounding the Star-Tetra field (red circle at center) and all aspects of the universe, on all scales.



www.floweroflife.org
www.onelotus.net

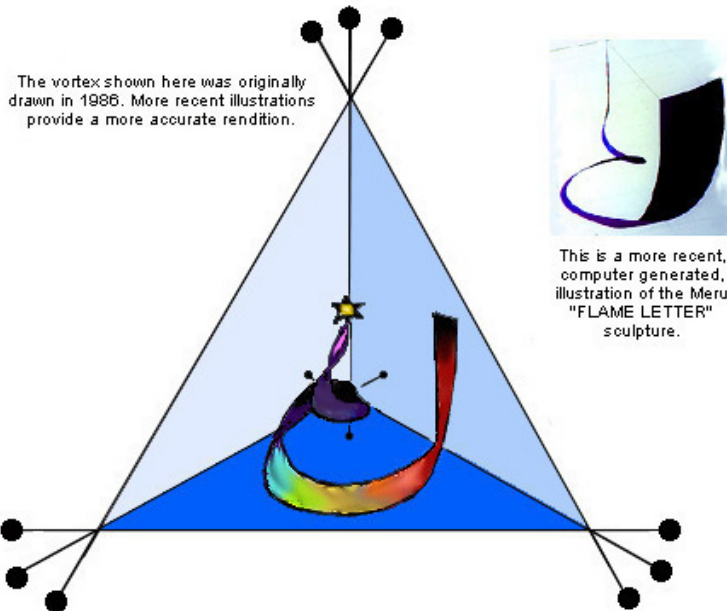
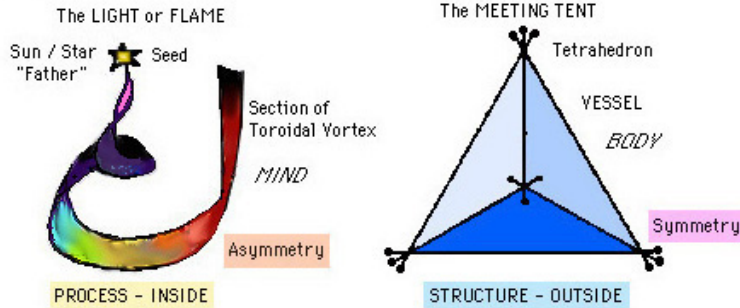
SG107.2.9.3 Star-Tetra & Merkaba (3)



forum.myspace.com



The LIGHT in the MEETING TENT

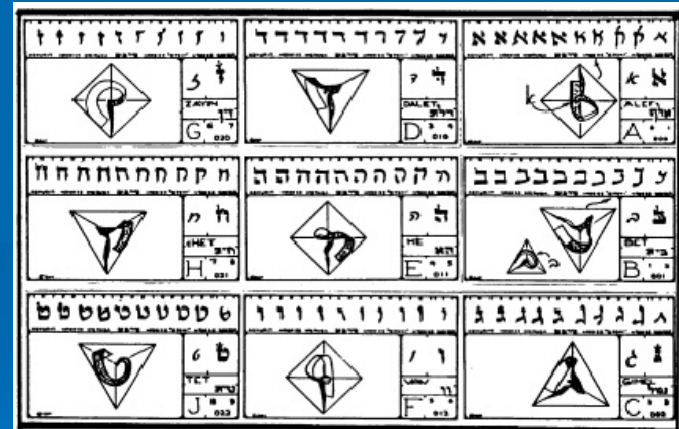


LightinTent Graphic R.3.0 ©'86,'96 S. Tenen / MERU Fdtn, P.O.B 503, Sharon, MA 02067 617 784-8902 meru1@well.com http://www.meru.org

SG107.2.10.1 Tetrahedron & Alphabets

In the 1980s, Stan Tenen of the Meru Foundation came up with the seminal idea that spinning the Golden Spiral (the "Flame") within a tetrahedron (the "Meeting Tent") was creating the shapes of the Hebrew letters.

Since then, the Meru Foundation has offered a variety of educational materials extending this original discovery to other languages as well.

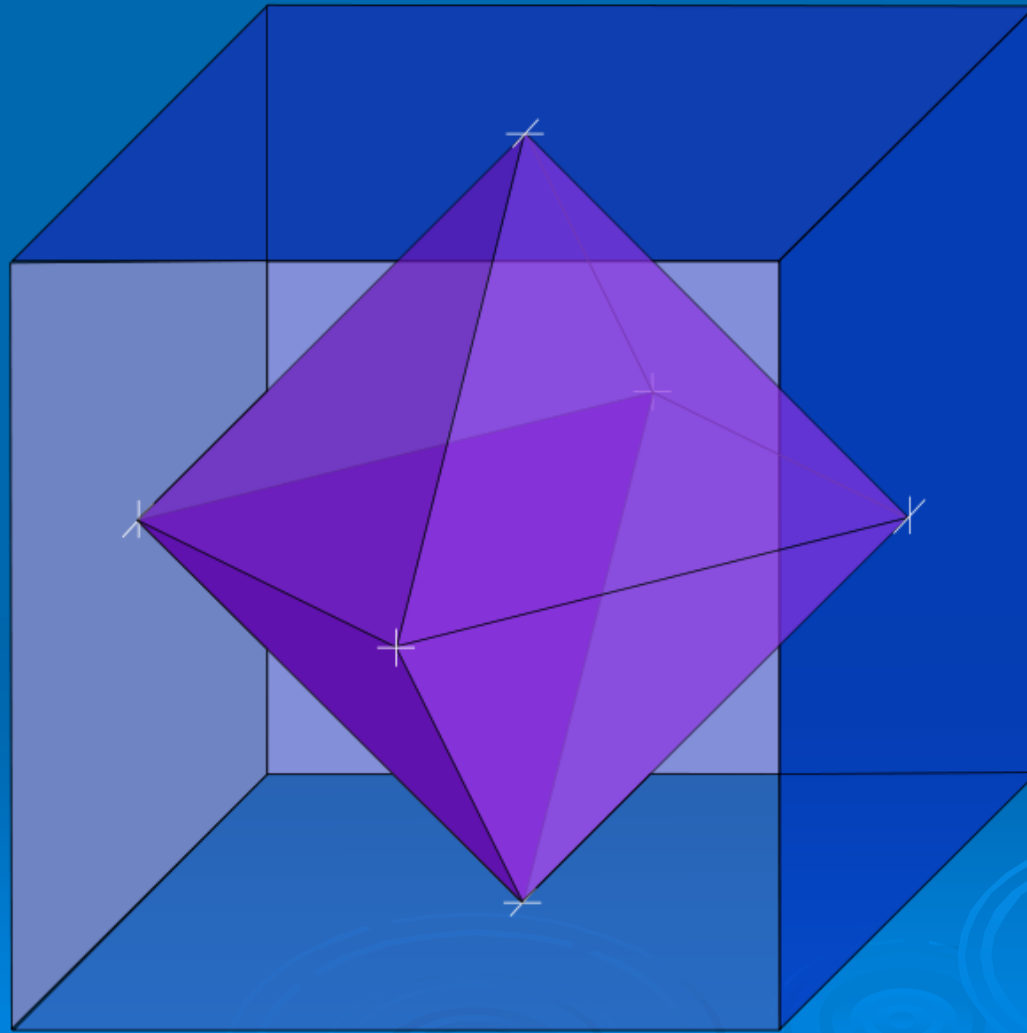


↑ Some of the original hand-drawn sketches of the 27 Hebrew letters as shadows of the Golden Spiral in a tetrahedral geometry. Stan Tenen. 1986.



www.meru.org

SG107.3. Chapter 3. The Cube & Octahedron

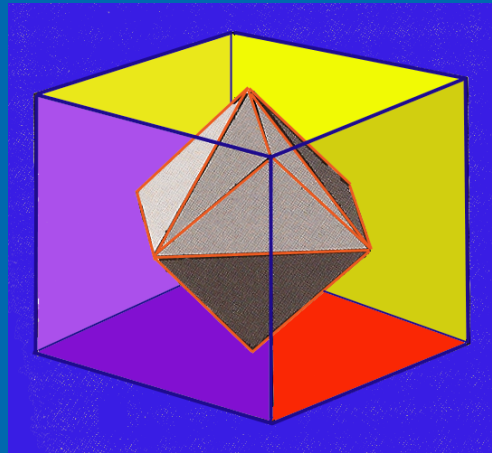


SG107.3.1 The Cube

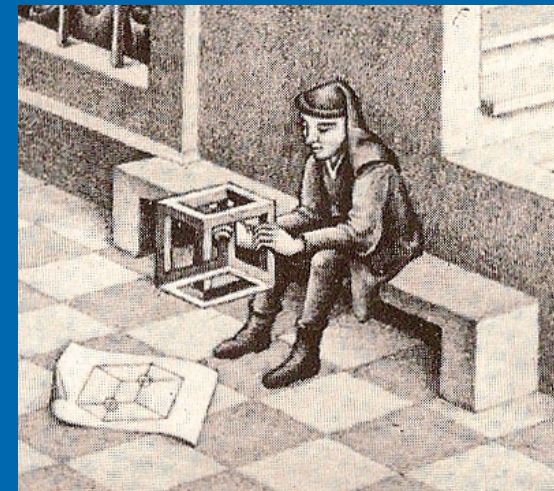
The Cube or Hexahedron can be thought of as the interface standard between the 5 Platonic solids.

It does inscribe:

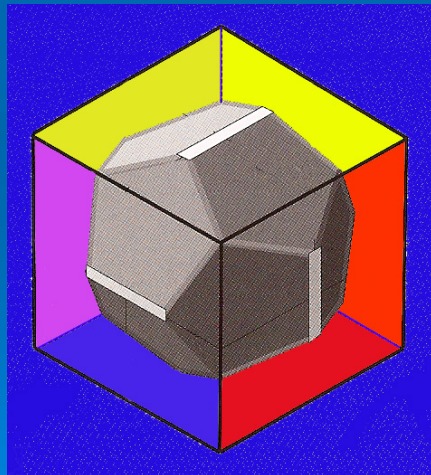
- Tetrahedron and Star-Tetra.
- Octahedron
- Icosahedron
- Dodecahedron



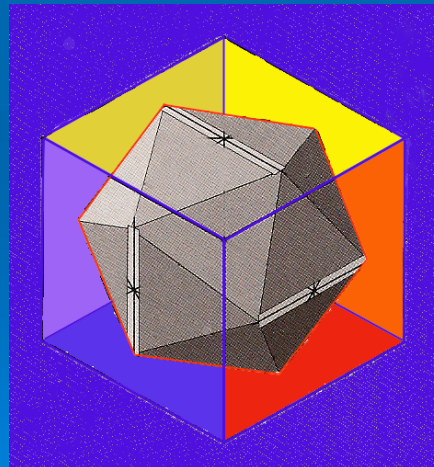
Octa in cube



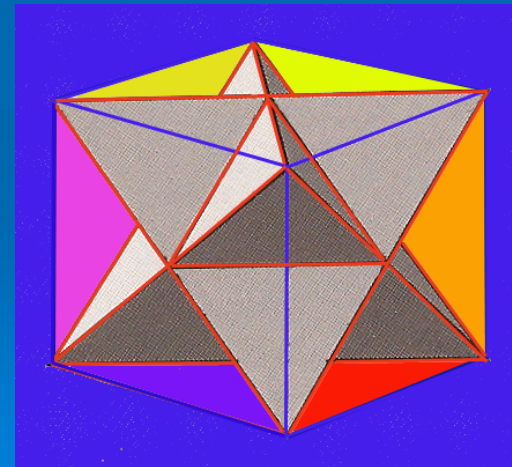
↑ Detail of Escher's
"Belvedere". 1958



Dodeca in cube



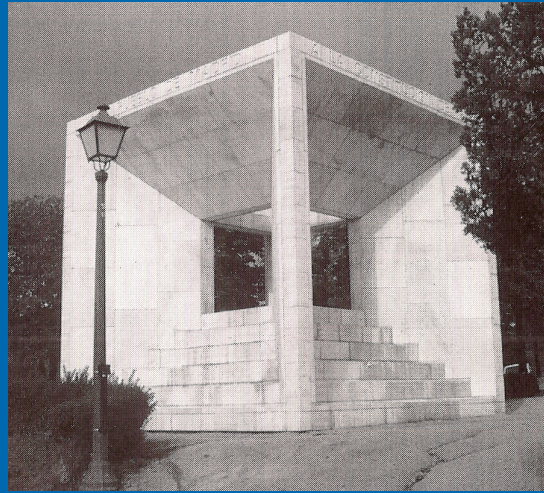
Icosa in cube



Star-Tetra in cube

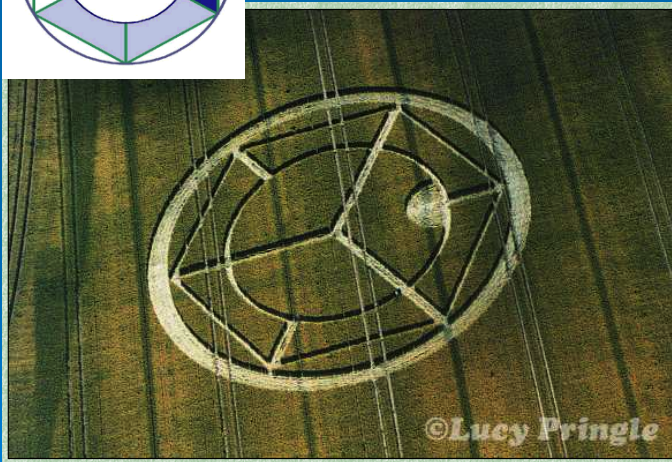
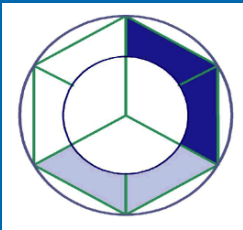
SG107.3.2.1 Cubes in Culture (1)

The Statue of →
the Constitution.
Madrid.



“The number Four, the first square number, reconciles the two forms of mathematical growth, being both $2+2$ and 2×2 , and it represents the human instinct for symmetry and order by dividing the compass onto four points and the year into four seasons. It is at the foundation of civilization, settlement and rectangular land division. It is the foursquare number of solid earth as opposed to formless heavens.”

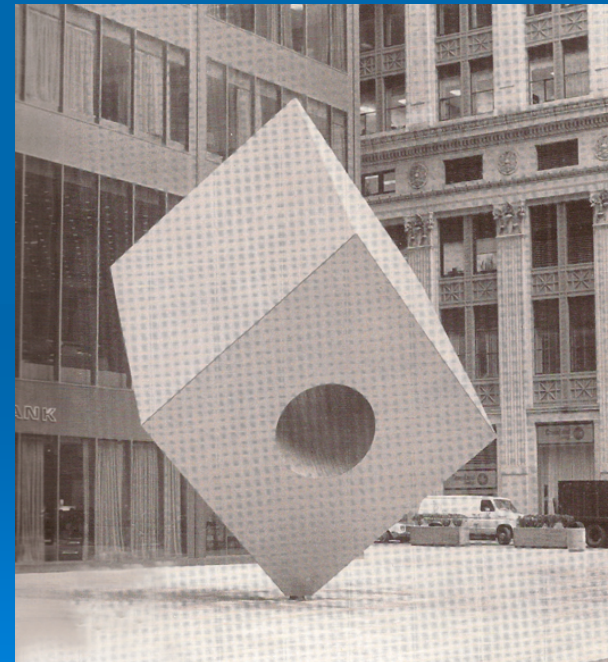
John Michell



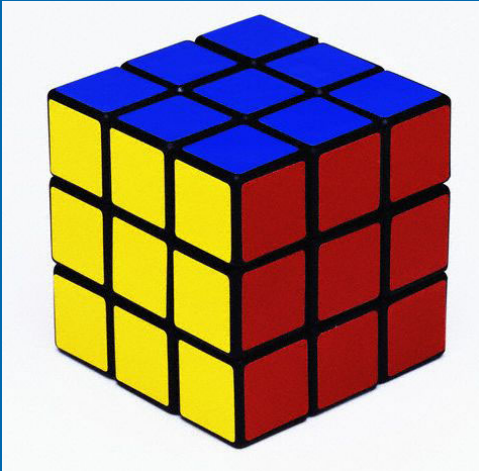
©Lucy Pringle

↑ Crop Circle. Honey Street, Wiltshire
July 17th, 1999

Cube sculpture →
in New York City.



SG107.3.2.2 Cubes in Culture (2) Rubik's Cube.



↑ A Rubik's Cube, solved.



↑ A Rubik's Cube, scrambled.

The Rubik's Cube, invented in 1974 by Hungarian sculptor and architect Ernő Rubik, is considered to be the world's best-selling toy. According to *Wikipedia*, 350 million cubes have sold worldwide, as of January 2009.

Permutations:

“The original (3×3×3) Rubik's Cube has eight corners and twelve edges. There are 8! (40,320) ways to arrange the corner cubes. Seven can be oriented independently, and the orientation of the eighth depends on the preceding seven, giving 3⁷ (2,187) possibilities. There are 12!/2 (239,500,800) ways to arrange the edges... Eleven edges can be flipped independently, with the flip of the twelfth depending on the preceding ones, giving 2¹¹ (2,048) possibilities.

$$8! \times 3^7 \times 12! \times 2^{11} = \text{approx. } 4.33 \times 10^{19}$$

There are exactly 43,252,003,274,489,856,000 permutations, which is approximately forty-three quintillion. The puzzle is often advertised as having only "billions" of positions, as the larger numbers could be regarded as incomprehensible to many.

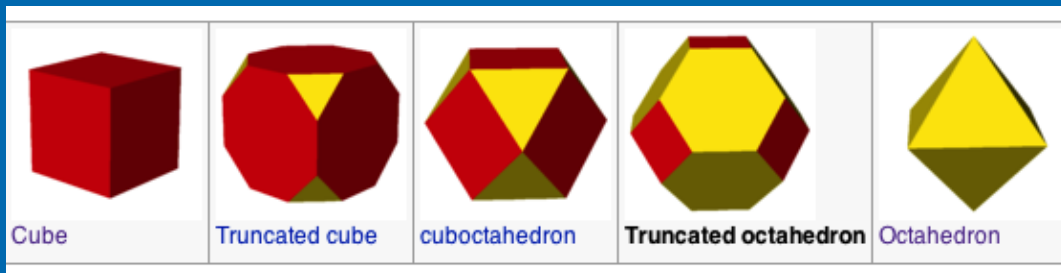
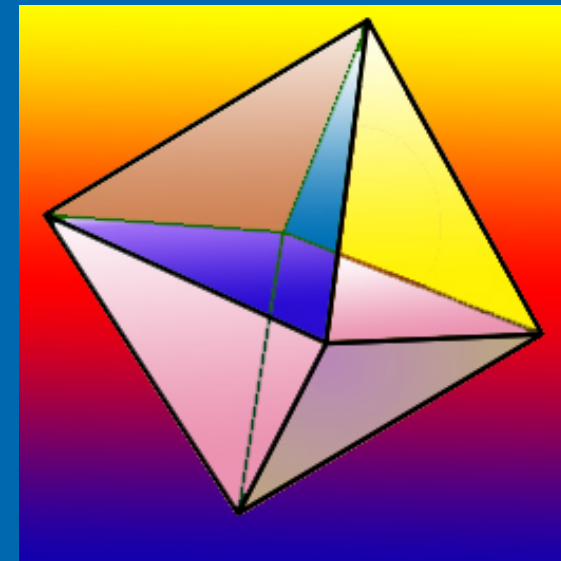
To put this into perspective, if every permutation of a 57-millimeter Rubik's Cube were lined up end to end, it would stretch out approximately 261 light years. Alternatively, if laid out on the ground, this is enough to cover the earth with 273 layers of cubes, recognizing the fact that the radius of the earth sphere increases by 57 mm with each layer of cubes.”

(Wikipedia)

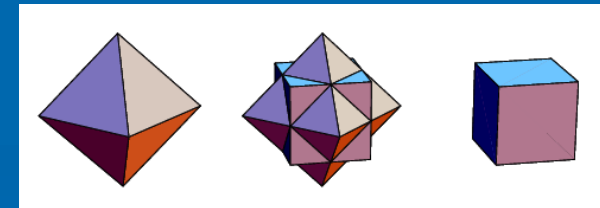
SG107.3.3.1 The Octahedron (1)

The octahedron is unique among the Platonic solids in having an even number of faces meeting at each vertex. Consequently, it is the only member of that group to possess mirror planes that do not pass through any of the faces.

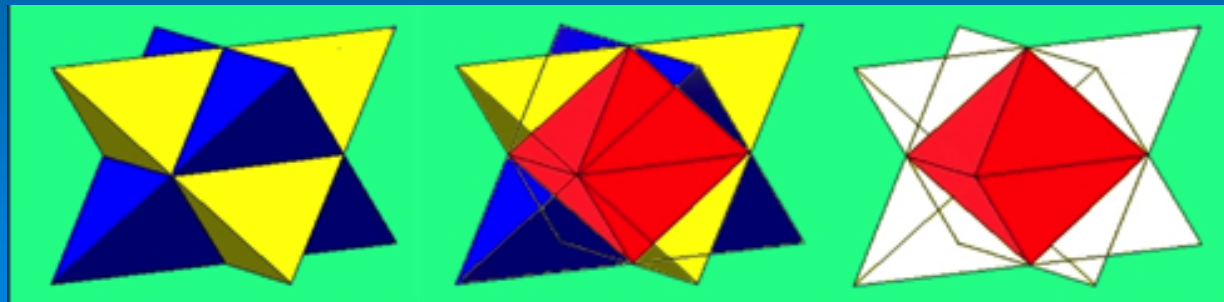
In terms of Sacred Geometry, one can also divide the edges of an octahedron in the Golden Ratio to define the vertices of an icosahedron.



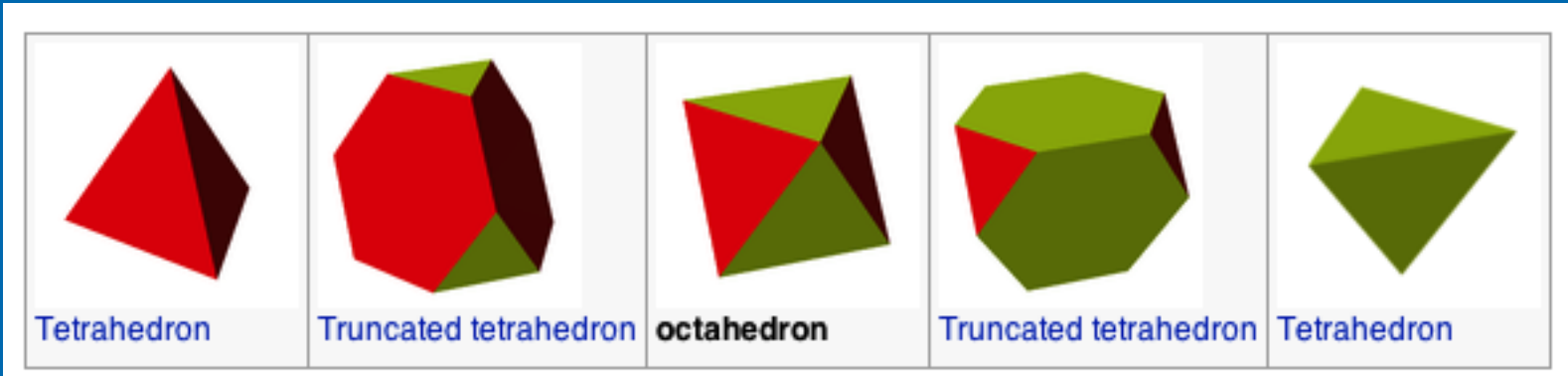
↑ Truncation sequence cube to octa. (Wikipedia)



↑ Octa & Cube



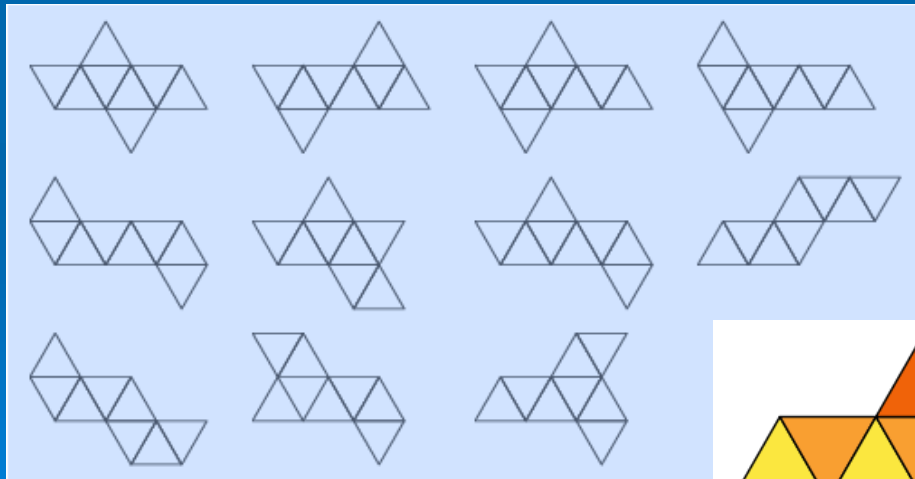
← The Octahedron within the Star Tetrahedron
(Graphics: Jay Goldner)
www.kornckler.dk



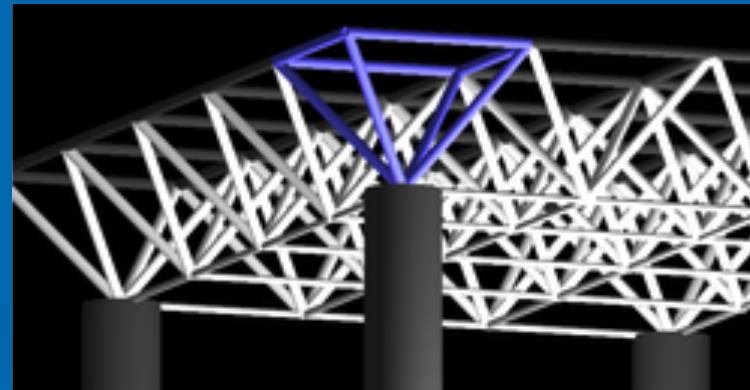
The octahedron can also be considered a rectified tetrahedron.

↑ Truncation sequence tetra to octa to tetra. (Wikipedia)

SG107.3.3.2 The Octahedron (2)



The 11 “nets” of the Octahedron.
Pick your favorite and built an Octahedron!

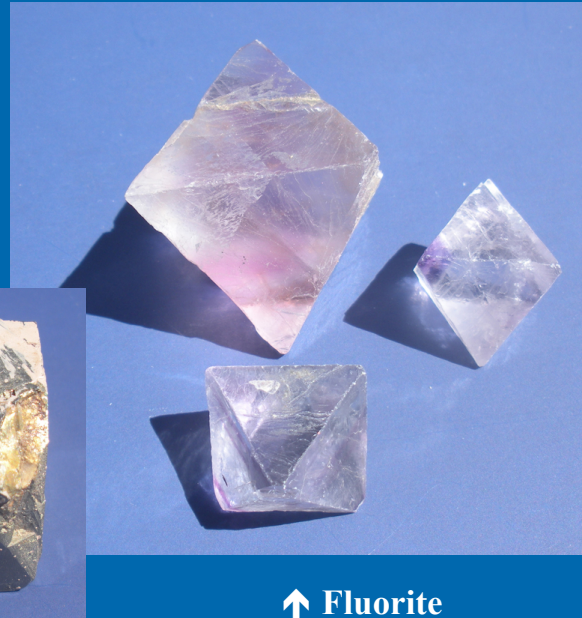


↑ Octahedra and tetrahedra can be alternated to form a vertex, edge, and face-uniform tessellation of space, called the *octet truss* by Buckminster Fuller. (Wikipedia)

SG107.3.4 The Cube & Octa in Nature



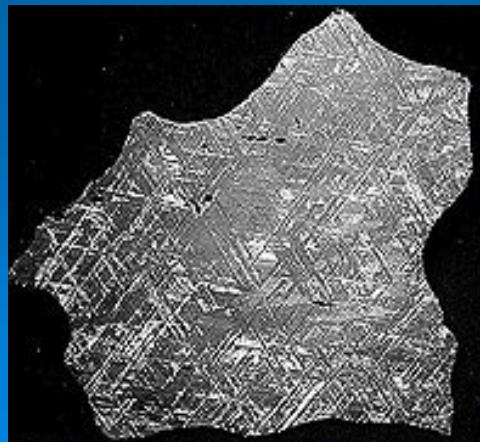
Pyrite ↑ →



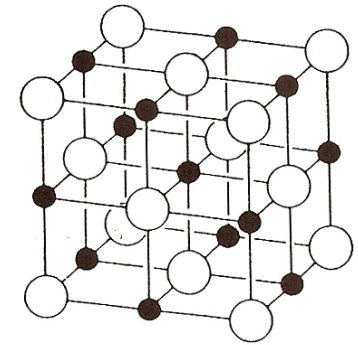
↑ Fluorite



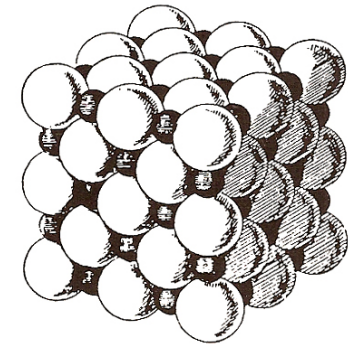
↑ The most common form of a rough diamond



← The most common type of iron meteorite is called “*octahedrite*” because of its structure: plates of the mineral *kamacite* are oriented parallel to the sides of an octahedron.

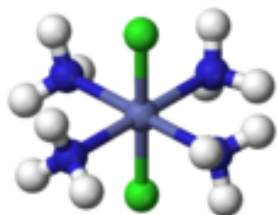


Two molecular models of sodium and chloride ions in the common salt crystal



SG107.3.5 Octahedral Molecular Geometry

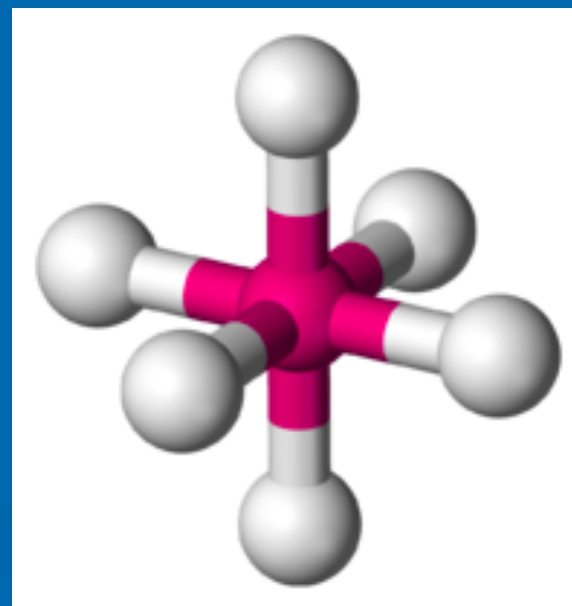
In chemistry, octahedral molecular geometry describes the shape of compounds where in six atoms or groups of atoms (ligands) are symmetrically arranged around a central atom, defining the vertices of an octahedron.



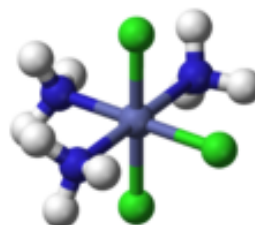
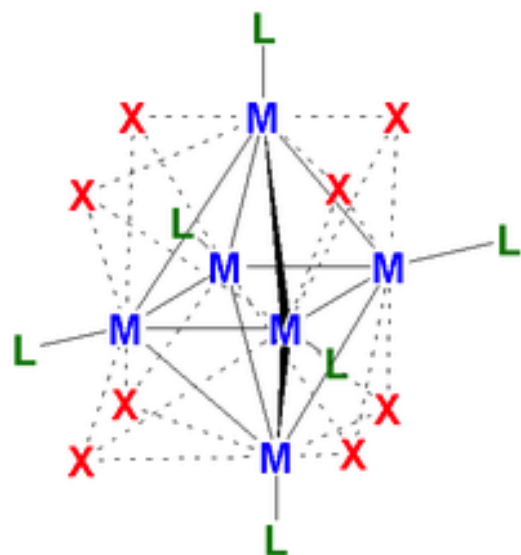
trans-[CoCl₂(NH₃)₄]⁺



fac-[CoCl₃(NH₃)₃]



↑ Idealized structure of a compound with octahedral coordination geometry.



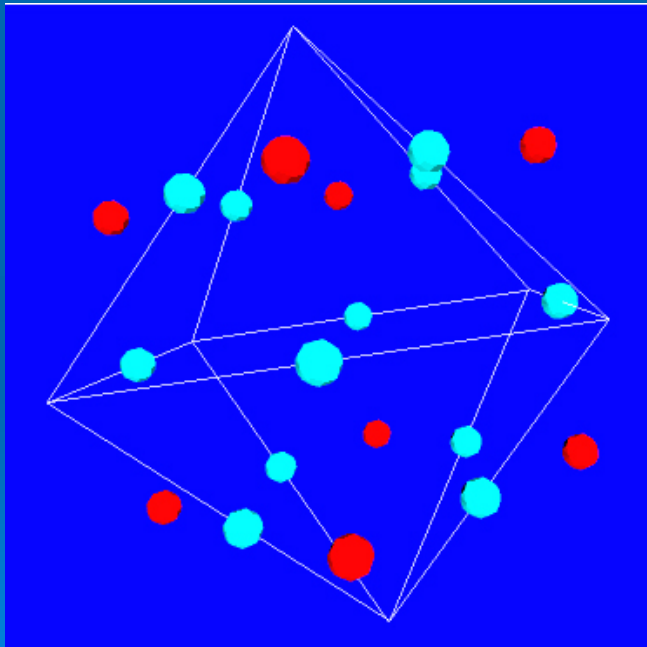
mer-[CoCl₃(NH₃)₃]

← Octahedral clusters are inorganic or organometallic cluster compounds composed of six metals in an octahedral array. Important classes of octahedral clusters are *chalcogenide* and *molybdenum* clusters.

SG107.3.6.1 Octahedral Music - the Hexany (1)

In music theory, the *Hexany* is a six-note just intonation scale, with the notes placed on the vertices of an octahedron. The notes are arranged so that every edge of the octahedron joins together notes that make a consonant dyad, and every face joins together the notes of a consonant triad.

This makes a "*musical geometry*" with the geometrical form of the octahedron. It has eight just intonation triads in a scale of only six notes, and each triad shares a pair of notes with each its three neighbors, arranged in a musically symmetrical fashion due to the symmetry of the octahedron. The chords fit together in exactly the same way as the faces fit together in the geometrical shape.



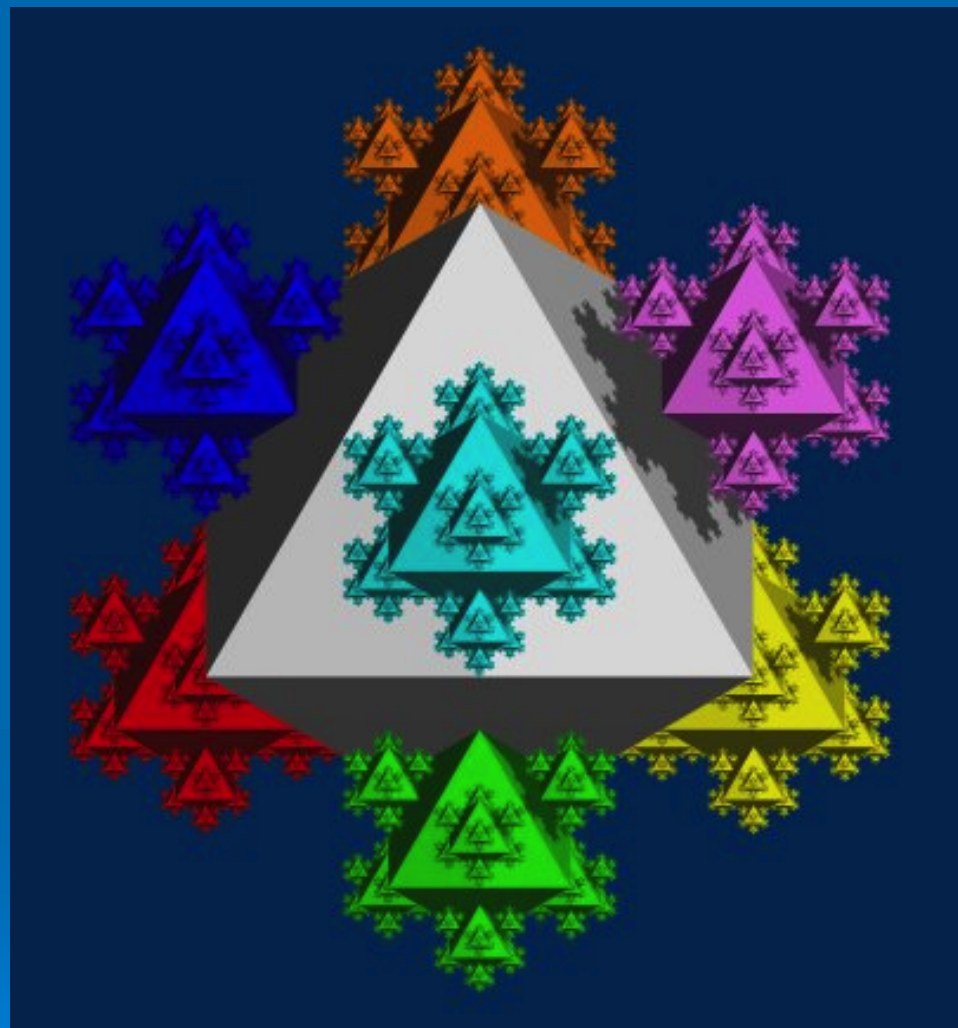
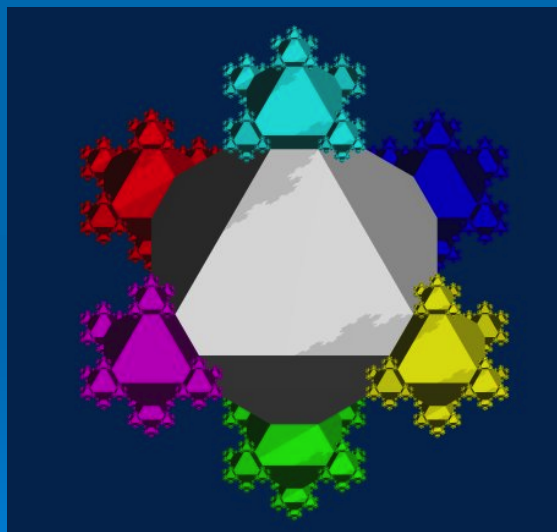
The *Hexany* is an idea due to Erv Wilson, a prolific Mexican-American music theorist, specialized in the areas of microtonal music and just intonation. The Hexany scales are obtained as successive cross sections of an n-dimensional cube, and the numbers of vertices follow the numbers in Pascal's Triangle. The octahedral Hexany is the third cross section of the four-dimensional cube.

There are other models of '*geometric music*' created by Erv Wilson and other researchers: *Octany* (8 notes), *Dekany* (10 notes, including tetrads), and *Eikosany* (20 notes in six dimensions, including pentads, tetrads, triads as well as hexanies).

← Digital animation keyboard of the Hexany.
The red spheres play triads, the blue spheres play dyads and the golden spheres at the vertices play single notes.

[//robertinventor.com/software/Hexany/hexany.htm](http://robertinventor.com/software/Hexany/hexany.htm)

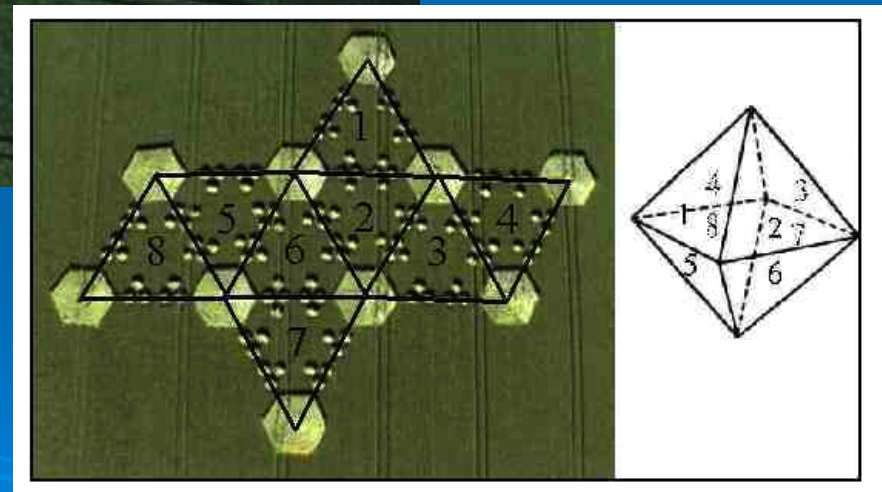
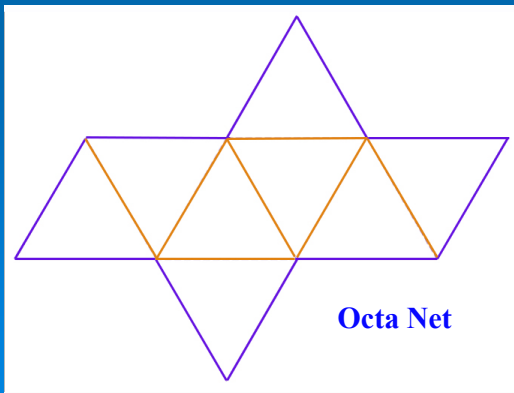
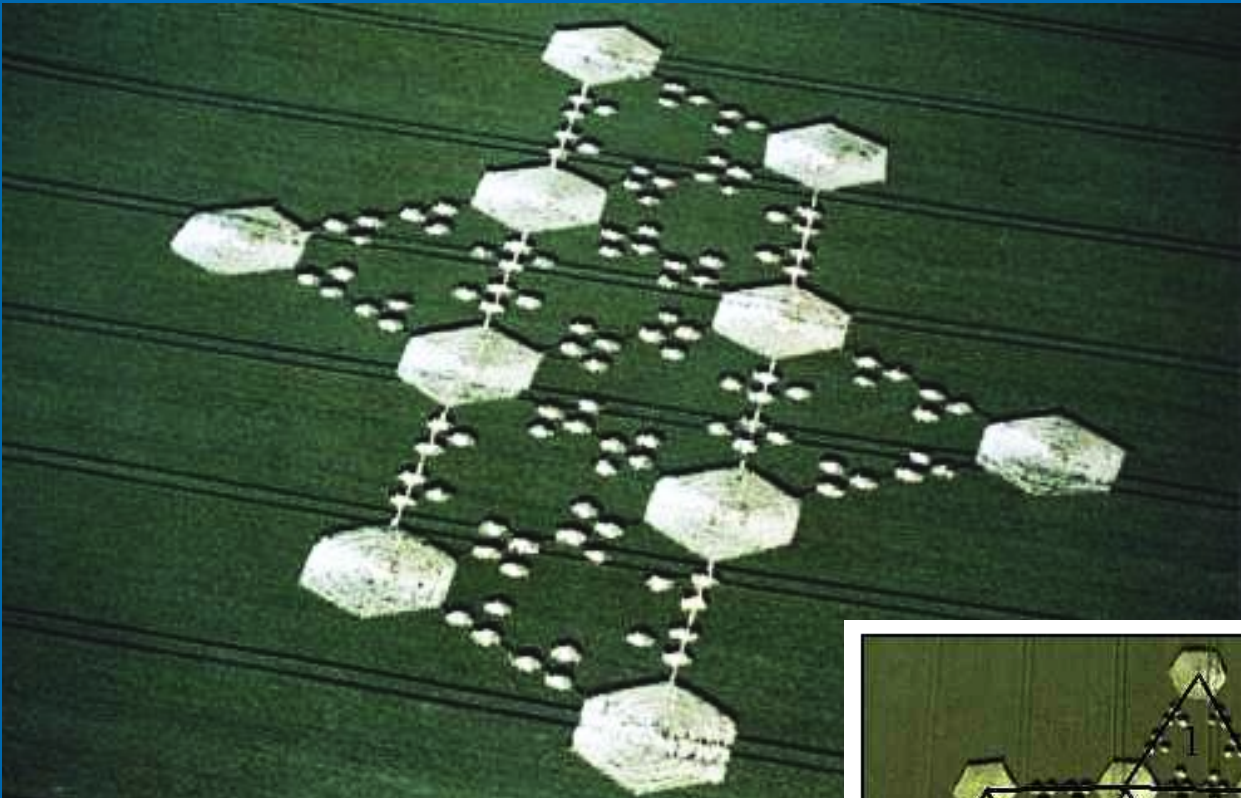
SG107.3.7 The Octahedron Fractal



www.enzedblue.com

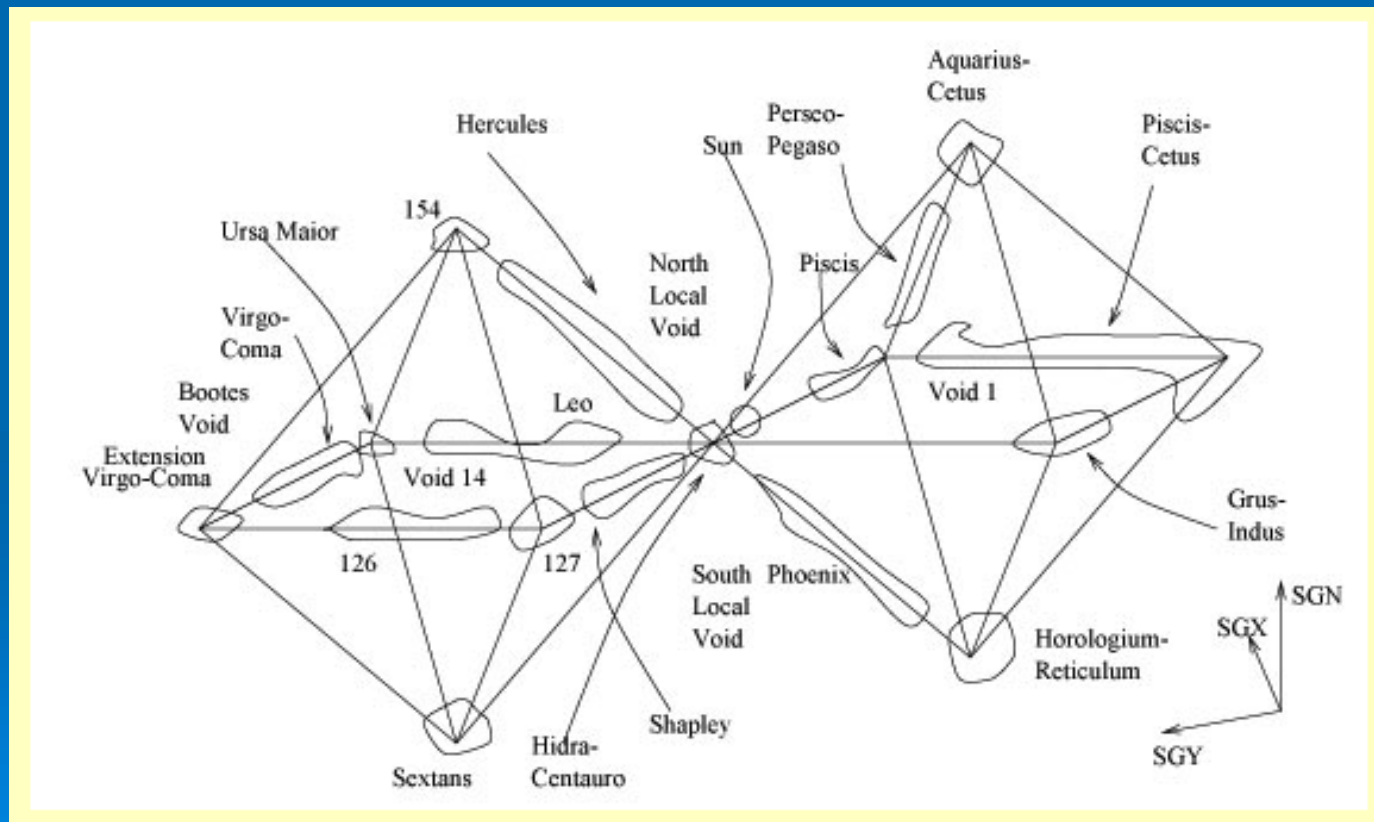
SG107.3.8
**Octahedron
Crop Circle**

← photo by Steve Alexander
West Overton, June 24, 1999



SG107.3.9 Galactic Octahedra

Since 2000, new data have allowed for the mapping of galactic super-clusters. It was found that they are gathered along lines & intersecting points which form at least 4 locally identified octahedra, Below are the two closest octahedra, seen from our sun's position near their meeting point. The super-clusters, which may each contain billions of galaxy, are found along the edges and at the corners of the shapes, and relative voids are indeed found within the shapes.



nedwww.ipac.caltech.edu/.../Figures/figure22.jpg

SG107.4 Chapter 4. The Dodeca & Icosahedron



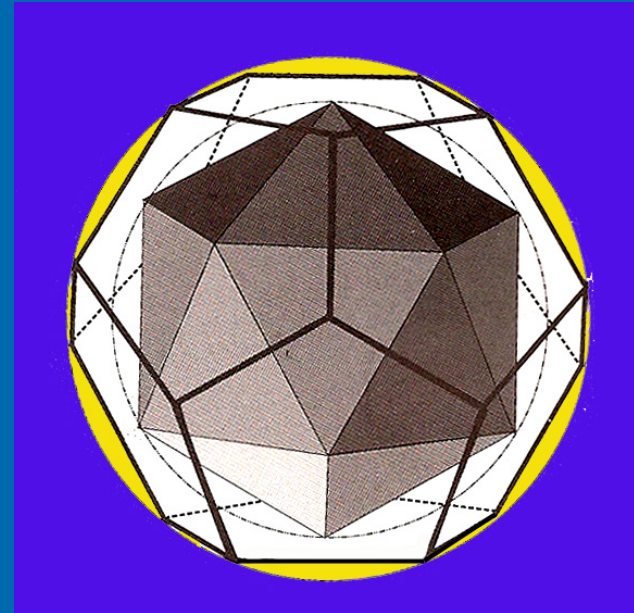
SW#84
"Sri Bindu"
by Aya

SG107.4.1 The Couple Dodeca-Icosa

Here we have yet another Platonic couple or “duals”: the Dodeca & the Icosa. They are partners in reciprocity because, while they have the same number of edges, they have a complementary number of vertices & faces:

DODECA: 30 edges - 12 faces - 20 vertices
ICOSA : 30 edges - 20 faces - 12 vertices

This means that these two Platonic Solids can morph into each other: you can mark the dodeca's 20 vertices and you will have the center of the 20 faces of the icosahedron and vice versa...



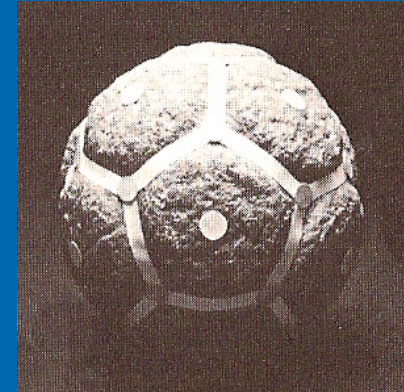
↑ Dodeca morphing into Icosa by progressive truncation. (Wikipedia).

The Dodecahedron has been an object of metaphysical reverence for millennia. We have dodecahedral spheres going back to the megalithic culture. In the Greek culture, Pythagoras and his school used the 2D Pentagram/Pentagon and the 3D dodeca as their secret recognition sign and, in the Classical Greece period, Plato gave the dodeca official recognition as the shape of the Earth “*seen from above*”.

The Dodeca has been associated with the Thought of God and the representation of the universe by many great thinkers and luminaries of the western culture. Fibonacci, Luca Pacioli, Da Vinci, Dürer, Kepler, Salvado Dali, Escher, Buckminster Fuller... have all been fascinated by the somehow mystical appeal of the dodeca.

Nowadays, cutting-edge scientists and computers artists are exploring the vast territories of the dodeca-icosa stellations and their 4D analogs: the 120-cell (made out of 120 dodecahedral cells) and the 600-cell polytopes.

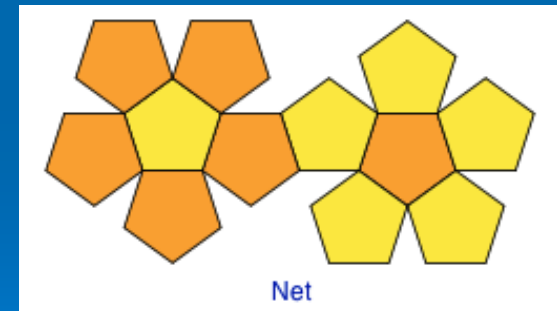
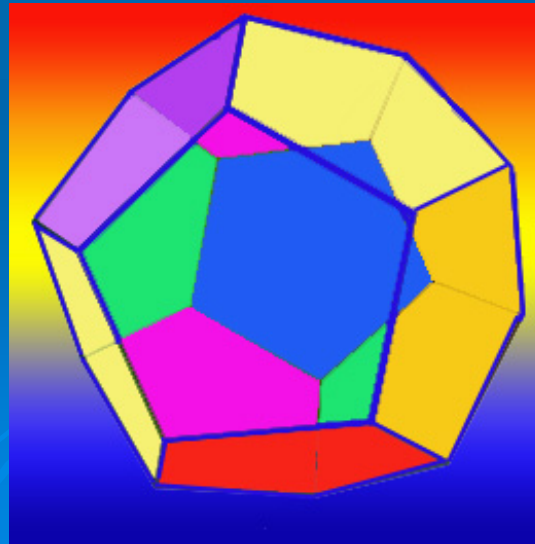
SG107.4.2.1 The Dodecahedron (1)



Neolithic dodeca sphere, Scotland.



↑ Roman dodeca



↑ “Net” for constructing a dodeca.

SG107.4.2.2 The Dodecahedron (2)



“The Sacrament of the Last Supper” by Salvador Dali is framed by a dodecahedron.

In Dali’s unique words : *”This is an arithmetic and philosophical cosmogony based on the paranoiac sublimity of the number twelve... the pentagon contains microcosmic man: Christ”*.

SG107.4.2.2 The Dodecahedron (3)



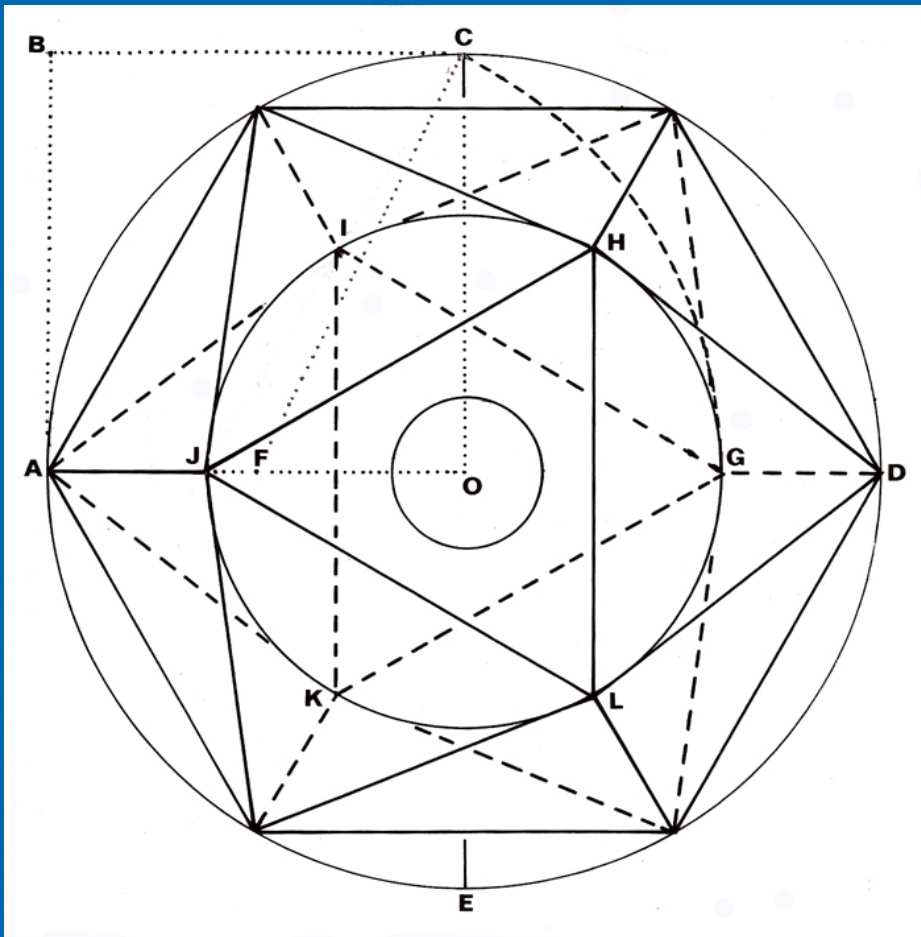
**“Reptiles” by M. C. Escher. 1943.
The high point of their journey in and out of “flatland”
is the dodecahedron.**

“(These Platonic Solids) symbolize man’s longing for harmony and order...”

*R e g u l a r
polyhedra are not
inventions of the
human mind. For
they existed long
before mankind
appeared on the
scene”.*

M. C. Escher

SG107.4.3 Construction of the Icosa



Step #1: Draw a circle with center O and mark the cross diameters AD & CE.

Step #2: With the same radius OA and center A, walk the compass 6 times around the circle to mark the 6 corners of an hexagon. Connect these corners with lines.

Step #3: Remember the construction of the Golden Section, based on the Square? We will follow the same process here. Mark the square ABCO. From the mid-base point F on AO, with radius FC, draw an arc intersecting OD in G.

G is the Golden Section point on OD, such as:
if $OD = 1$, $OG = 1/\Phi$ (or $GD / OG = OG / OD$).

Step #4: From center O, with radius OG, draw a circle. From G, with the same radius OG, walk the compass 6 times around this smaller circle and mark the points HIJKL.

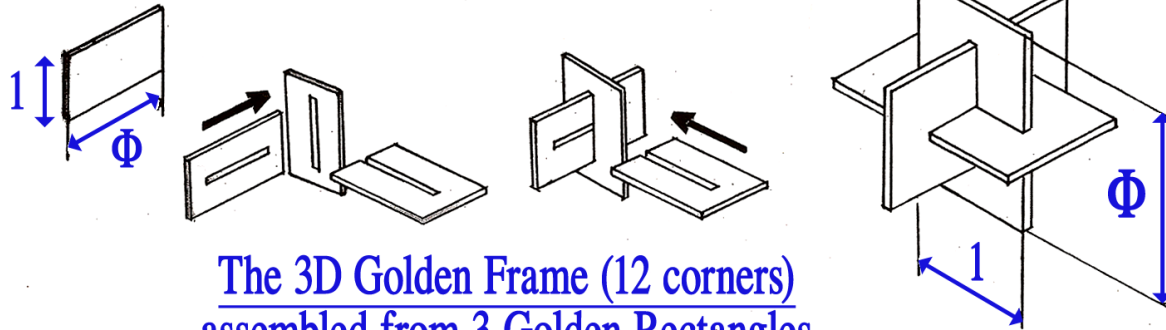
Step #5: connect the 6 points GHIJKL to form an hexagram.

Step #6: Connect the corners of the outer hexagon to the corners of the hexagram, as per drawing.

You then have the 12 vertices of the icosahedron.

The icosahedron is born from the Golden Ratio,
the 'Divine Seed' of PHI Harmony.

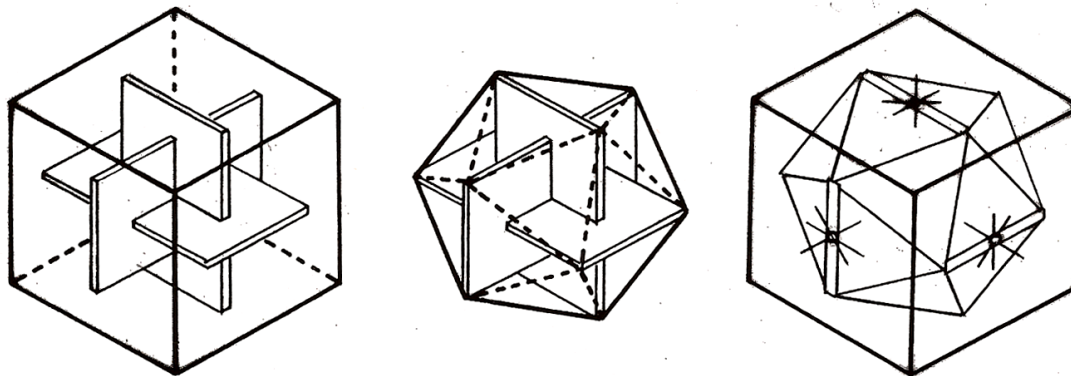
SG107.4.4 The Golden Frame



The 3D Golden Frame (12 corners)
assembled from 3 Golden Rectangles
(short side = 1, long side = Φ)

The 3D Golden Frame is the inner seed, golden scaffolding or *golden print* structure for the icosahedron & dodecahedron.

A 3D cross of Golden Rectangles, the Golden Frame in-forms the two large Platonic Solids.



Connecting the 12 corners of the Golden Frame
to create an Icosahedron
(12 vertices, 20 faces, 30 edges)

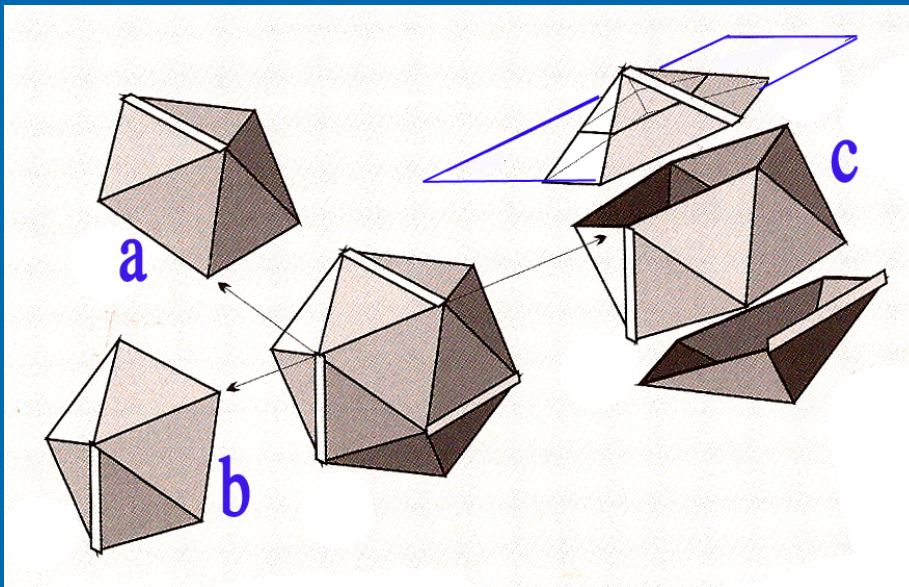
Retraced from R. Vincent. *Géométrie du Nombre d'Or*.
Chalagam, France. 2003. <http://www.chalagam.com>

SG107.4.5.1 PHI Harmonics - Icosa

The icosahedron is composed of two pyramids with pentagonal bases (in opposite direction) and a hoop of 10 alternating equilateral triangles.

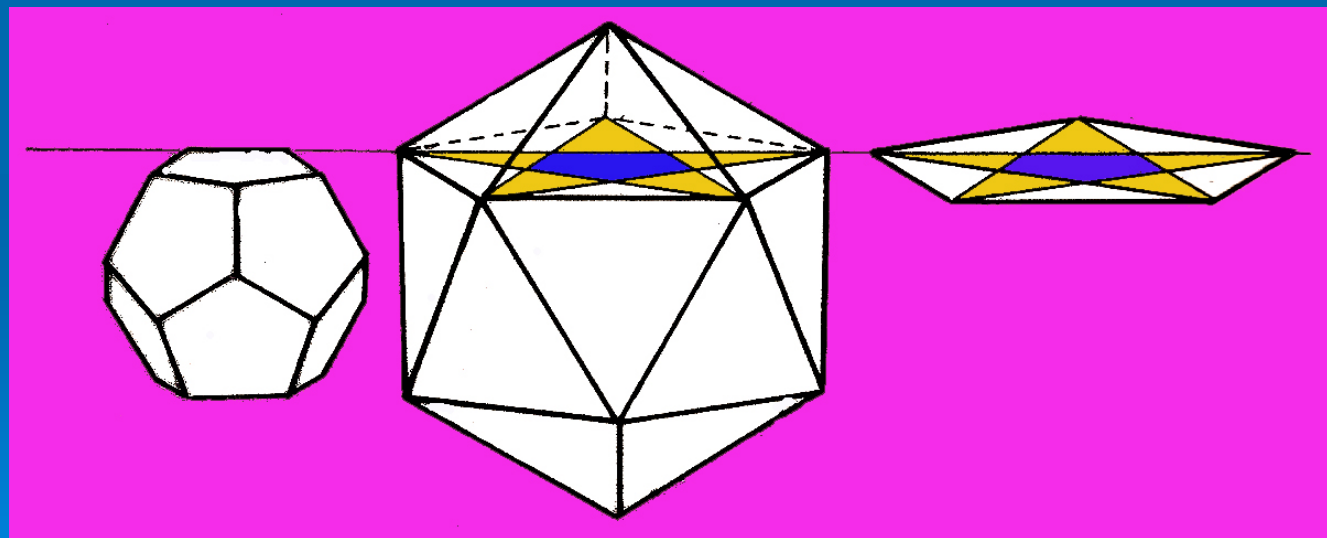
a and **b**: pyramids with pentagonal bases that can be cut from the ico by any of its 12 vertices.

c: Hoop of 10 equilateral triangles.

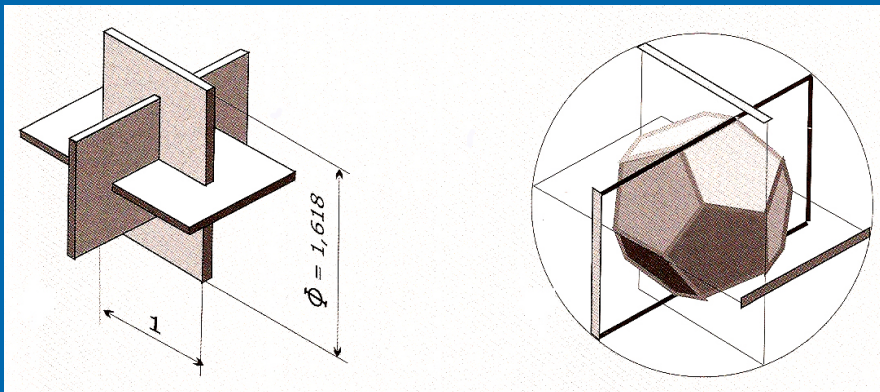


The PHI Penta-Star →
hidden in the ico.

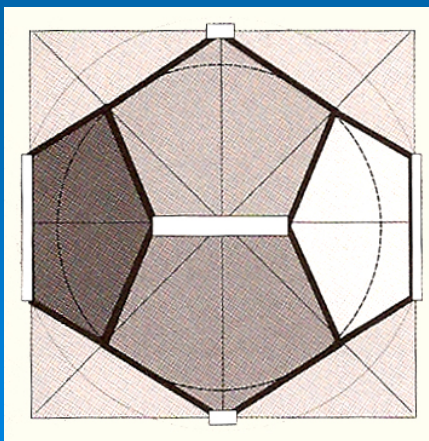
Any plane, parallel to
the pentagonal base of
the pyramid, cuts the
pyramid in a
pentagonal shape.



SG107.4.5.2 PHI Harmonics - Dodeca

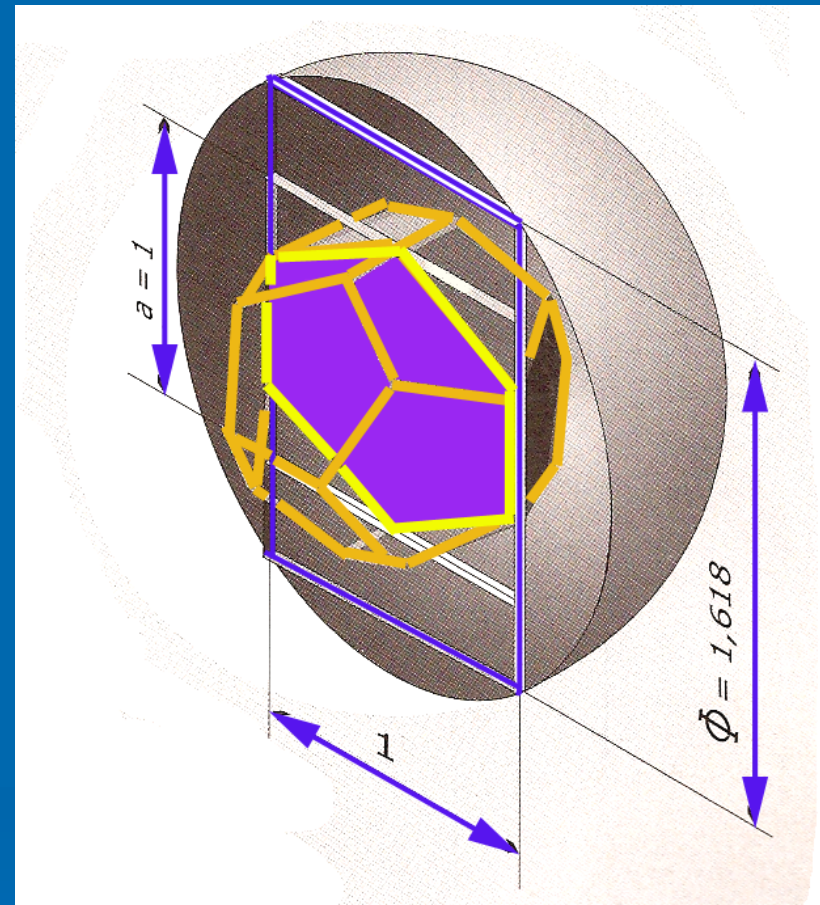


↑ The dodeca placed in the center of the sphere inscribing the 3 Golden Rectangles making up the icosahedron.



← Dodeca seen edge-on.

(The sphere inscribed in the dodeca is in contact with the centers of its 12 faces.)

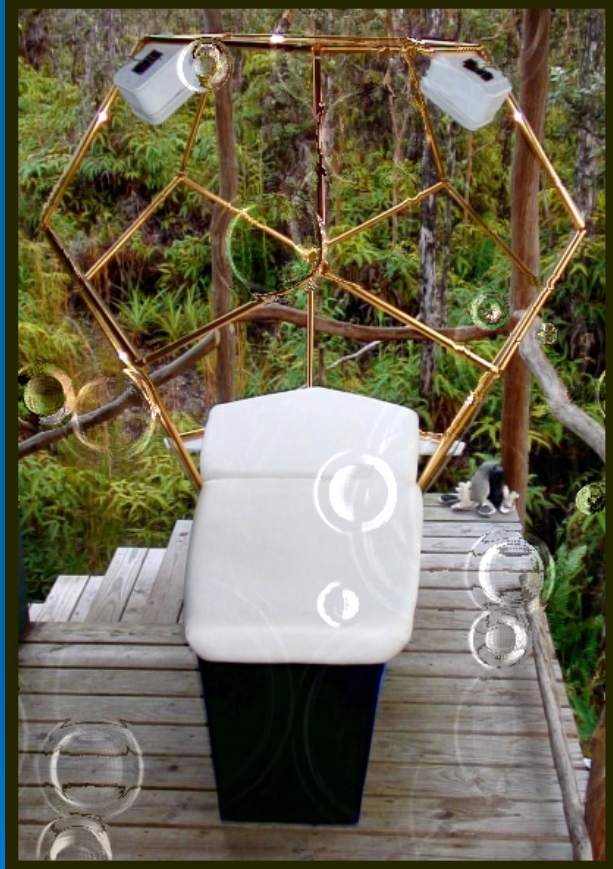


↑ The cross section of the sphere, along the vertical golden rectangle, reveals the dodecahedron inscribed in a cube of side = 1.

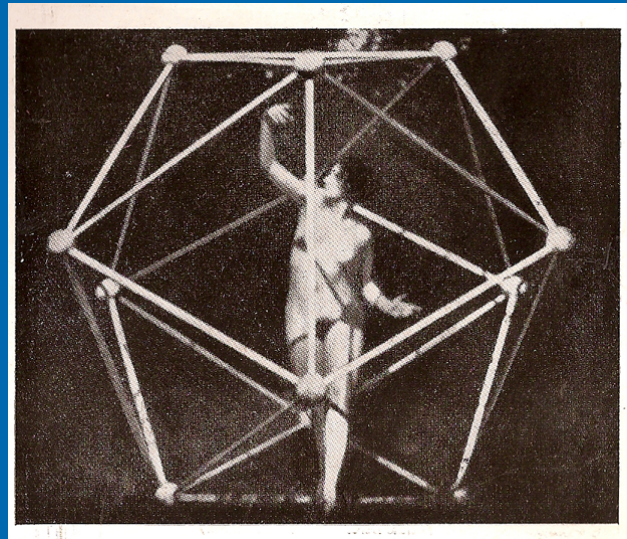


www.chalagam.com

SG107.4.5.3 PHI Healing Ico-Dodeca Chambers



↑ The DreamWeaver by B. Howard.
www.dreamweaving.com



← Icosa frame
for
choreography
training.
Von Laban.

Matila Ghyka.
Le Nombre d'Or.
Paris, 1933

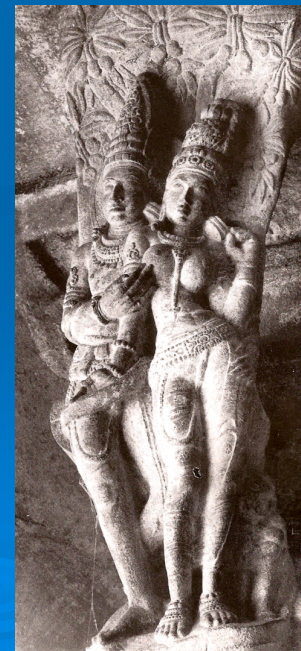


↑ www.thegenesiscenter.net



SG107.4.6.1 The Icosa-Dodeca Genesis (1)

In Hindu cosmology, Purusha is the unmanifested Source-Monad, the “Original Being” of pure consciousness. Prakriti is the Great Mother power of creation, the womb of manifestation. The Hindu mandala of creation sees Purusha as the Icosahedron, while Prakriti is the Dodecahedron. The Dance of these original parents begets the universe.



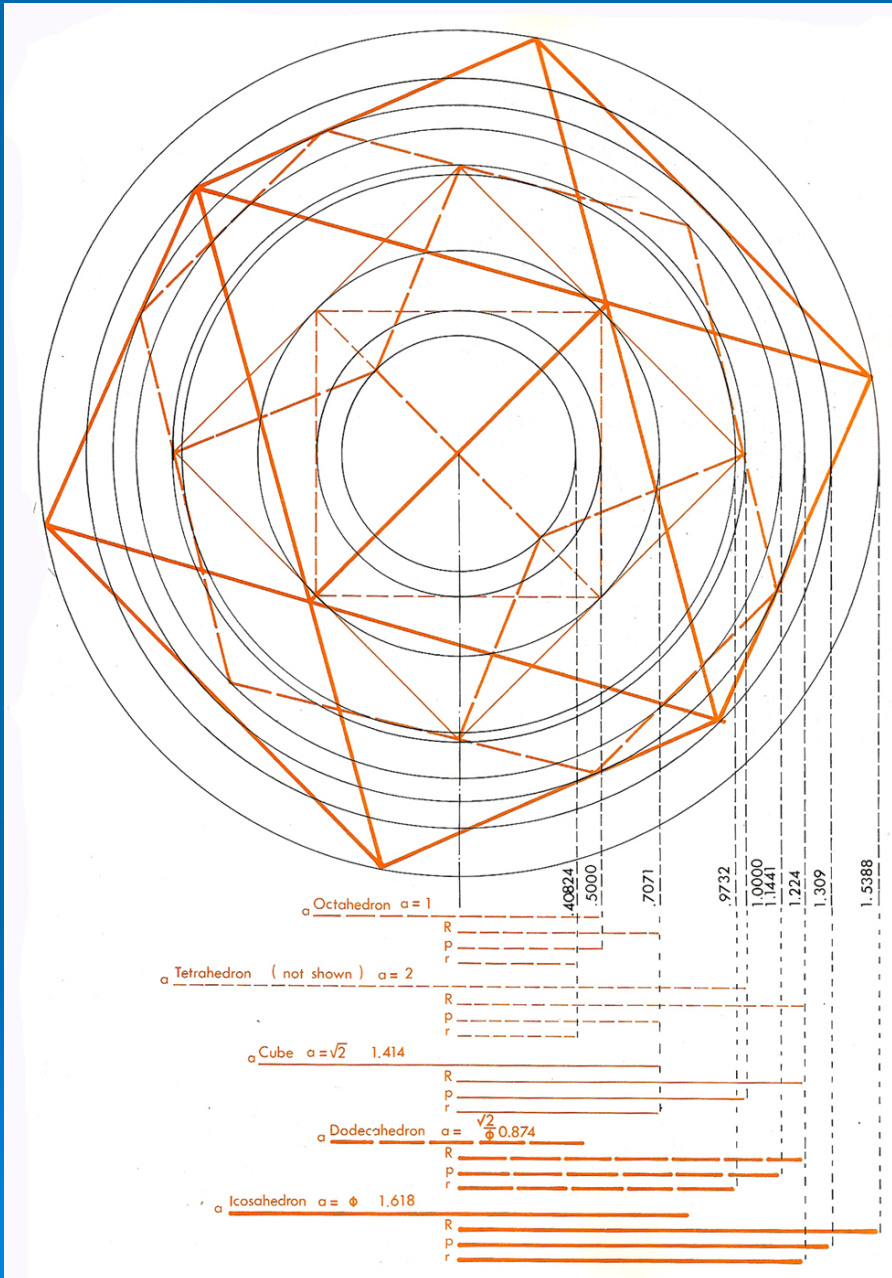
SG107.4.6.2 The Icosa-Dodeca Genesis (2)

In the process of geometrical construction, the Dodeca is generated organically, as a result of the inner diameters of the Icosa. The tetrahedron interfacing with the Dodeca generates the Cube. Both the Cube and the Tetra are touching the Dodeca. And the heart of the Tetra is the Octahedron.

Robert Lawlor explains further:

“The octahedron symbolizes the crystallization, the static perfection of matter. It is the diamond, the heart of the cosmic solid... Icosahedron is the Purusha, generating the dodecahedron, the Prakriti, and within Prakriti the full play of manifested existence. The whole coagulation (process) is begun by the secret seed which contracts the circle, the infinite, undifferentiated spirit, into the icosahedron. The seed is PHI, the fire of spirit.”

Design & quote from:
R. Lawlor. *Sacred Geometry*. Thames, 1982.



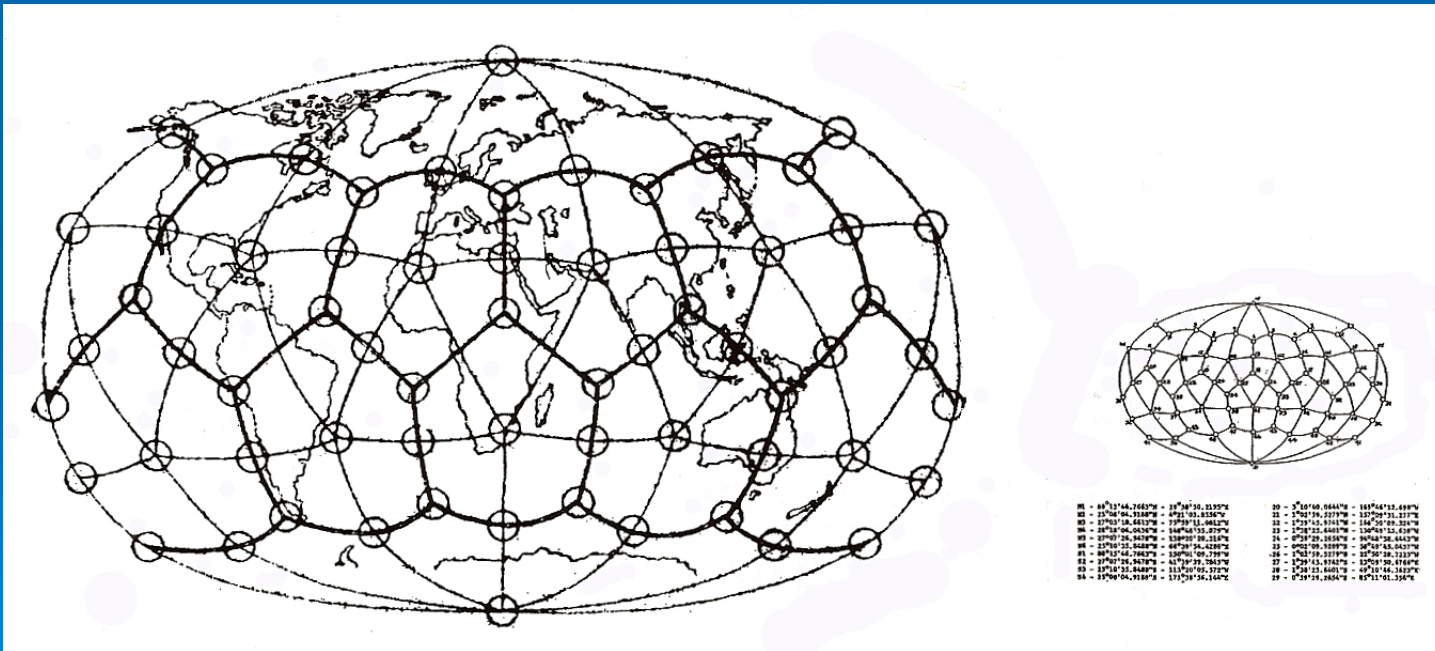
SG107.4.7.1 The Grids - Earth (1) Icosa-Dodeca

In the 1970s, a trio of Russian researchers (Goncharov, Morozov & Makarov) published a seminal article suggesting that the Earth is structurally based upon a dual geometric grid combining the icosahedron and the dodecahedron.

This grid acts as the energy matrix or network lattice directing the main planetary events, whether natural or cultural. The nodes and geometries of the Earth grid correspond to significant features. They are correlated by data from fields as diverse as history, sacred sites locations, geology and plate tectonics, magnetic anomalies, ornithology and paths of bird migrations, meteorology etc...

This research has since been greatly expanded upon and, while still somehow 'controversial', is now an integral part of the new paradigm recognizing the Web of Life and its sacred geometry. [◆SG107.2.4] Moreover, the view of the Earth as a 'geometric crystal' and 'sacred solid' resembles closely the Pythagorean vision as expounded by Plato who said:

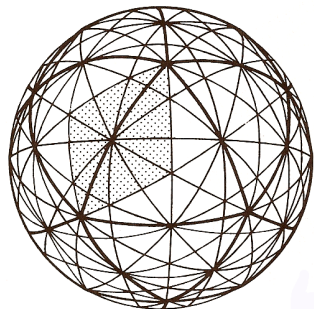
"The Earth, seen from above, resembles a ball sewn from twelve pieces of skin".



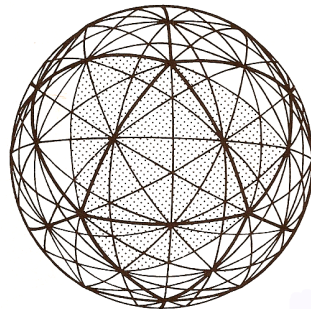
SG107.4.7.2 The Grids - Earth (2) UVG

The husband-and-wife team of Becker (Professor of Industrial Design at the University of Illinois, Chicago) and Hagens (Professor of Anthropology at Governors State University), while considering that the grid proposed by the Russian team is essentially correct, worked with other researchers (Ivan P. Sanderson and Chris Bird) to develop a more comprehensive Earth grid with the overlay of an icosahedrally-derived, spherical polyhedron developed by R. Buckminster Fuller. In his book *Synergetics 2*, he called it the "Composite of Primary and Secondary Icosahedron Great Circle Sets", shortened to Unified Vector Geometry (UVG).

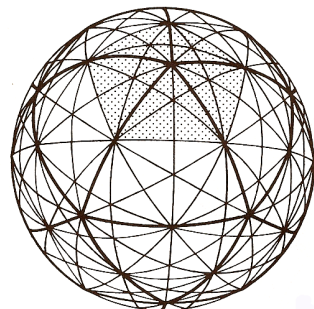
The UVG geometric grid has 120 identical triangles - all approximately 30, 60 and 90 in composition.



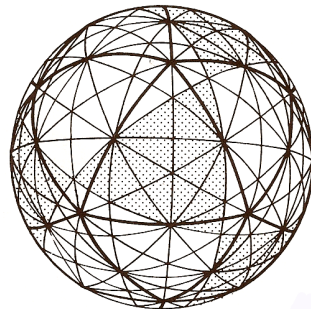
Rhombic Triacontahedron



Octahedron

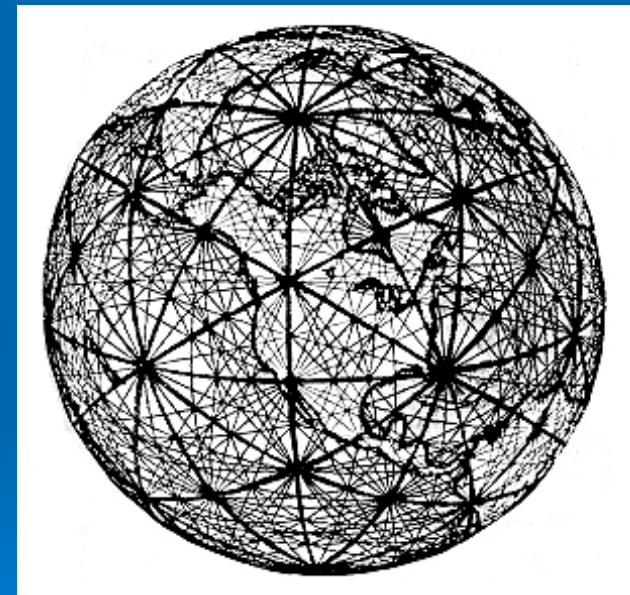


Dodecahedron



Vector Equilibrium

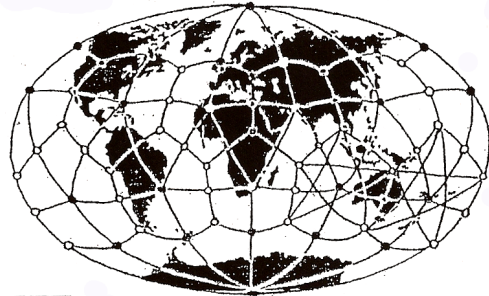
← The UVG Grid (Vector Equilibrium) is a spherical icosahedron providing edges for 3 other polyhedra. (B. Fuller)



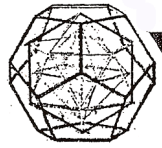
↑ The UVG120 Earth Grid



The Geometry of the Universe



Earth crystalline energy grid

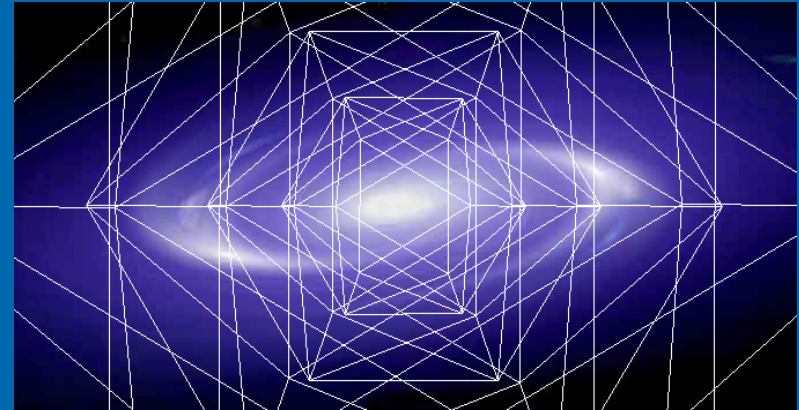


Socrates told his pupil Simmias:
"The real earth, viewed from above,
looks like a ball made out of twelve
pieces of skin sewn together."
(*Timaeus*)

LATEST !!! An article in *Nature* (October 9, 2003 www.nature.com) by astrophysicist J. P. Luminet and al. suggests, from new WMAP data, that the universe exhibits a dodecahedral space topology.



SG107.4.7.3 The Grids - Universe Dodeca?



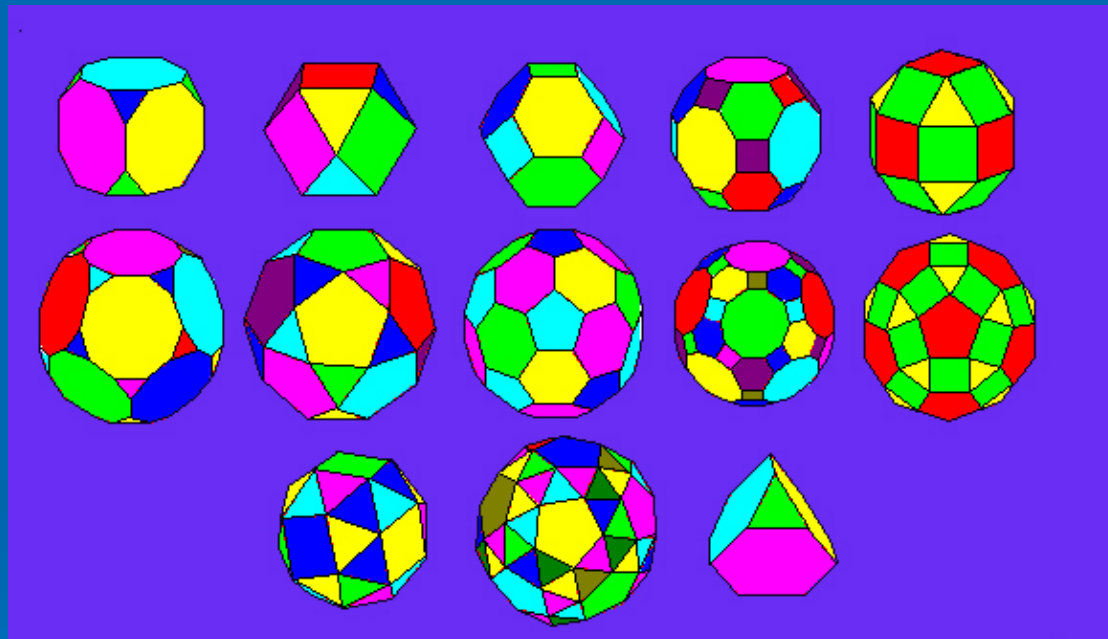
The question of the '*shape and structure of the universe*' is an age-old quest. Explanations have come and gone, from descriptions of shamanic journeys to string theory...

In October 2003, new data about the cosmic background radiation brought by NASA's Wilkinson Microwave Anisotropy Probe (WMAP) may hint at a possible answer along the line of ancient Sacred Geometry: the universe is finite and resembles a dodecahedron.

Jeffrey Weeks and a team of French cosmologists came to that model after careful measurements of the WMAP data. The density fluctuations of the cosmic background radiation can tell a lot about the physical shape & structure of space. The conclusion was that the mathematics add up nicely to support a finite dodecahedral topology.

Mathematician George Ellis wrote: "*Can this theory be confirmed? Yes, indeed*", explaining that WMAP's successor will provide even more precise key data on the cosmic background radiation that will confirm or disprove Weeks & colleagues' theory.

SG107.5 Chapter 5. 13 Archimedean Solids

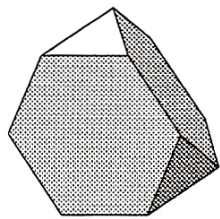


SG107.5.1.1 The Archimedean Solids (1)

Whereas the 5 Platonic Solids have only one type of regular polygon as their face, the 13 Archimedean Solids have 2 or more types of regular polygon (triangle, square, pentagon, hexagon, octagon or decagon) as faces.

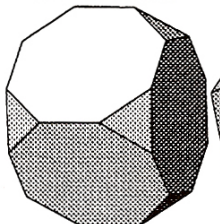
The names commonly used for the Archimedean Solids were given by Johannes Kepler.

The 13 Archimedean classified by progressive truncation

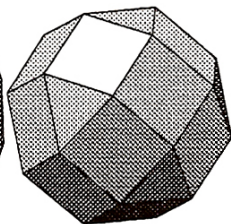


Truncated Tetrahedron

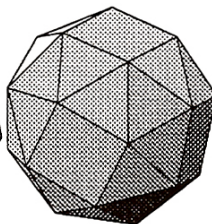
The truncated tetrahedron shows the "progression" from a tetrahedron to another tetrahedron, since the tetrahedron is a dual to itself, i.e., connecting the midpoints of the faces yields another tetrahedron pointing in the opposite direction from the original. The row below shows the progression from a hexahedron (cube) to an octahedron. The bottom row shows the progression from a dodecahedron to an icosahedron, as corners are trimmed off and turned into other regular polygons.



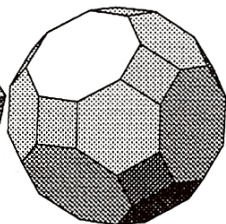
Truncated Cube



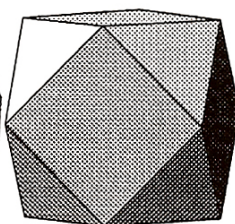
Rhombicub-octahedron



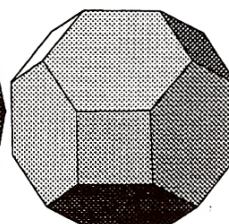
Snub Cube



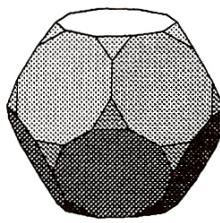
(Rhombi)truncated Cuboctahedron



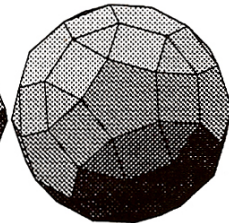
Cuboctahedron (Dymaxion)



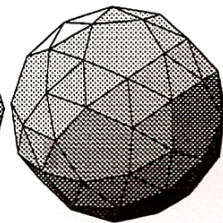
Truncated Octahedron (Mecon)



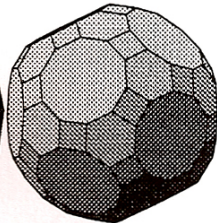
Truncated Dodecahedron



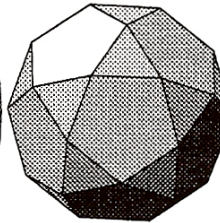
Rhombicosi-dodecahedron



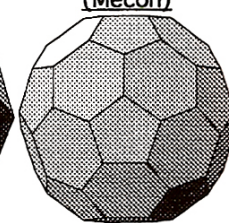
Snub Dodecahedron



(Rhombi)truncated Icosidodecahedron



Icosidodecahedron



Truncated Icosahedron

← In the Archimedean Solids, the arrangement of the faces surrounding each vertex must be the same for all vertices.

(Graphics & text from B. Rawles. *Sacred Geometry Design Sourcebook*. Elysian, 1997.)



www.geometrycode.com

SG107.5.1.2 The 13 Archimedean (2)

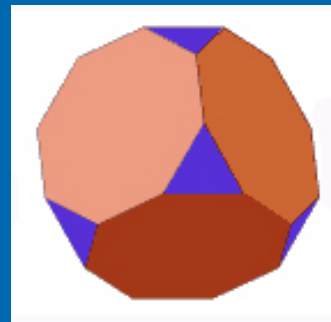
If we classify the 13 Archimedean by the number of edges meeting at their vertex, we have:

3 edges per vertex (*simplicial Archimedean*).

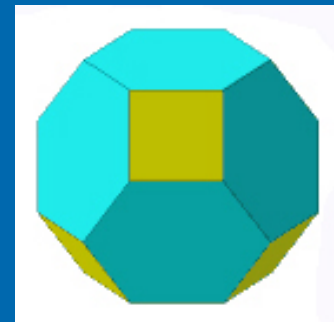
Five are first degree truncations of a Platonic and two are second-degree truncations.



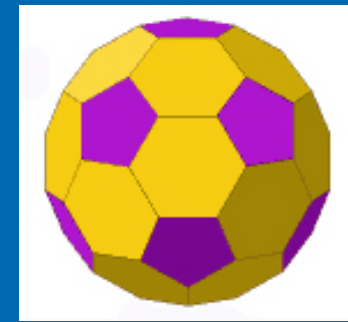
Truncated
Tetrahedron



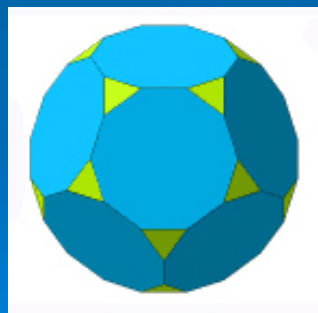
Truncated
Cube



Truncated
Octahedron



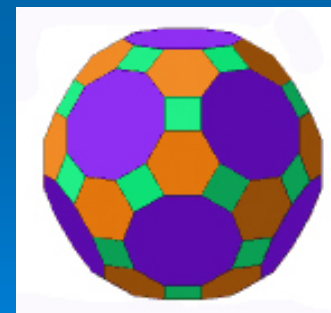
Truncated
Icosahedron



Truncated
Dodecahedron



Truncated
Cuboctahedron

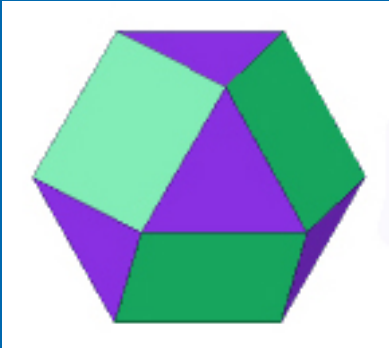


Truncated
Icosidodecahedron

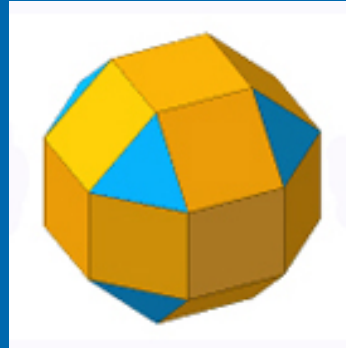
SG107.5.1.3 The 13 Archimedean (3)

4 edges per vertex.

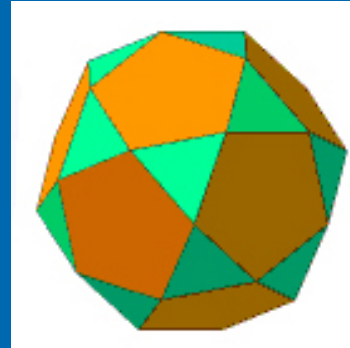
These are *non-chiral*.



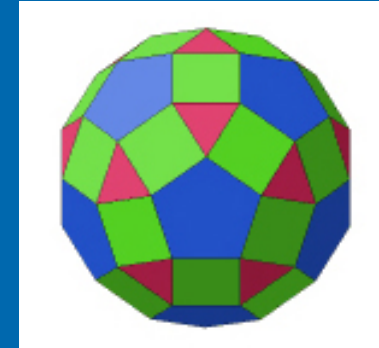
Cuboctahedron



Rhombicub-
octahedron

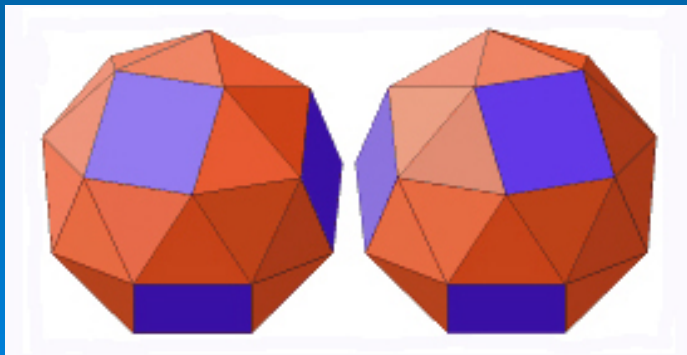


Icosi-
dodecahedron

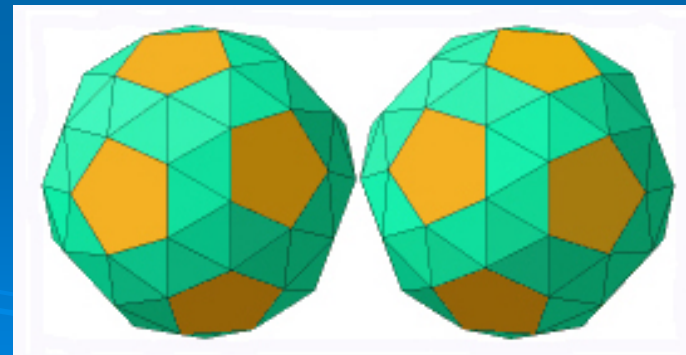


Rhombicosi-
dodecahedron

5 edges per vertex. These two last Archimedean are *chiral*,



Snub Cube

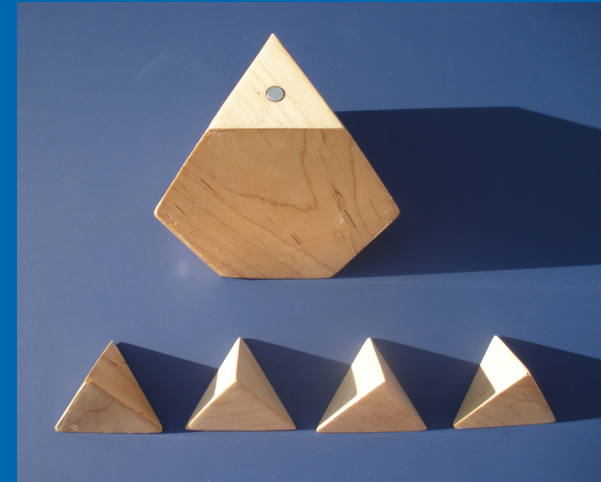
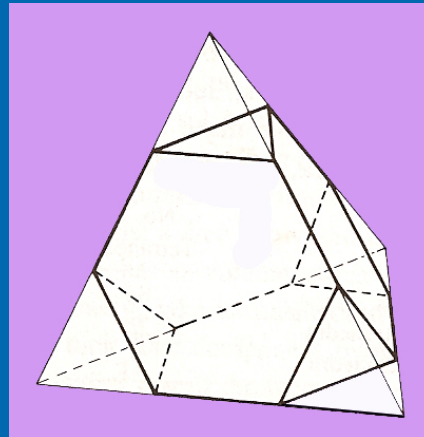


Snub Dodecahedron

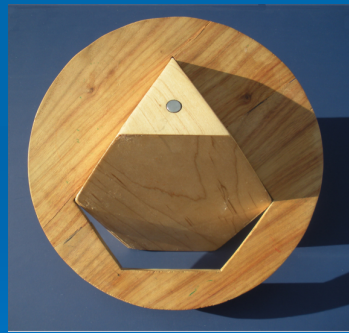
SG107.5.2.1 The Truncated Tetrahedron



The 1st Archimedean solid:
The Truncated Tetra.



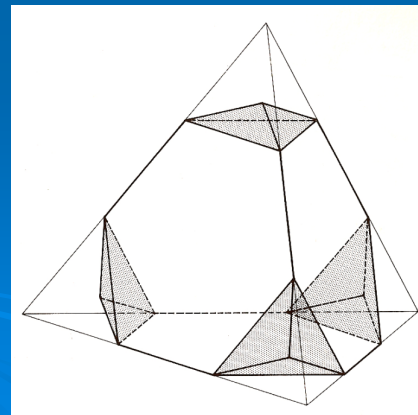
↑ The 4 tetrahedra taken off a larger tetra to create a Truncated Tetra
(model by Fred Rusher)



↑ The Truncated Tetra forms a diamond shape when taken through the 2D plane, as discovered by Fred Rusher.



www.geocosmicarts.com



← The “*Diamond Point Complex*” occurring in metallic alloys. Model by Arthur L. Loeb. The domain consists of a truncated tetra with four 1/4 tetra attached to each triangular face.

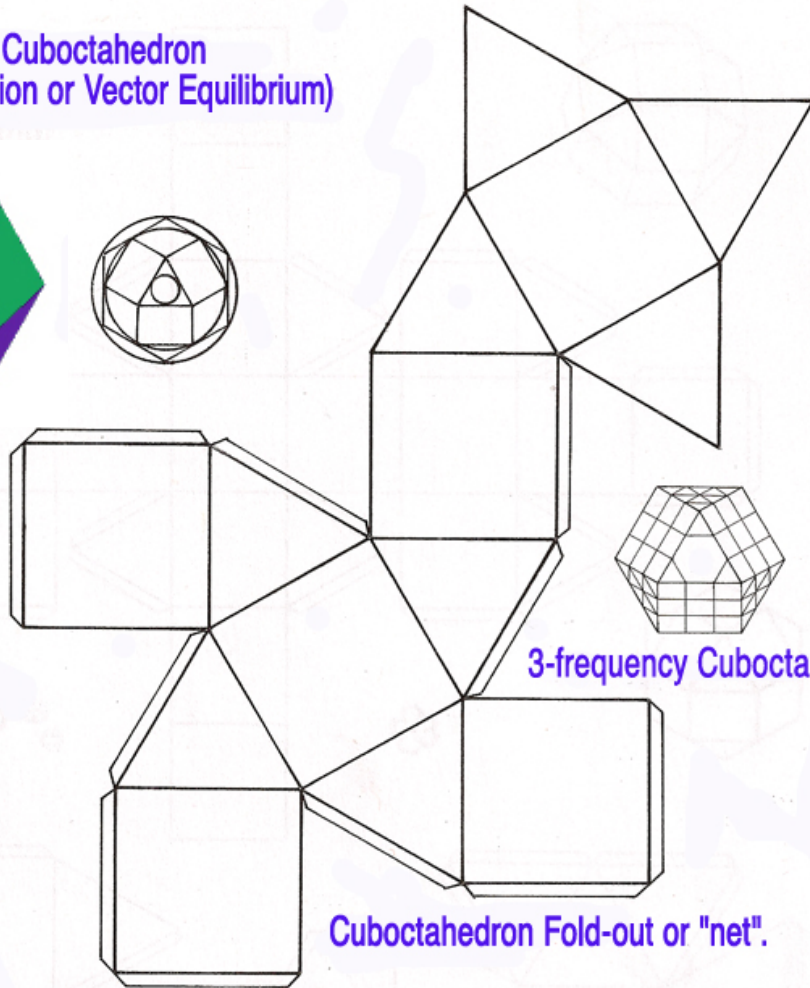
SG107.5.2.2 The Truncated Cube



Left: Cube. Right: Truncated Cube.
Models by Fred Rusher. www.geocosmicarts.com

SG107.5.3.1 The Cuboctahedron (1)

The Cuboctahedron
(a.k.a. Dymaxion or Vector Equilibrium)



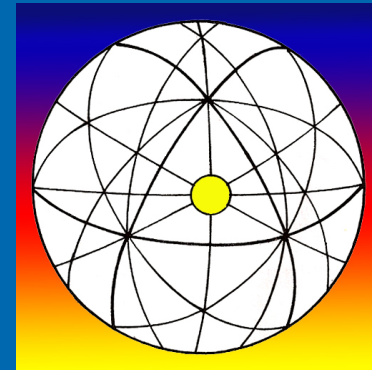
3-frequency Cubocta

Cuboctahedron Fold-out or "net".

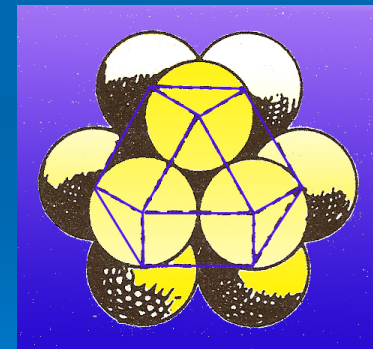


The *Cuboctahedron* has 6 squares and 8 equilateral triangles.

In terms of dynamics, the cubocta is an equilibrium of forces - hence the name *Vector Equilibrium*.

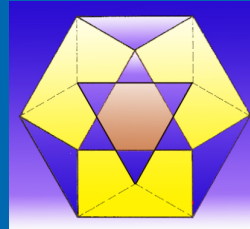


↑ Spherical cubocta with some great-circle diameters.



↑ Closest-packing of spheres: 12 balls surround a 13th ball. If their centers are joined by lines, they form a cubocta.

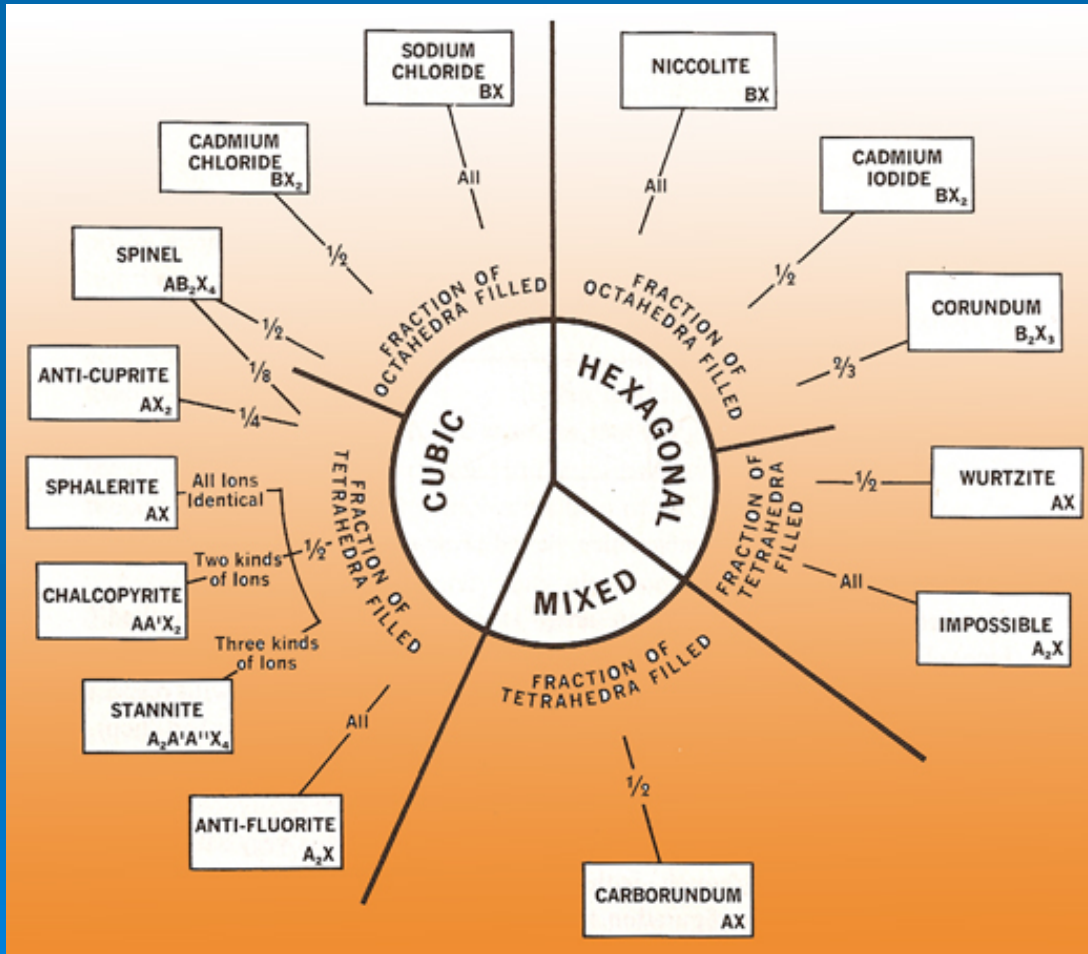
SG107.5.3.2 The Cuboctahedron (2) Crystal Structures



Arthur L. Loeb, in his appendix to B. Fuller's *Synergetics*, shares his investigation of the architecture of crystal structures on a microscopic scale that correlates with Fuller's macroscopic geometric and geodesic systems.

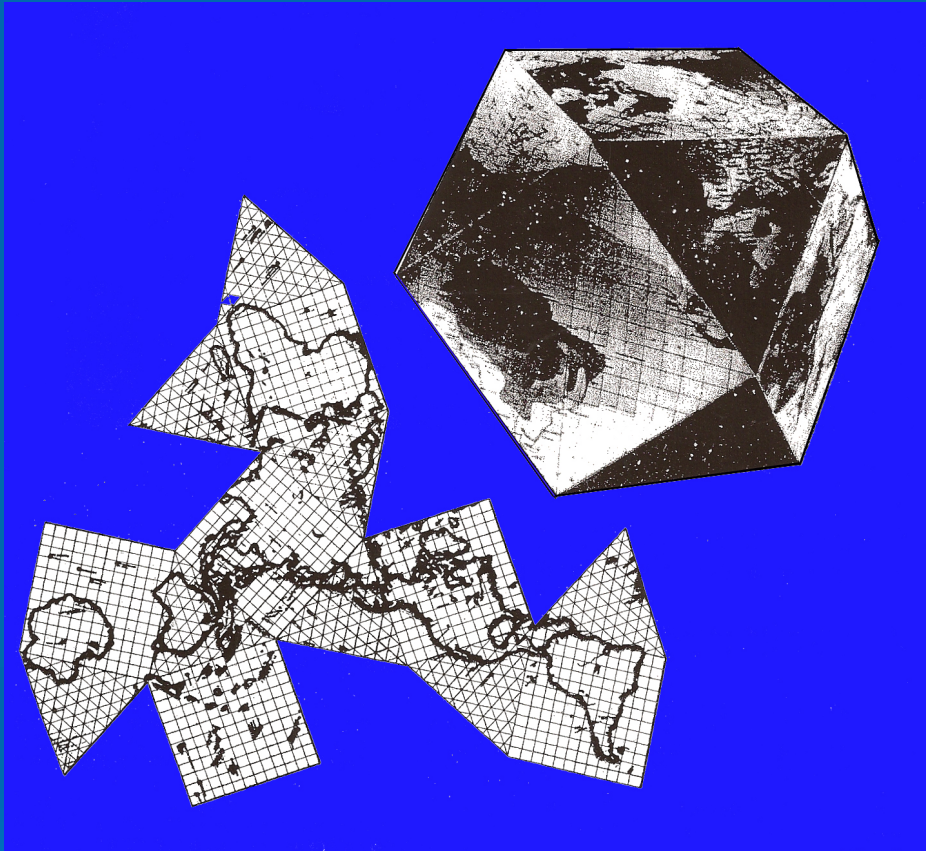
"In many metals, e.g. copper, the atoms arrange themselves as if they were closely packed spheres. They form triangular layers stacked in such a way that any atom is cuboctahedrally surrounded by its nearest neighbors..."

The cuboctahedron provides a n hexagonally close-packed array... Many crystals are represented by a model in which large, closely packed spheres correspond to larger ions, and smaller spheres in the voids between correspond to smaller ions... The voids bounded by 8 spherical surfaces are octahedral and the voids bounded by 4 spherical surfaces are tetrahedral."



Common minerals represented by sphere-packing geometries.

SG107.5.3.3 The Dymaxion Map



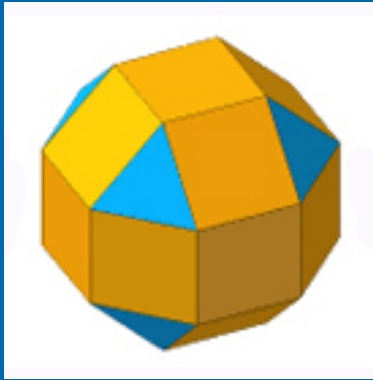
↑ Patented in January 1946 (#2,393,676), the *Dymaxion Map* was the first of a “United Nations” representation of the world whereby the non-western countries & continents are given their just comparative areas and presence.

Here, Sacred Geometry & Design Science touch upon geo-politics and global economics.

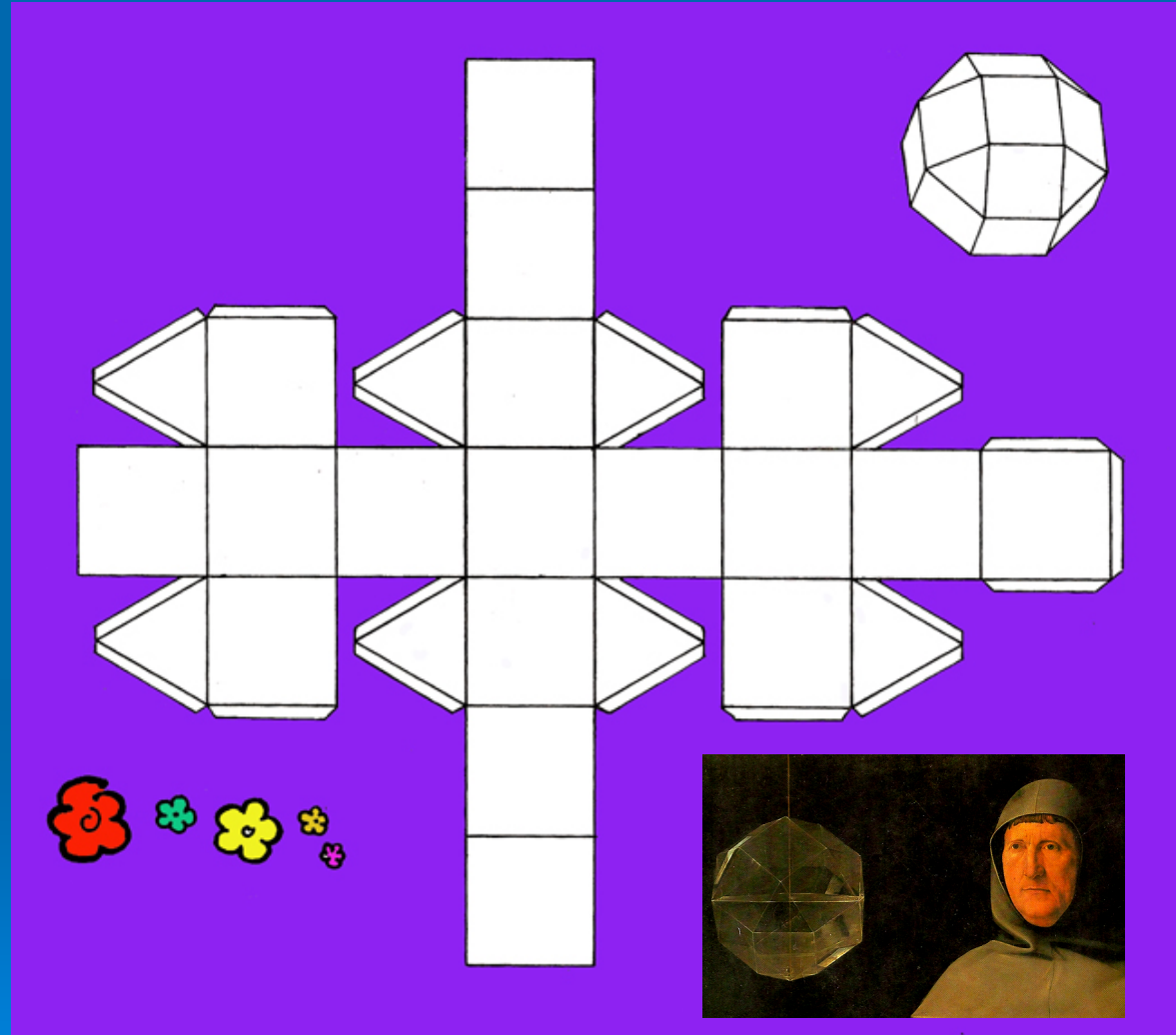
“*Dymaxion*” is one of Buckminster Fuller’s two names for the Cuboctahedron (the other name is “*Vector Equilibrium*”).

In the book recording his many inventions, B. Fuller explains his motivations:

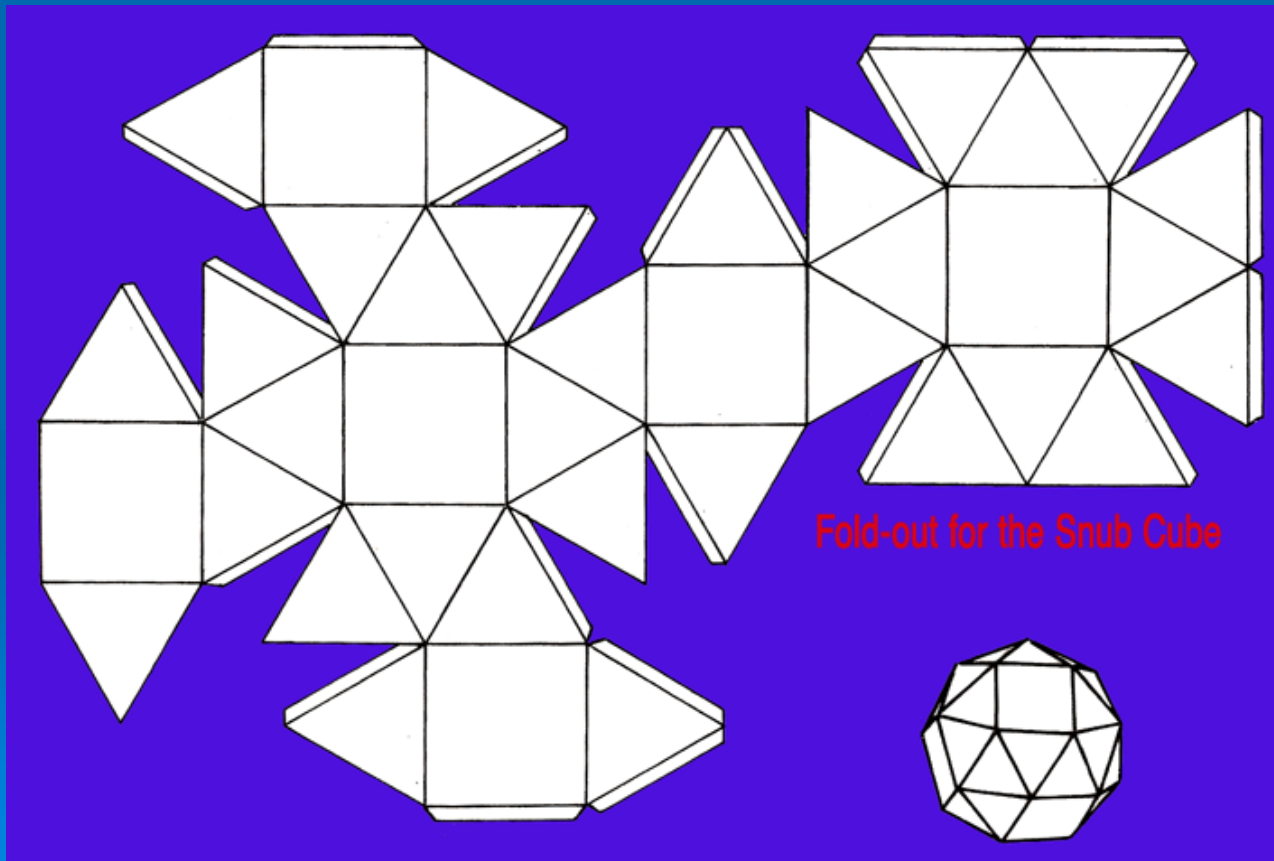
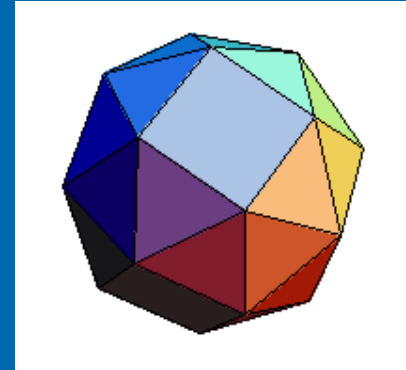
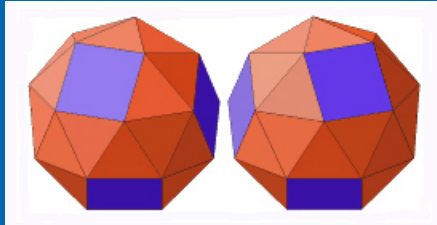
“My 1927 commitment to deal henceforth only with total planetary physical & metaphysical resources, employed only in technology useful for all people around the surface of Spaceship Earth, called for a non-distorted map of the world upon which to identify the resources and the people.”



SG107.5.4 The Rhombicuboctahedron

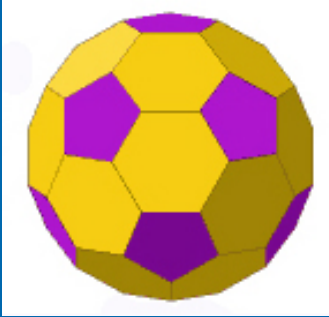


SG107.5.5 The Snub Cube



The Snub Cube is also called the "*cubus simus*" (Kepler 1619) or *snub cuboctahedron*.

It has 38 faces (32 triangular and 6 square faces), 60 edges, and 24 vertices.



SG107.5.6.1. Geodesics (1)



The Geodesic Dome

(US patent June 1954 - #2,682,235)

‘When I invented and developed my first clear-span, all-weather geodesic dome, the two largest domes in the world were both in Rome and were each 150 feet in diameter. They are St. Peter’s, built around A.D. 1500, and the Pantheon, built around A. D. 1. Each weighs approximately 30,000 tons.

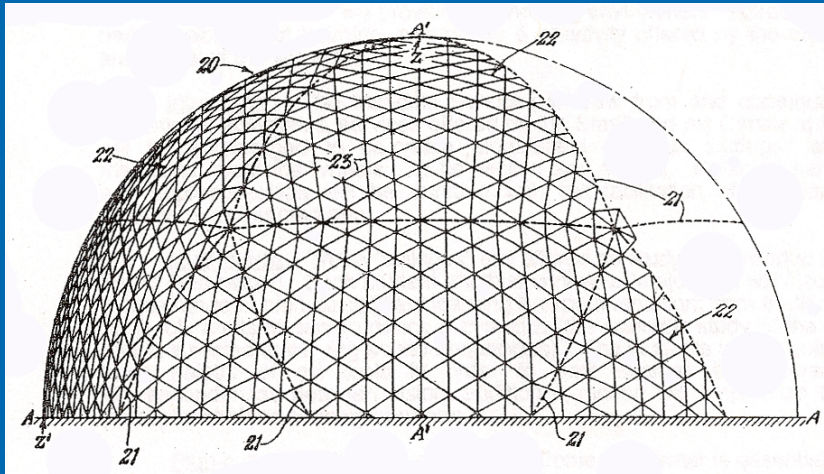
In contrast, my first 150 ft diameter geodesic dome installed in Hawaii weighs only 30 tons - 1/100th the weight of its masonry counterpart. An earthquake would tumble both the Roman 150-footers, but would leave the geodesic unharmed.’

‘At no time during my last 56 years have I paid any attention to conventional architecture’s ‘orders’ about the superficial appearance of my structures...

When the whole installation and assembly is complete and tested, and I can stand off and look at it as an operating reality, if it does not look beautiful to me, I know that I have failed...

(R. Buckminster Fuller.
Inventions. 1983)

SG107.5.6.2 Geodesics (2)



Epcot's Spaceship Earth Dome.



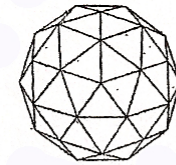
[//bluegrassplaygrounds.com](http://bluegrassplaygrounds.com)



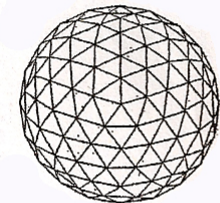
www.pacificdomes.com



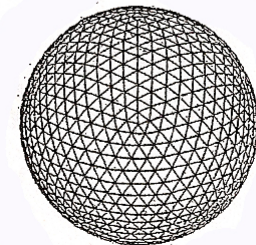
www.domeincorporated.com



Frequency-2



Frequency-4



Frequency-9

SG107.5.6.3 Geodesics (3) and Tensegrity

Buckminster Fuller's geodesic domes are based on the underlying concept of *tensegrity*. In his studies of self-organizing systems, Fuller, like water-wizard Schauberger, came to this wise conclusion: "Don't fight forces, use them!". The result is '*Tensegrity*', the synergetic, complementary, harmonious balance between tension and compression, two forces usually thought to be opposite. Thus the extreme flexibility and yet immense strength of geodesic domes.

Tensegrity structures are found in a variety of natural self-assembled systems, including carbon atoms, water molecules, proteins and viruses.

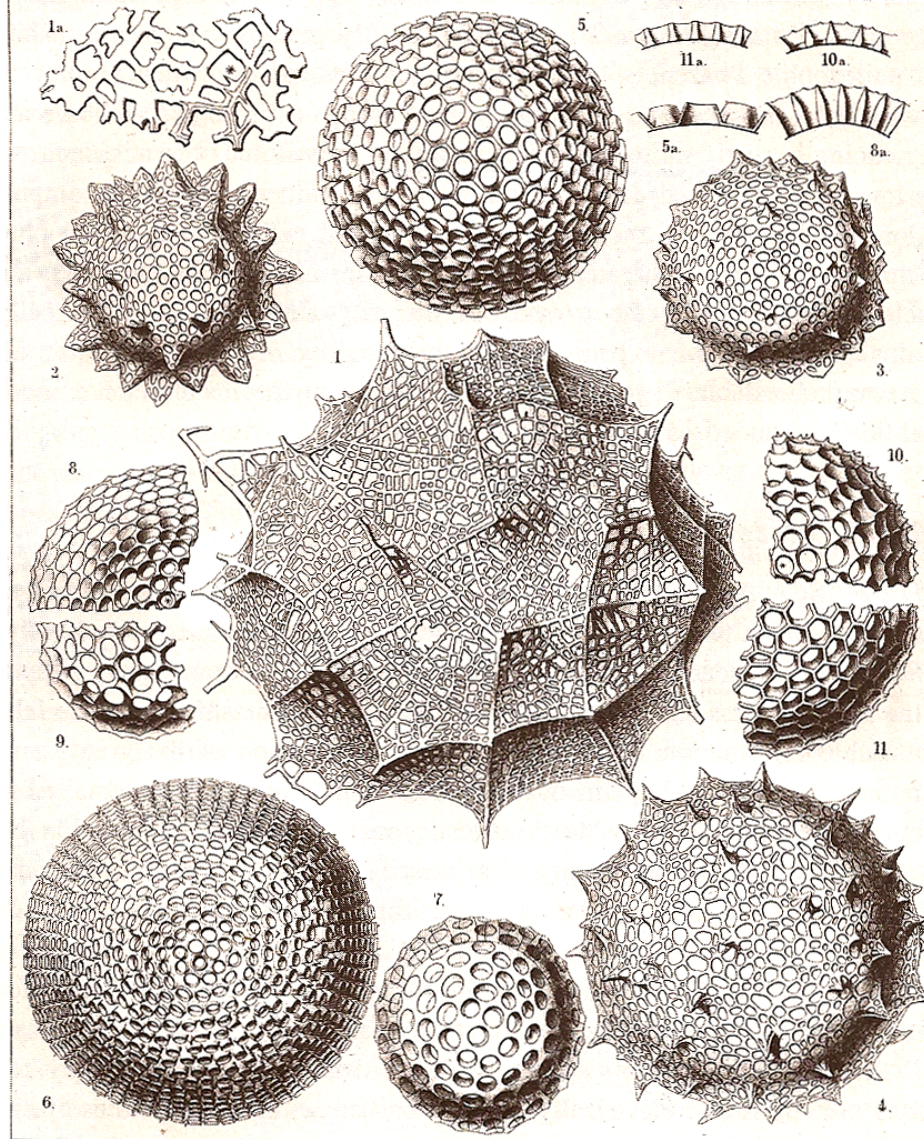
It so happens that the same understanding of *tensegrity* was also arrived at by medical researcher Donald Ingber who found a prevalence of pre-stressed tensegrity structures in the human body, on an anatomical, cellular, and molecular level.

"We are 206 compression-resistant bones that are pulled up against the force of gravity and stabilized through a connection with a continuous series of tensile tendons, muscles, and ligaments" (Ingber Lab).

"Similarly, on a cellular level, like Fuller's geodesic domes, the cell's durability is derived from its capacity to distribute tension throughout its structure as well as respond to and neutralize tensional forces with specialized compression.

Several geodesic tensegrity structures naturally occur on the molecular level. Basement membrane proteins, polyhedral enzyme complexes, clathrin-coated transport vehicles, viral capsids, lipid micelles, individual proteins, and RNA and DNA molecules all employ hexagonal arrangements. The cell's microfilament network itself is a geodesic tensegrity structure."



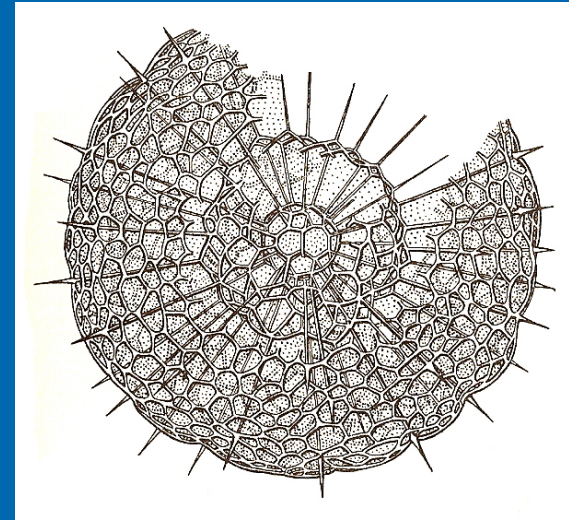


H. Beckel and A. Schmidt, del.

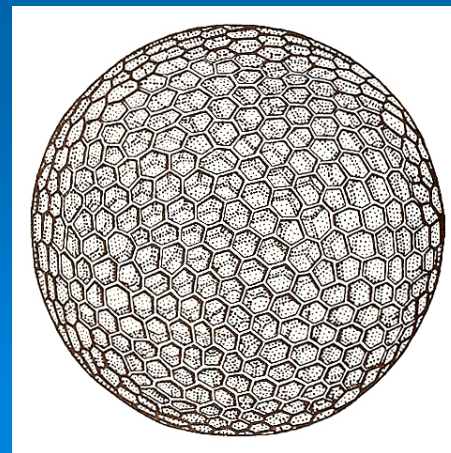
E. Steuch, Jena, Lithogr.

1 OROSPHAERA, 2-4. CONOSPHAERA, 5, 6. ETHMOSPHAERA,
7-11. CERIOSPHAERA.

SG107.5.6.4 Geodesics (4) in Nature



↑ Actinomma arcadophorum



← Aulonia hexagona

SG107.5.6.5 Geodesics (5) C₆₀ The Buckyball

The Truncated Icosahedron became famous in 1985 with the discovery of the C₆₀ Buckminsterfullerene (“*Buckyball*”) molecule.

B. Fuller’s ideas guided the chemists who discovered the substance to theorize that its shape was a truncated icosahedron. And in 1991 this was proven correct. [◆SG203.2]
Science history note: the same hypothesis was published in 1970 by Japanese scientist E. Osawa and in 1973 by Russian scientists Bochvar & Gal’pern, but not in English.

Named “*Molecule of the Year*” by Science magazine in 1991, the Buckyball has a 5-fold rotational symmetry based on the pentagon structure.

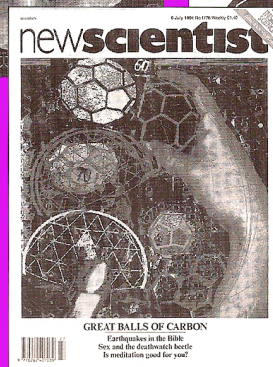
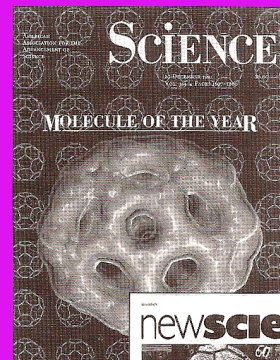
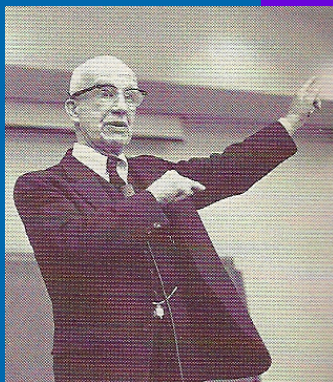
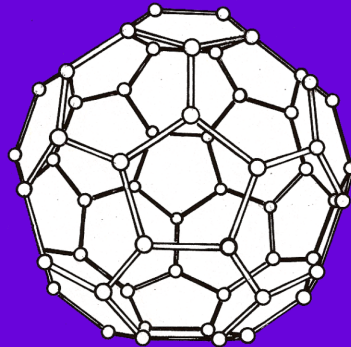
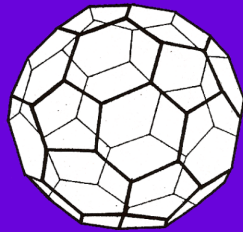
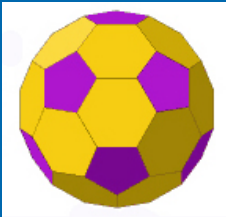
This species of carbon has been linked with soot formation and possible existence in interstellar space.

C₆₀ is a very stable molecule maintaining the tetravalency of all carbon atoms. Many purely theoretical papers have already been published on this structure alone.

[◆SG203A]

(Images from Hargittai. *Symmetry*. 1994)

98



SG107.5.6.6 Geodesics (6) C20

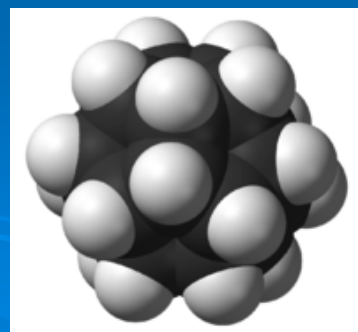
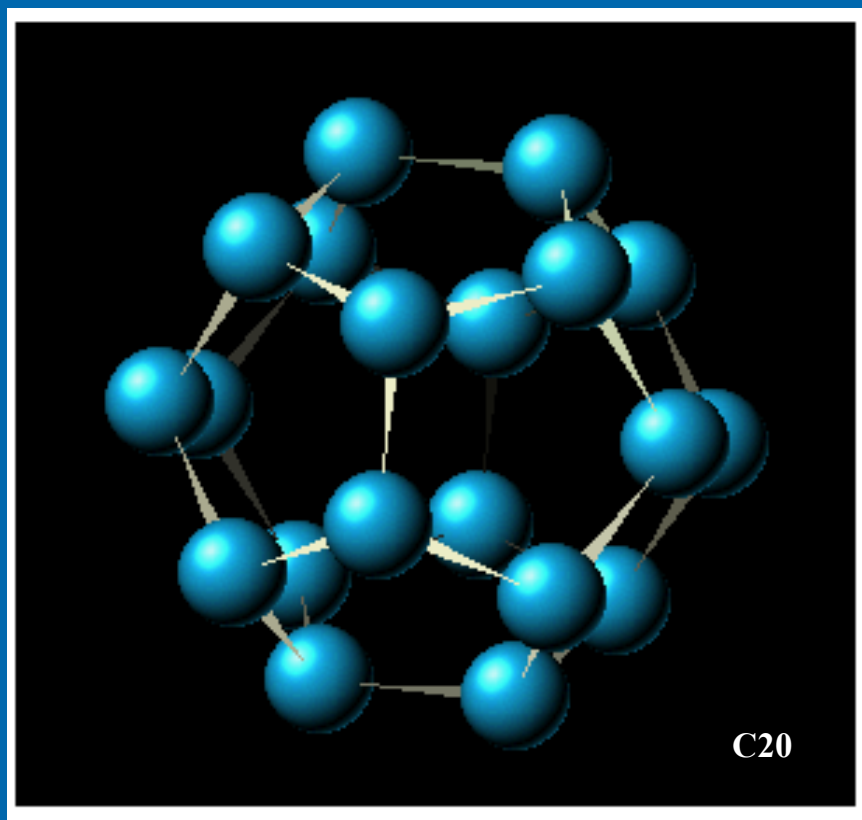
In its September 7th, 2000 issue, Nature heralded the discovery of the C₂₀, the smallest molecule in the Fullerene family.

Horst Prinzbeck (Albert Ludwigs U, Friburg, Germany) made a dodecahedron-shaped molecule out of carbon atoms.

All fullerenes incorporate exactly 12 pentagonal rings of carbon atoms, with some number of hexagonal rings filling in the rest of the structure. The most famous member of the family is C₂₀ - *Buchminsterfullerene* (1985).

Uniquely, the C₂₀ Fullerene has NO hexagons.

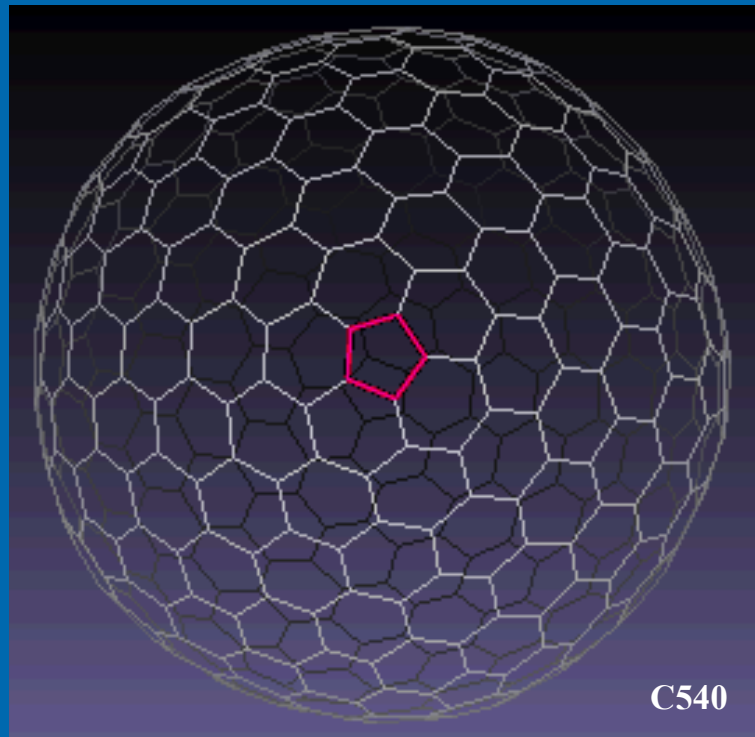
Prinzbeck and team started with a molecule known as *dodecahedrane* (C₂₀Heo), first synthesized in 1982. The Hydrogen atoms were ruptured by bromine atoms that were then stripped away, leaving a “*delicate dodecahedral cage of pure carbon*”.



← Dodecahedrane

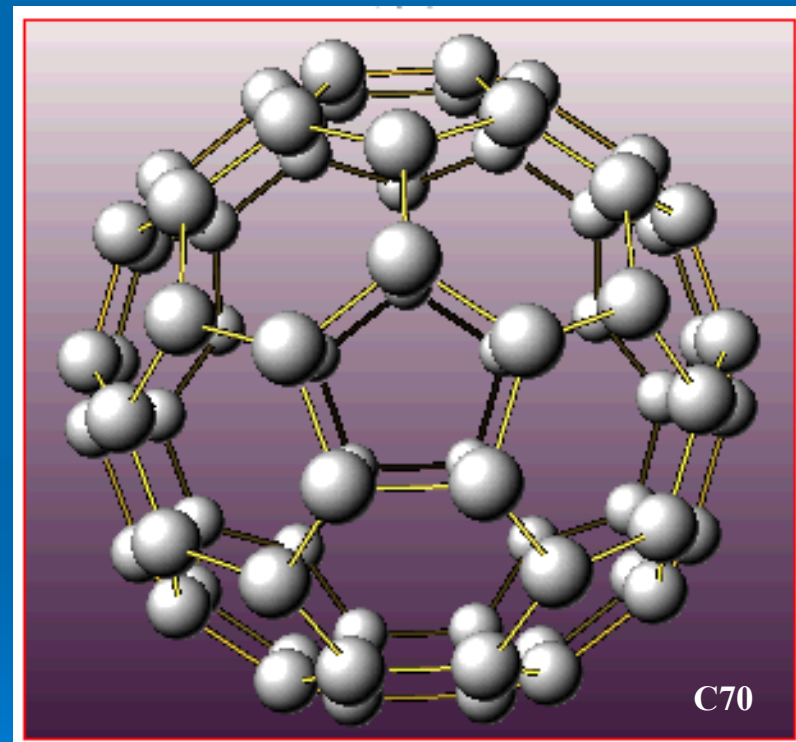
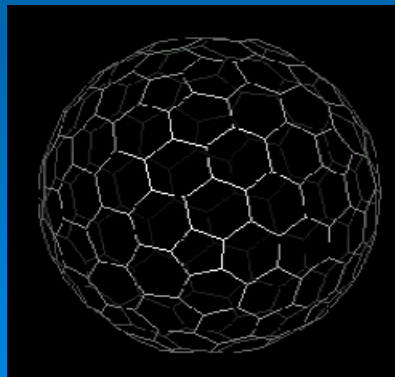
SG107.5.6.7 Geodesics (7) Higher Fullerenes

The Fullerene family comprises higher frequency members.



All images:

<http://www.univ-lemans.fr/.../01/weber/fullerenes.html>

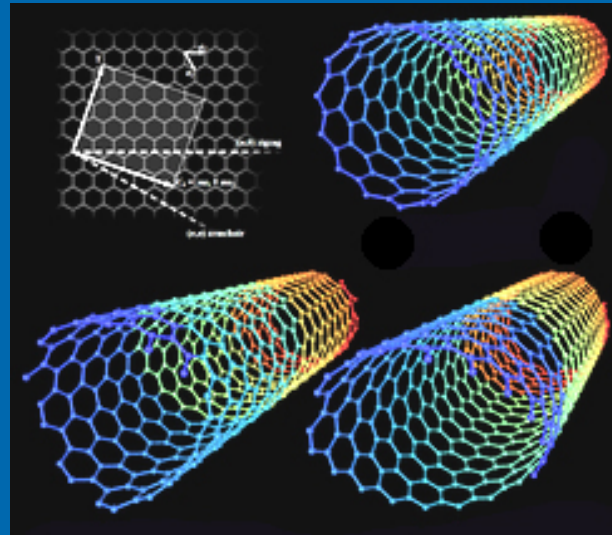


“Carbon nanotubes (CNTs) are allotropes of carbon. Their name is derived from their size, since the diameter of a nanotube is on the order of a few nanometers (approximately 1/50,000th of the width of a human hair), while they can be up to several millimeters in length, with a nanostructure that can have a length-to-diameter ratio of up to 28,000,000:1.

Nanotubes are members of the fullerene structural family, which also includes the spherical buckyballs. The ends of a nanotube might be capped with a hemisphere of the buckyball structure.

These cylindrical carbon molecules have novel properties that make them potentially useful in many applications in nanotechnology, electronics, optics and other fields of materials science, as well as potential uses in architectural fields. They exhibit extraordinary strength and unique electrical properties.”

(Wikipedia)

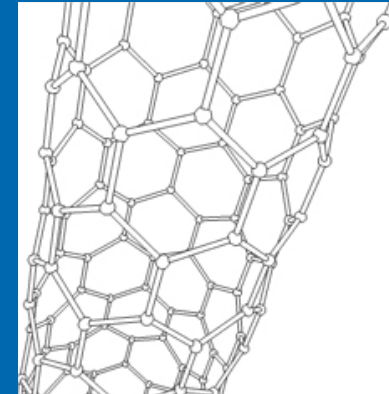


The question of the toxicity of nanotubes has been raised as NTs can cross cell membrane barriers.



↑ A Nanobud combining a nanotube and a fullerene. Nanobuds are good field emitters.

SG107.5.6.8 Geodesics (8) Nanotubes



Potential applications of NTs

GDVs (gene delivery vehicle) -
Nano batteries - Nano flowers &
meadows - Nano radios - Nanotori
(NTs bent into a torus shape) -
Nano transistors - Solar cells -
Space elevators - Ultra capacitors...

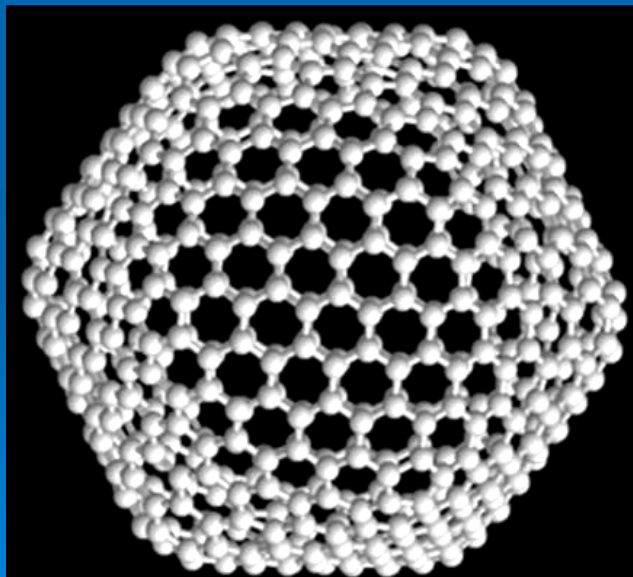
SG107.5.6.9 Geodesics (9) Buckypaper

Buckypaper is a thin sheet made from an aggregate of carbon nanotubes. The nanotubes are approximately 50,000 times thinner than a human hair.

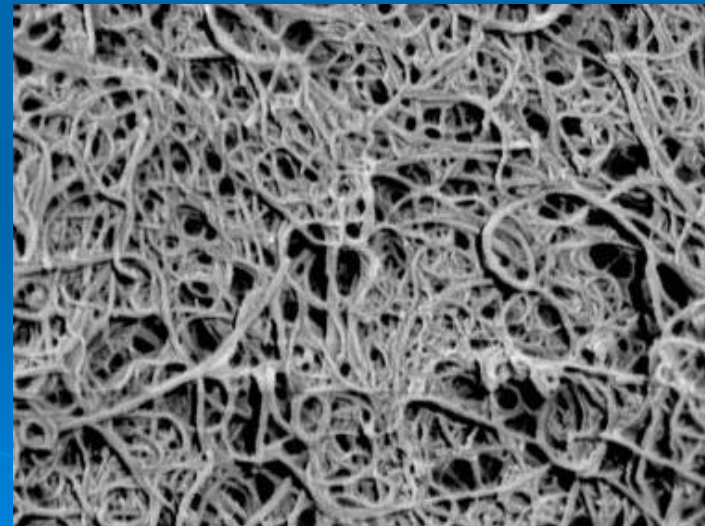
Buckypaper is one tenth the weight yet potentially 500 times stronger than steel when its sheets are stacked to form a composite. It could disperse heat like brass or steel and it could conduct electricity like copper or silicon.



↑ Naked-eye size Buckypaper

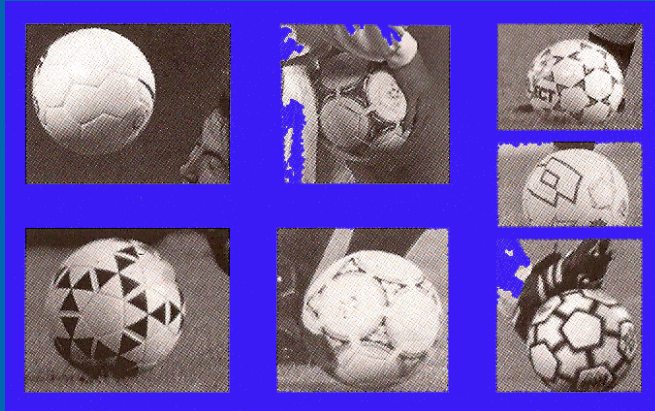


↑ Nano size.



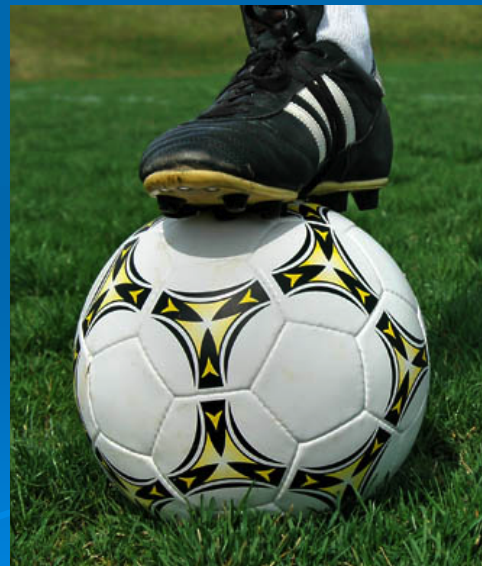
↑ Intermediate size

SG107.5.6.10 Geodesics (10) in Football

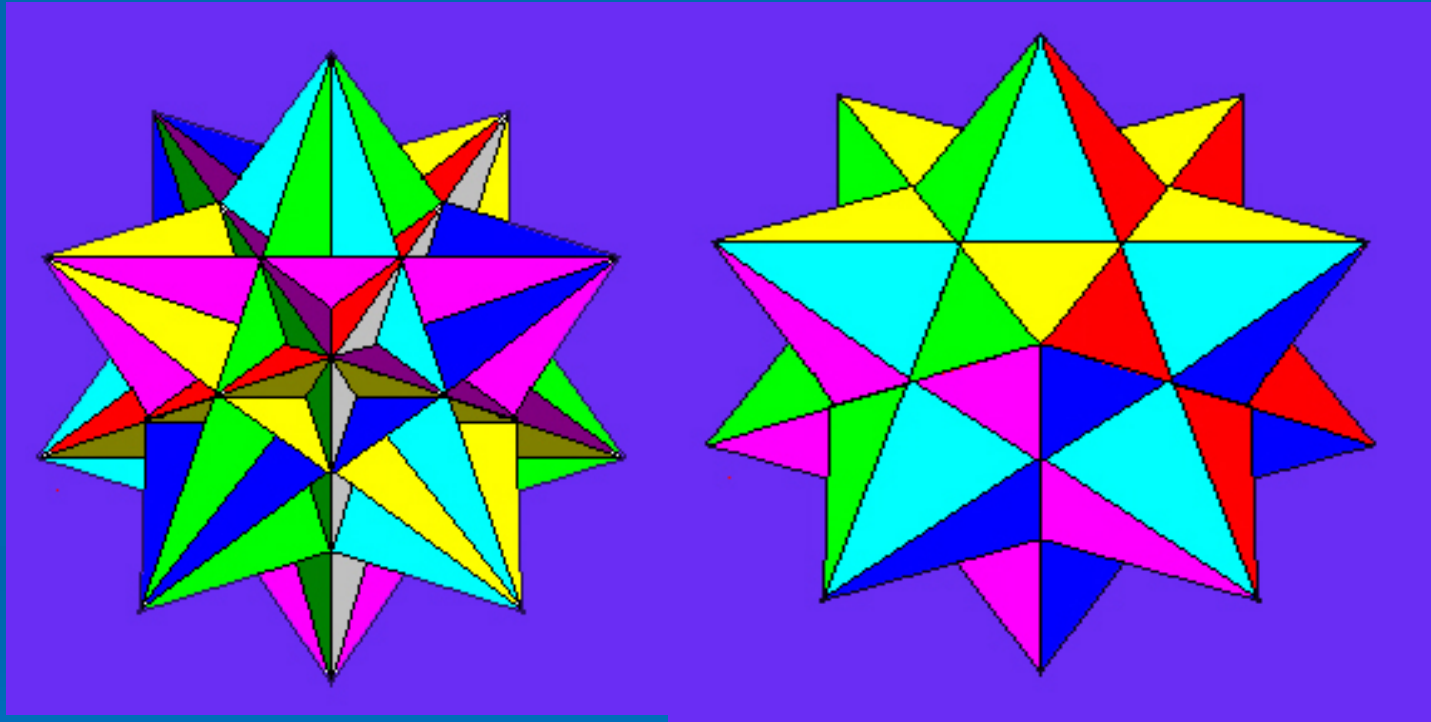


Beneath the fancy decorations, the balls always show a geodesic structure composed of a pentagon surrounded by five hexagons.

This is Sacred Geometry art in full action.



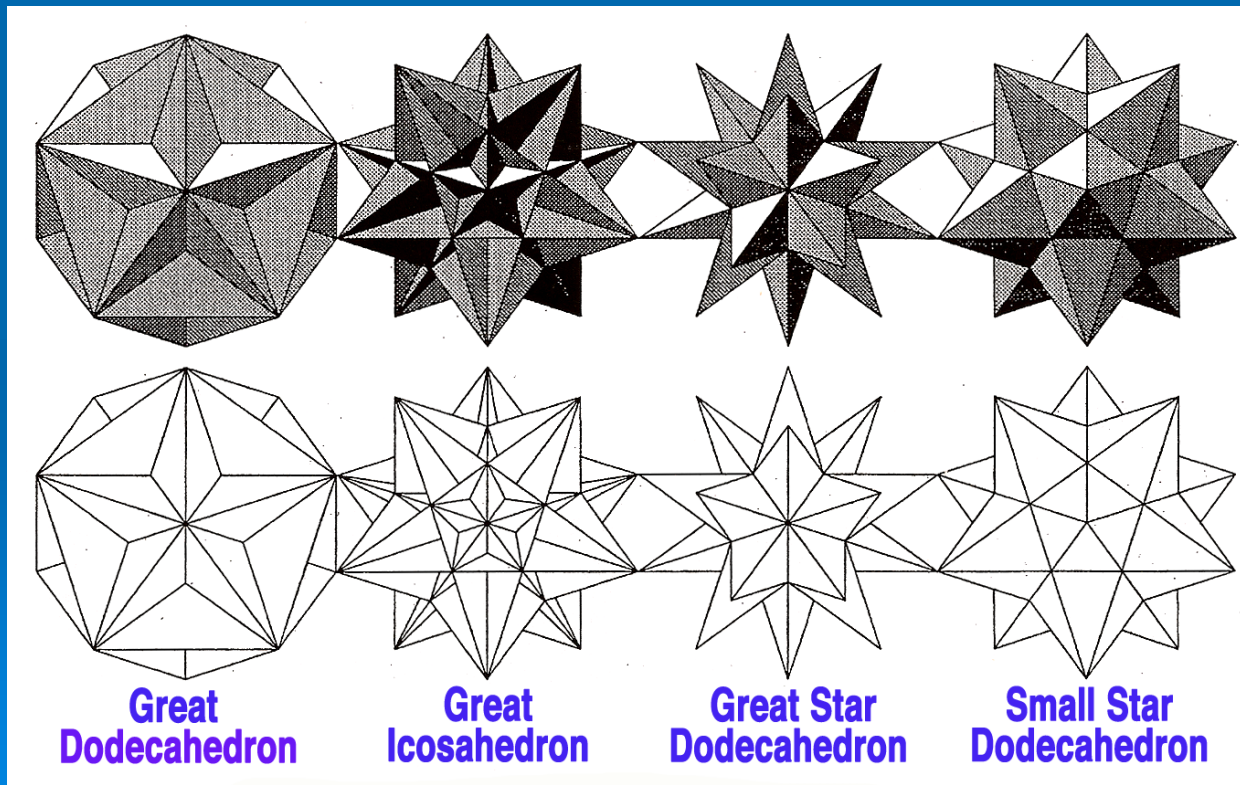
SG107.6 Chapter 6. Other Polyhedra



SG107.6.1.1 The 4 Kepler-Poinsot Solids (1)

While the 5 Platonic and the 13 Archimedean Solids are convex, the 4 Kepler-Poinsot Solids are concave: some of their vertices do not touch the circumscribed sphere. Johannes Kepler (*Harmonices Mundi*, 1619) is recorded to have discovered two star-polyhedra: the small and large star-dodecahedron. In 1809, in addition to re-discovering Kepler's two star-solids, Louis Poinsot discovered two more: the Great Icosahedron and the Great Dodecahedron.

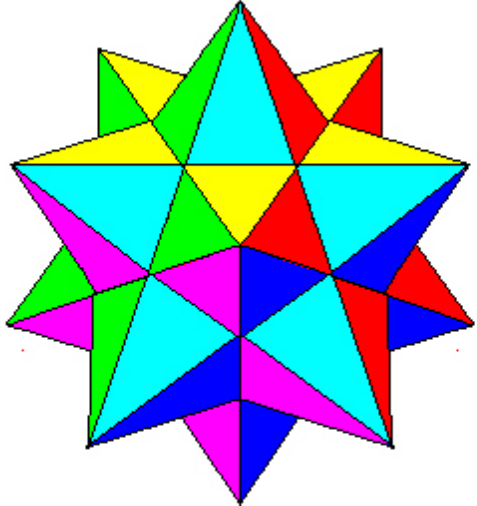
Historically speaking, a small stellated dodecahedron appears in a marble tarsia (inlay panel) on the floor of St. Mark's Basilica, Venice, Italy. It dates from the 1400s and is sometimes attributed to Paolo Uccello. In his *Perspectiva corporum regularium* (1500s), Wenzel Jamnitzer depicts the great dodecahedron and the great stellated dodecahedron.



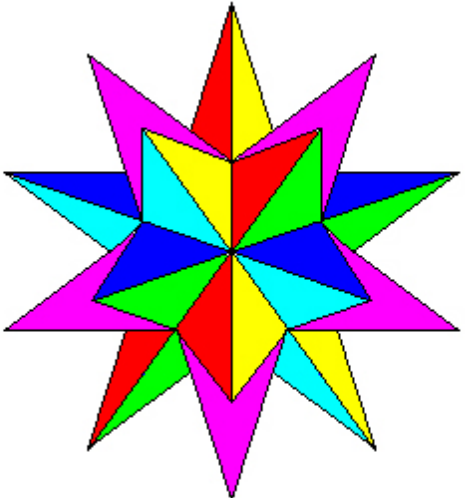
← Designs from
Bruce Rawles,
Sacred Geometry
Design Sourcebook.

www.geometrycode.com

SG107.6.1.2 The 4 Kepler-Poinsot Solids (2)



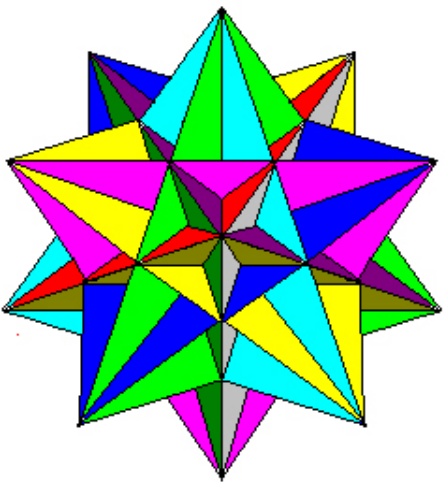
Small Stellated Dodecahedron



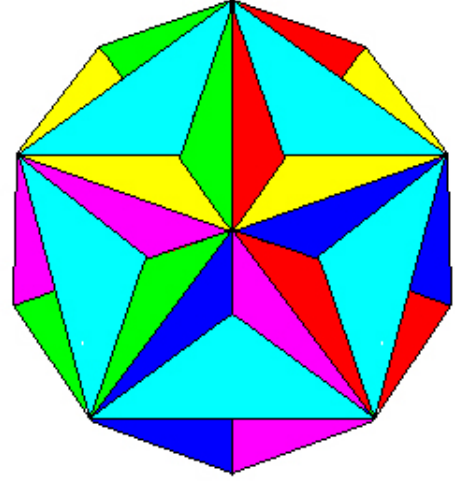
Great Stellated Dodecahedron

↑ Kepler's Solids

Poinsot's Solids →



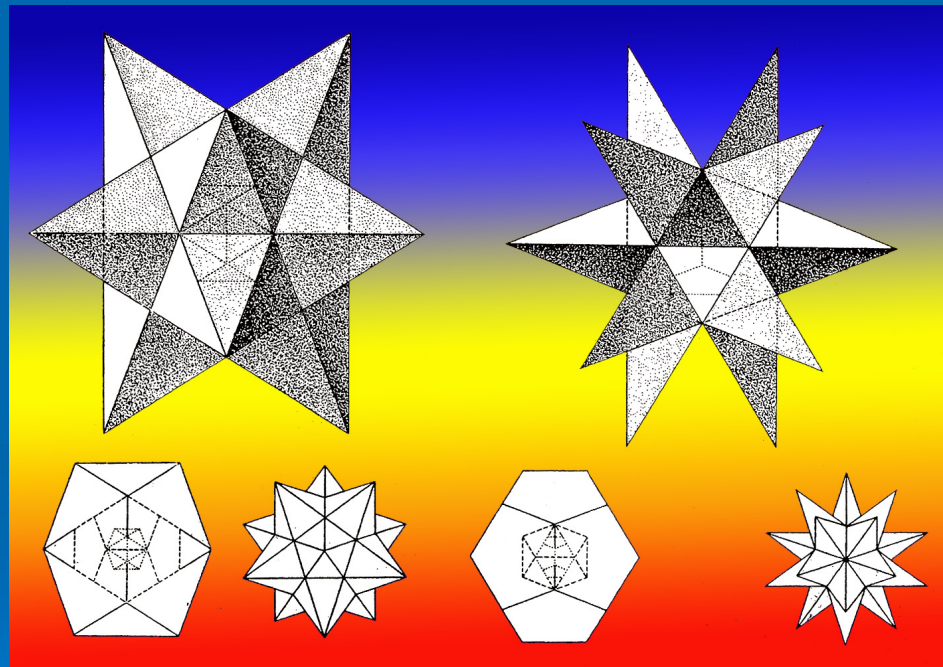
Great Icosahedron



Great Dodecahedron

SG107.6.1.3 Kepler's (3) Two Star Polyhedra

Kepler's two symmetrical star-solids are 3D transformations of the pentagram.
They are directly linked to the Golden Ratio Φ .



↑ The Small Star-Dodeca
(12 vertices)
obtained by extending
the faces
of a regular dodeca,

↑ The Great Star-Dodeca
(20 vertices)
obtained by extending
the faces
of a regular icosahedron



↑ Engraving from *Harmonices Mundi*. 1619.

[Graphics on left from Matila Ghyka.
The Geometry of Art and Life.
1946.]

SG107.6.2.1 Stellation (1)

Stellation is a process of constructing new polygons (in two dimensions), new polyhedra in three dimensions, or, in general, new polytopes in n dimensions. The process consists of extending elements such as edges or face planes, usually in a symmetrical way, until they meet each other again. The new figure is a stellation of the original.

Matila Ghyka, a European pioneer of Sacred Geometry in the first half of the 20th century, explains:

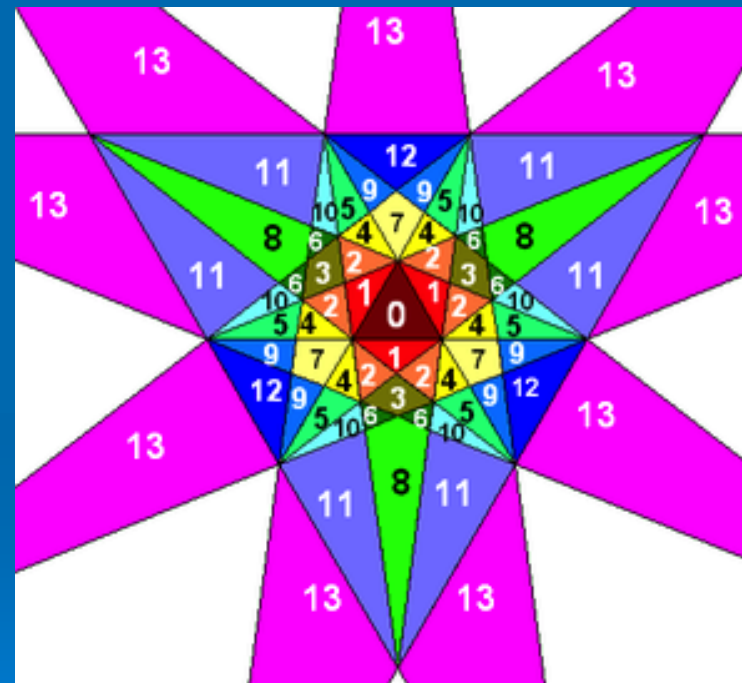
“There is another way of passing from dodecahedron to icosahedron, and from icosahedron to dodecahedron: it is to lengthen out all the sides of either of these solids until they meet.

This operation, on the dodeca, produces the 12 vertices of an enveloping icosahedron; on the icosahedron, it produces the 20 vertices of an enveloping dodecahedron.

These operations can be repeated indefinitely, producing alternating ever-growing dodecahedra and icosahedra, and we obtain thus a ‘pulsation of growth’ in which lines, surfaces and volumes are ruled by the Golden Section or Φ proportion.

These same operations, in their first step, produce the two Star-Polyhedra of Kepler.”

[M. Ghyka. *Geometry of Art and Life*. 1946.]



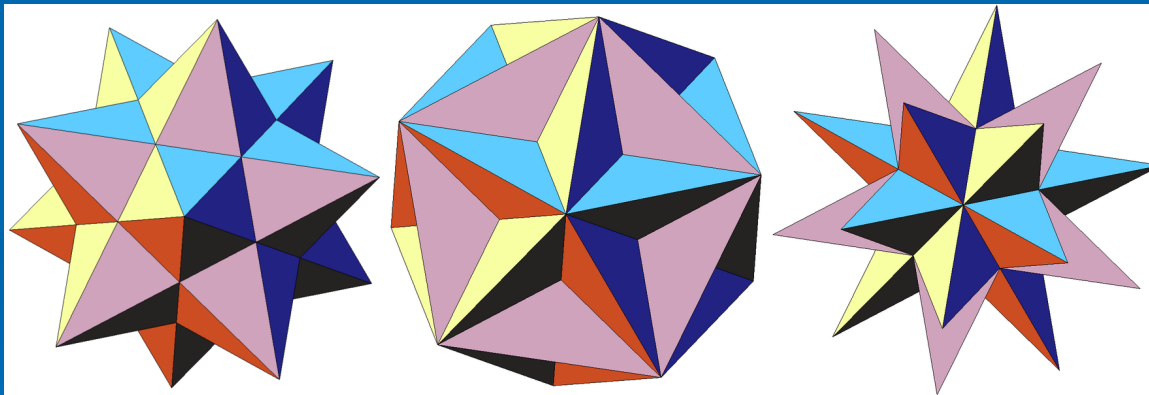
↑ Stellation diagram of the icosahedron
(Wikipedia)

SG107.6.2.2 Stellation (2)

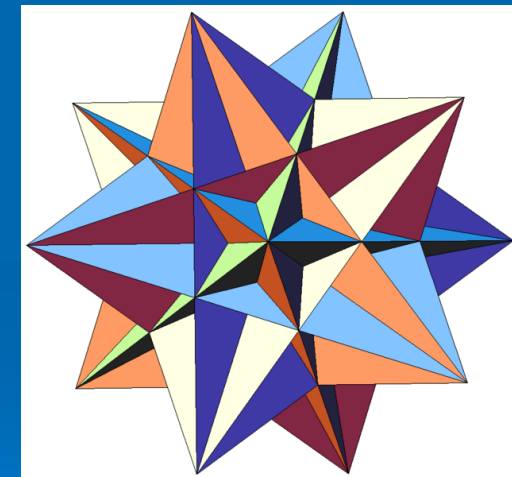
In 1938 (3rd ed, 1999), H.S.M. Coxeter, P. Du Val and others published *The Fifty-Nine Icosahedra* which enumerated the stellations of the icosahedron, according to the rules by J.C.P. Miller.

Polyhedronists (a growing professional & amateur group) are still discussing the acceptable rules of stellation but as a field of scientific research and artistic applications, polyhedron stellations are of immense interest.

Archimedean Solids can also be stellated: *Wikipedia* mentions that there are 187 stellations of the triakis tetrahedron and 358,833,097 stellations of the rhombic triacontahedron. Lots to explore and enjoy!



↑ The 1st, 2nd and 3rd stellation of the dodecahedron.
(*Wikipedia*)

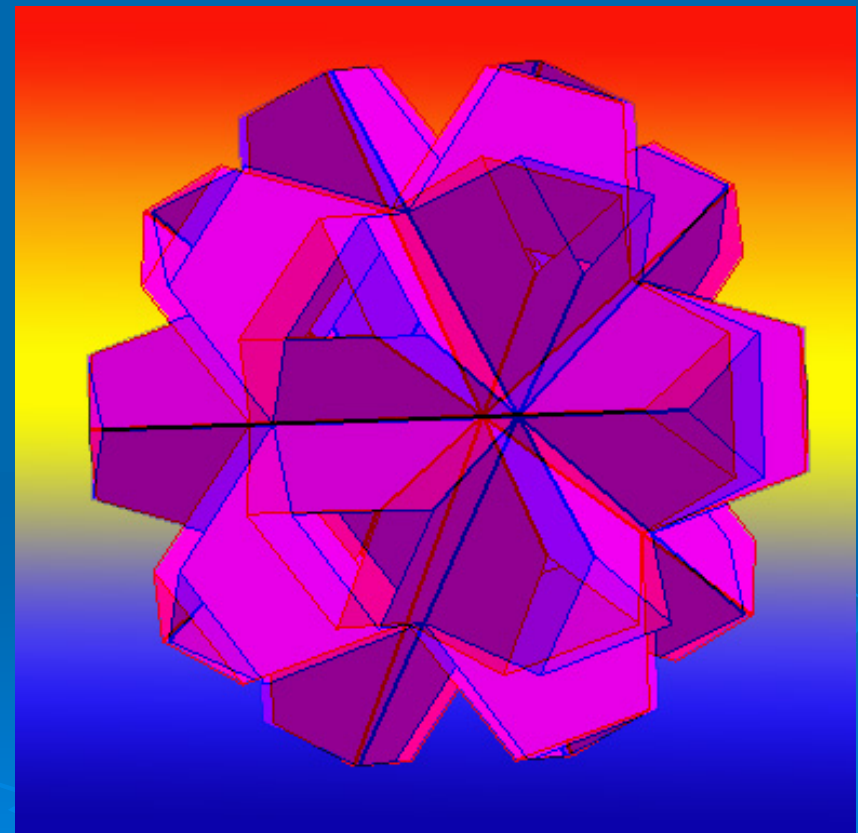
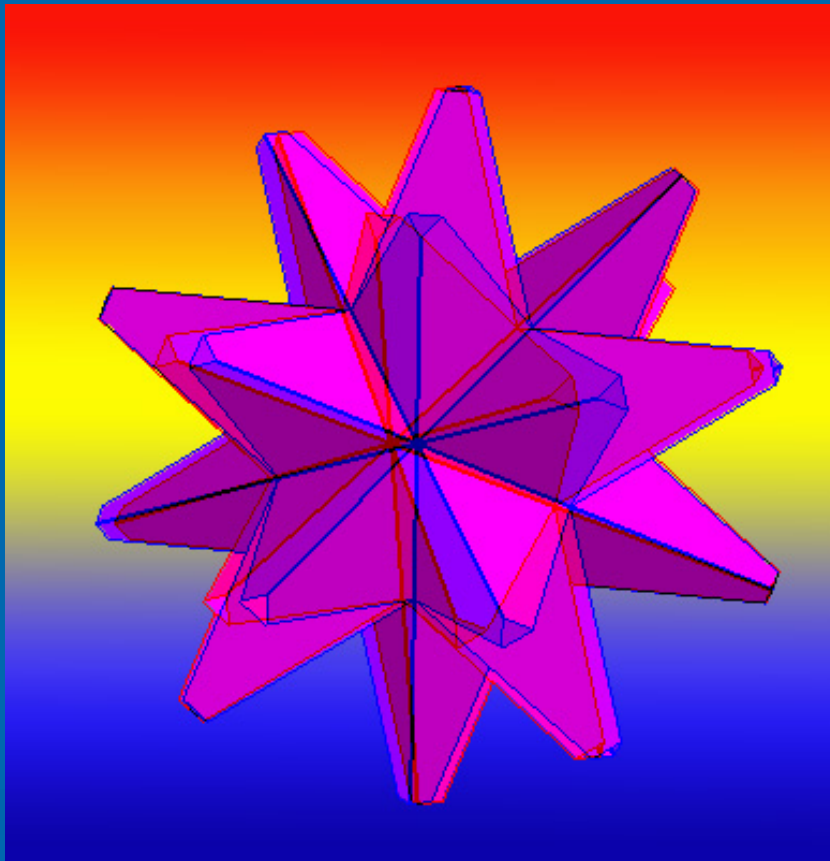


↑ The 16th stellation of the icosahedron.

Animated stellations of the Dodecahedron:
<http://dogfeathers.com/java/stardodec.html>



SG107.6.2.3 Stellation (3)



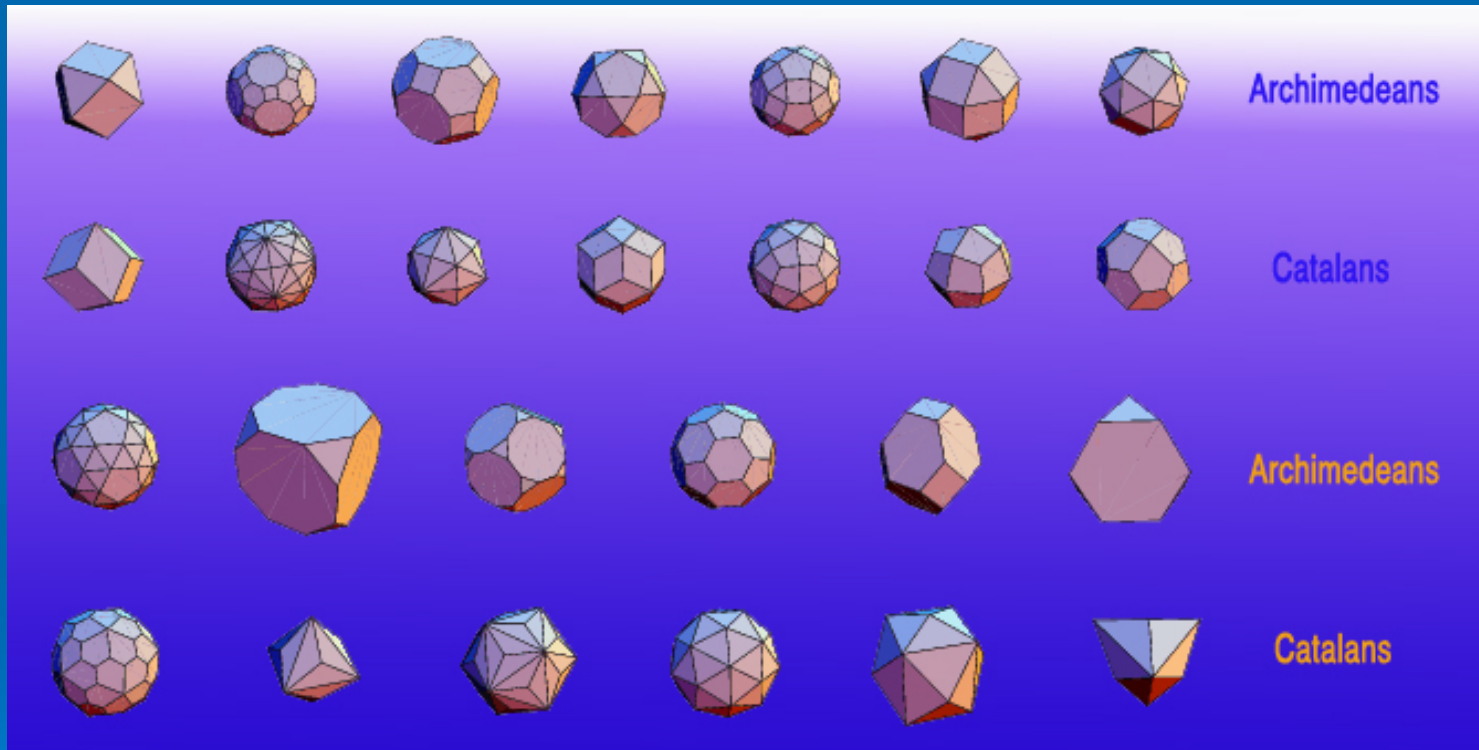
Animated stellations of the Dodecahedron:
<http://dogfeathers.com/java/stardodec.html>

The Catalan Solids are the duals of the Archimedean Solids. They are named after Belgian mathematician, Eugène Catalan, who first described them in 1865.

A dual of a polyhedron is created by replacing each face by a vertex, and each vertex by a face.

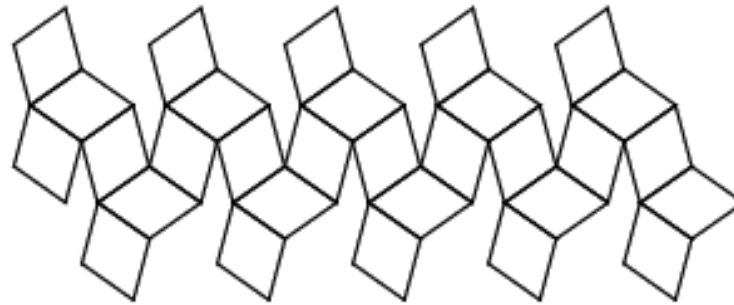
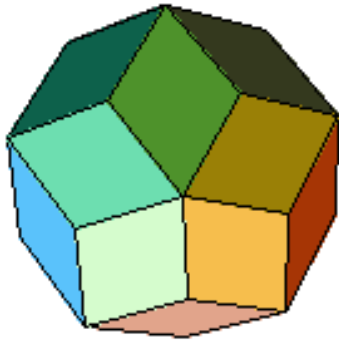
Example: the dual of the icosahedron is the dodecahedron.

Unlike the Platonic & Archimedean solids, the faces of the Catalans are not regular polygons.

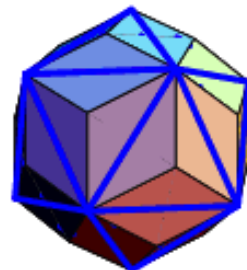
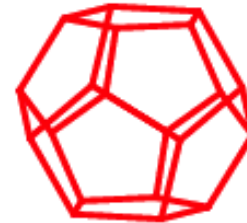
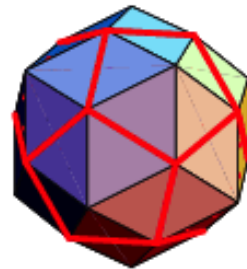


the 13 Catalans with mouse rotation
<http://www.ibiblio.org/e-notes/3Dapp/Catalan.htm>

SG107.6.3.2 The 13 Catalans (2) Rhombic Triacontahedron (1)

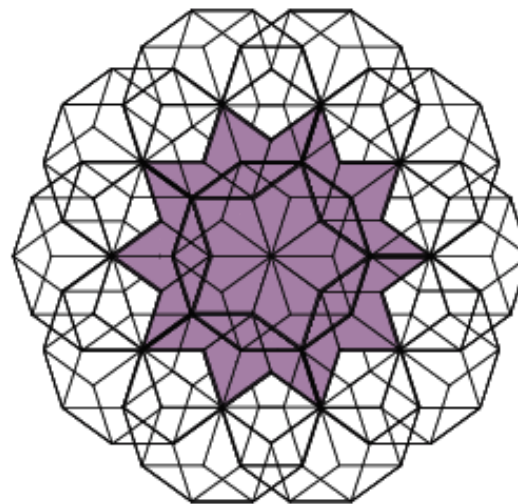
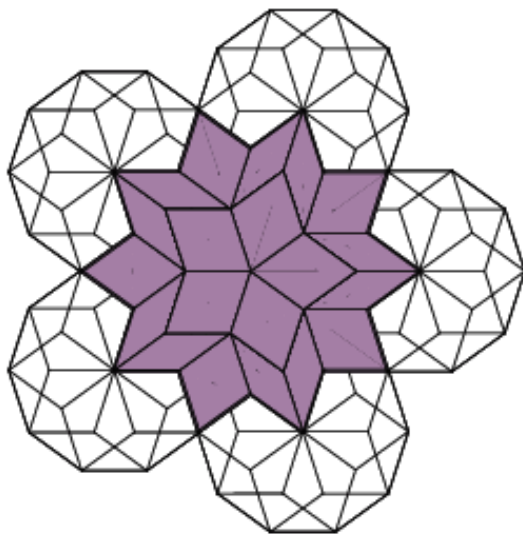
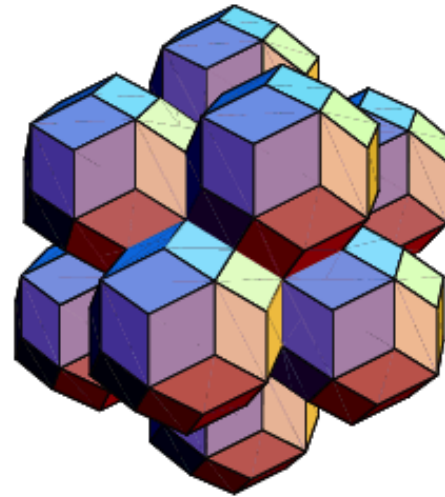
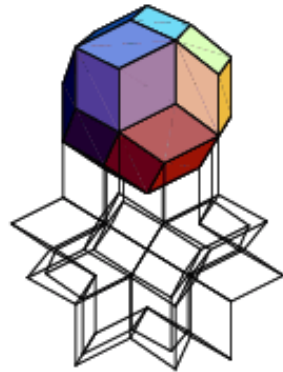


← The *rhombic triacontahedron* and its net.



mathworld.wolfram.com

← The short diagonals of the faces of the *rhombic triacontahedron* give the edges of a dodecahedron, while the long diagonals give the edges of the icosahedron.

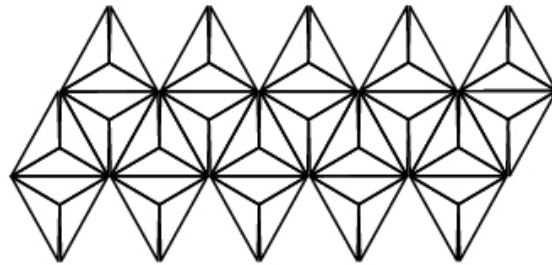


SG107.6.3.3
**The 13
Catalans
(3) Rhombic
Triaconta
(2)**

← 12 rhombic
triacontahedra
fit together
around a central
rhombic
hexecontahedron

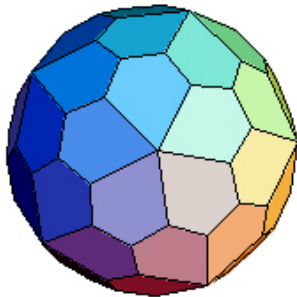
SG107.6.3.4 More Catalans (4)

Triakis Icosahedron

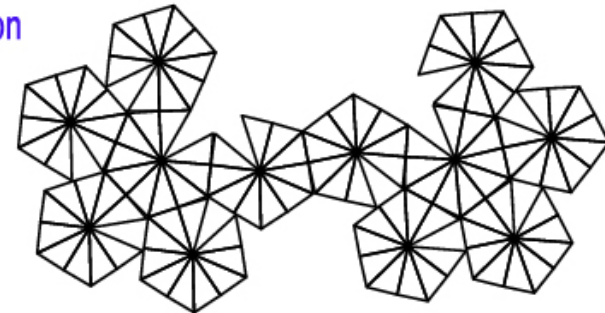


[//mathworld.wolfram.com](http://mathworld.wolfram.com)

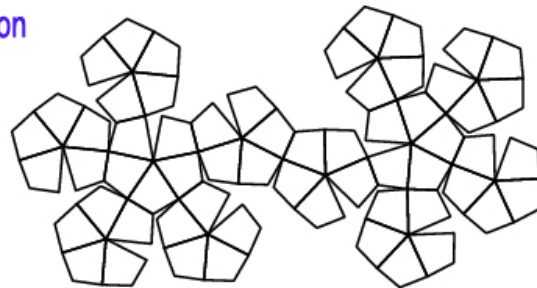
Pentagonal Hexeconta



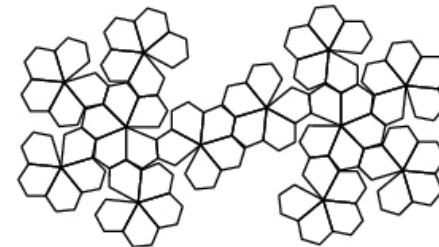
Disdyakis Triacontahedron



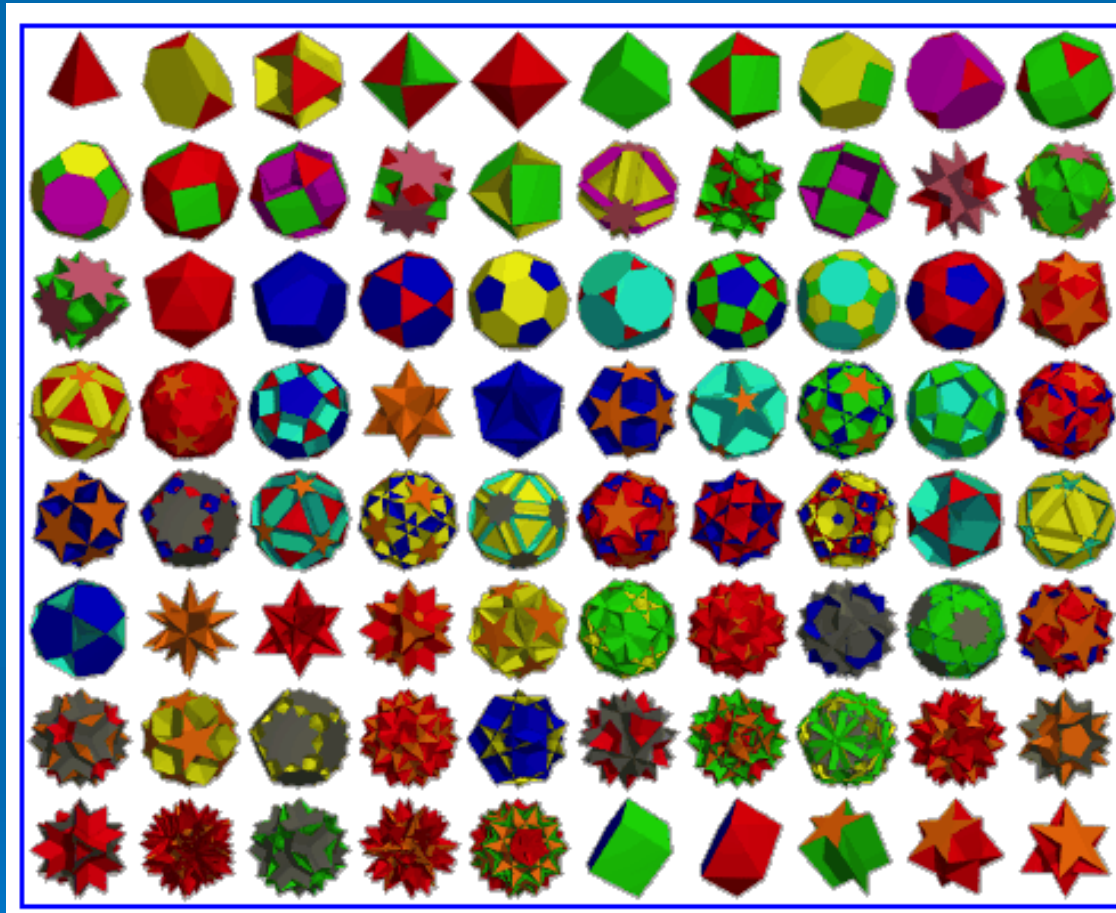
Deltoidal Hexecontahedron



Pentagonal Hexacontahedron Net



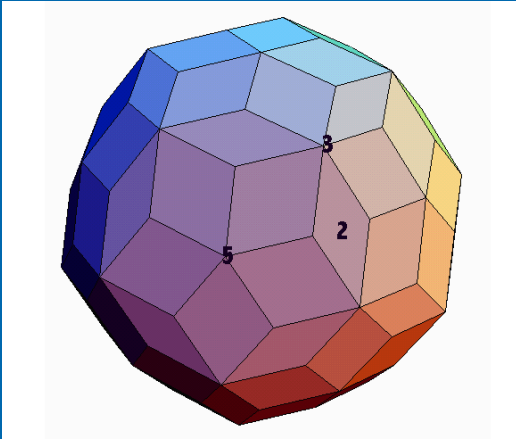
SG107.6.4 All 80 Uniform Polyhedra



<http://www.mathconsult.ch/>. Visit this site for Visual Index, Guided Tour, List of Thumbnails, hi-res image + geometrical info for each polyhedron.
Roman E. Meader is the developer of *Mathematica* software, a wonderful 3D visual tool.

A *Zonohedron* is defined as a convex polyhedron in which every face is a parallelogram.

SG107.6.5 Zonohedra: The RE



The Rhombic Enneacontahedron a 90-faced polyhedron composed of two shapes of rhombic faces.



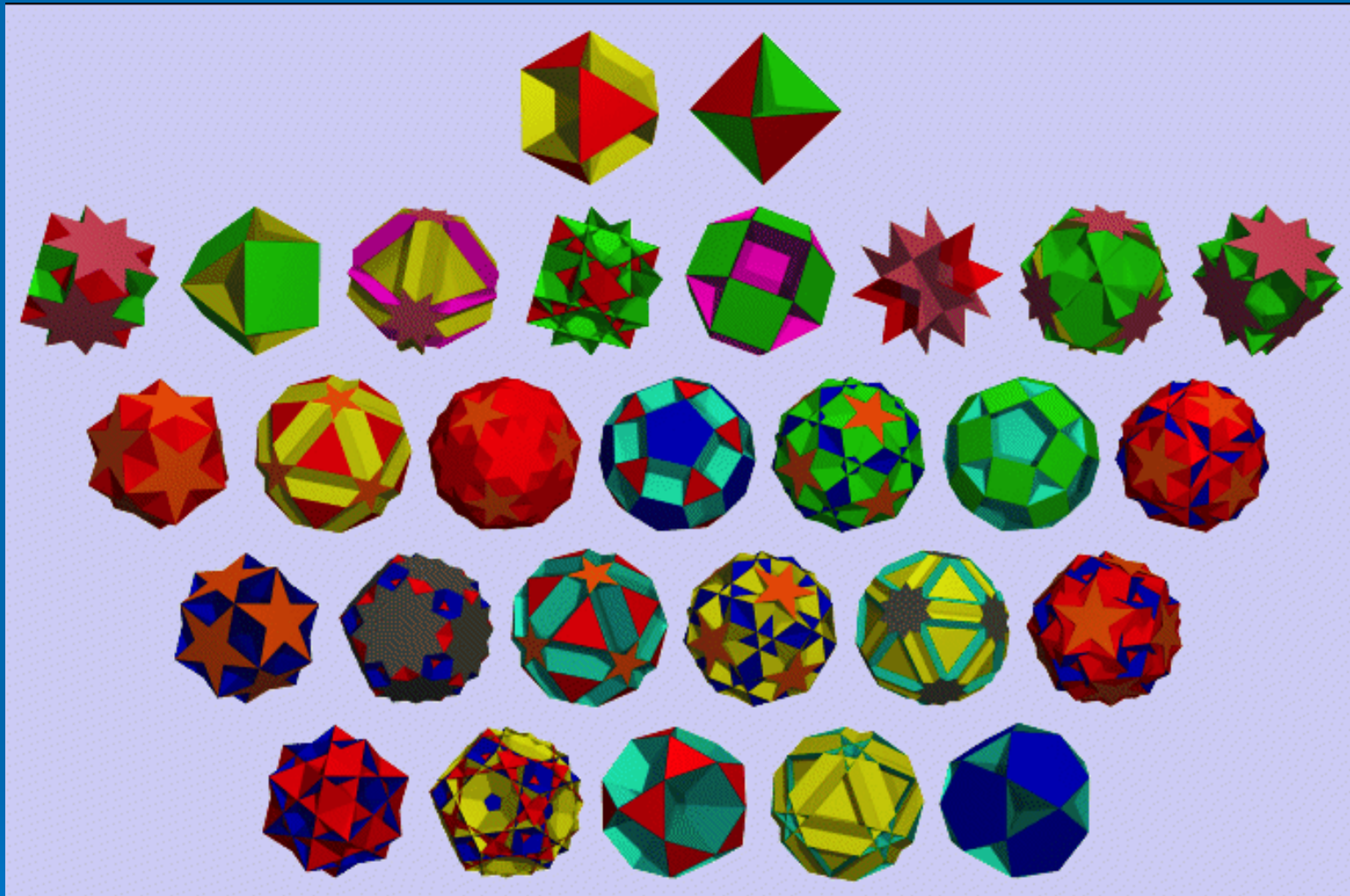
↑ The assembled RE



↑ The empty plexi shell (left) and the 120 colored parallelepipeds. The components assemble in a color-matching manner to fill the interior of the 90-faced volume. The result can be viewed as a sculpture, a puzzle, or a mathematical model.

This is the beautiful model created by George W. Hart www.georgehart.com

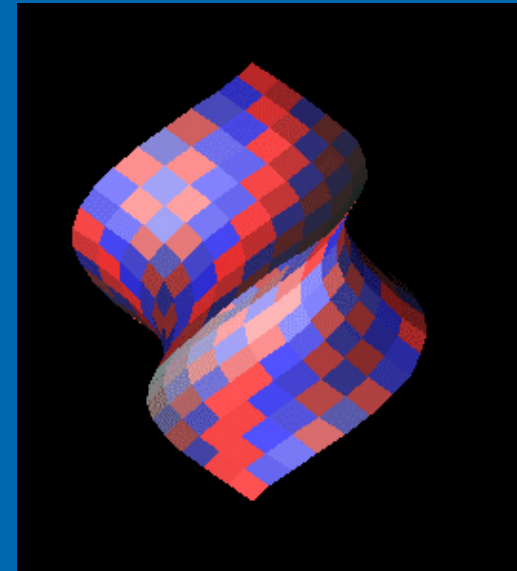
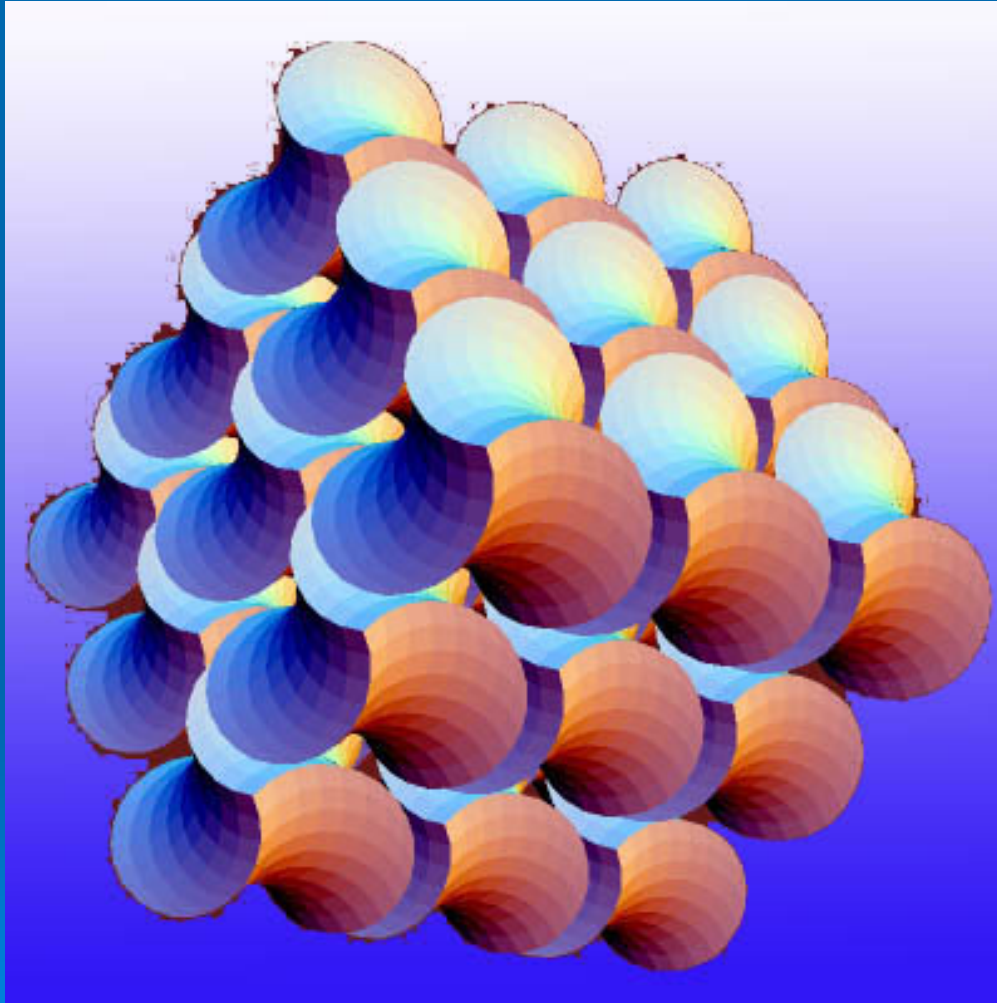




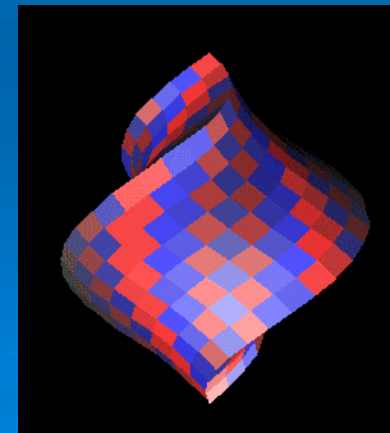
SG107.6.6 Some Non-Convex Polyhedra

www.ac-noumea.nc/maths/amc/polyhedr/p_unif_.htm

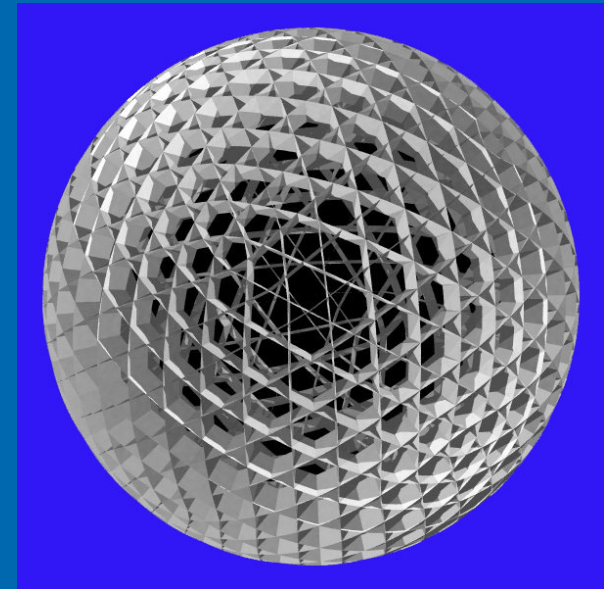
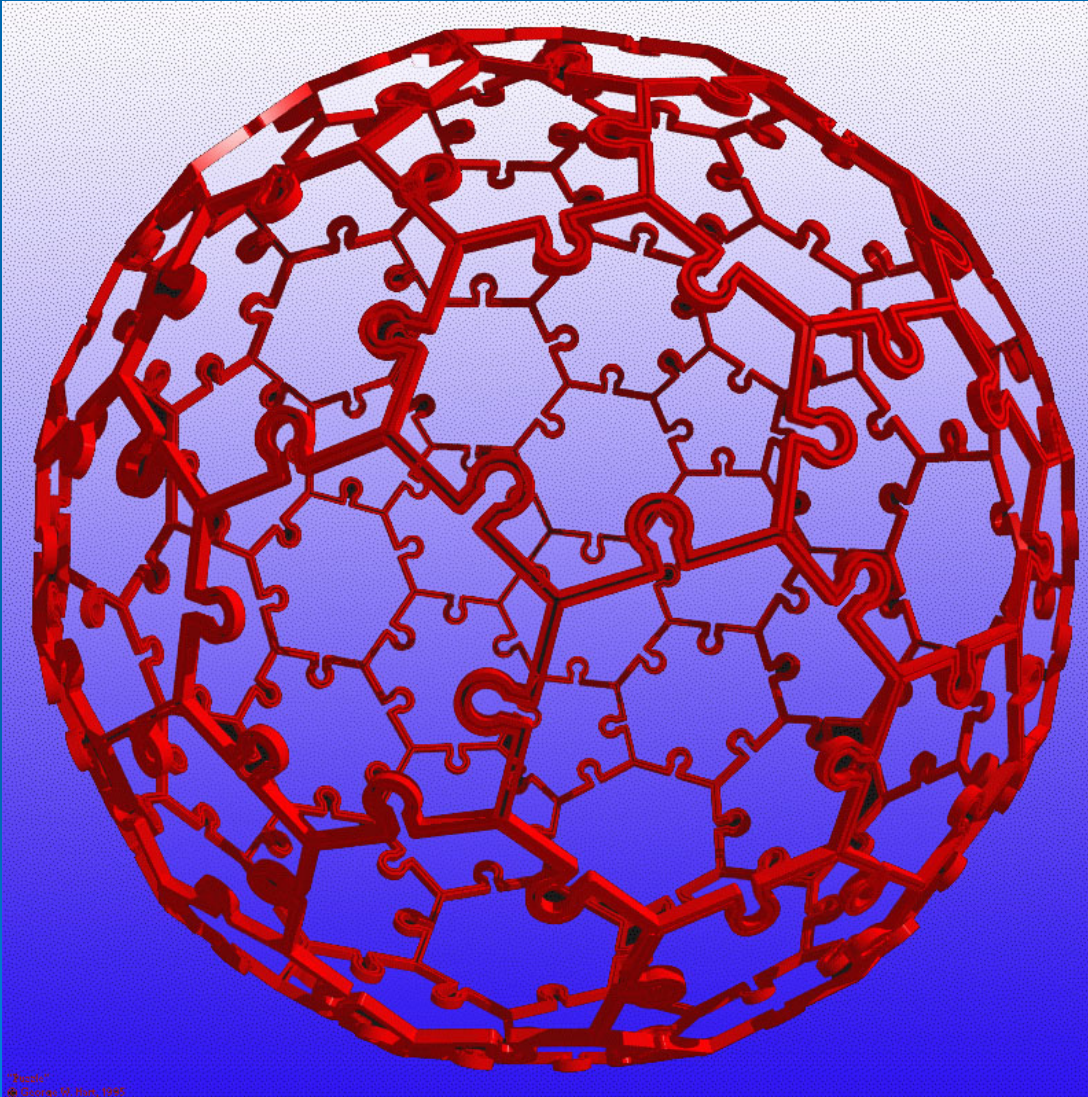
SG107.6.7.1 Other Polyhedra in Art (1)



Spira-hedra
(Unknown web origin)



SG107.6.7.2 Other Polyhedra in Art (2)



← ↑ Two works by George Hart.
www.georgehart.com

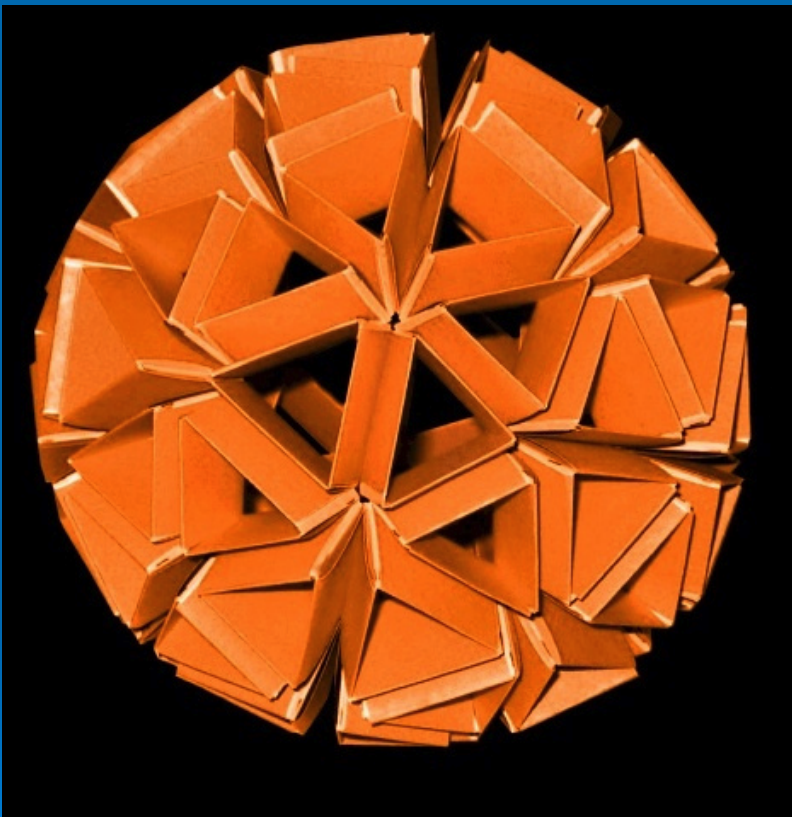


SG107.8 Polyhedral Dice

SG107.9 Polyhedra Origami (1)

Modular Origami creates paper forms entirely by folding, but using more than one piece of paper.
(Works on this page by Arturo Pascalín).

<http://www.aleakybos.ch/sha.htm>



↑ Geosphere frequency-2
Built out of 200 modules

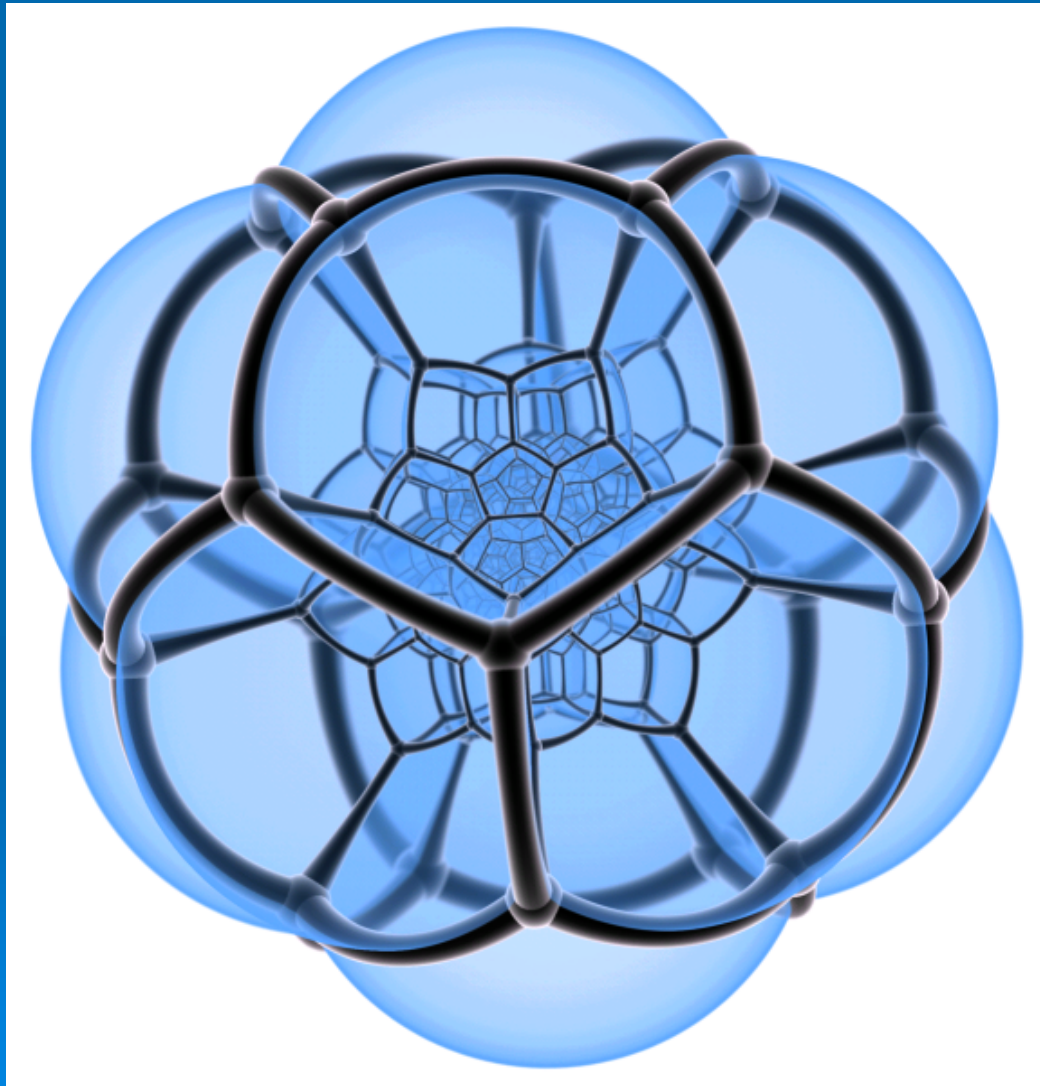


↑ Geosphere v3



← Snub
Cuboctahedron

SG107.7 Chapter 7. Higher-Dimensional Polyhedra



SG107.7.1.1 Hyper Platonics (1)

In 200 BCE, Euclid proved that there are exactly five regular solids in 3 dimensions. In 1852 (published in 1901), Swiss mathematician Ludwig Schläfli proved that there are exactly six regular solids in 4 dimensions.

The 6 convex regular 4-dimensional solids are also called *4-polytopes* or *polychora*. They are analogs of the 3D Platonic Solids with a newcomer: the 24-cell. They can be called *Hyper-Platonics*.

The mathematical reasoning for 4-dimensional solids is the following:

Just as, in three-dimensional space, there are regular *polyhedra* bounded by congruent faces, in four-dimensional space, there are regular *polytopes* bounded by congruent polyhedra. In three dimensions, one can place *three equilateral triangles* around a vertex with room left over. Then the gap can be closed by folding up in the faces into the 3D until the edges touch. In four dimensions, one can place *three equilateral tetrahedra* around an edge with room left over. Then the gap can be closed by folding up in the solids into the 4D until the faces touch.

Using trigonometry, the dihedral angle of a regular tetrahedron is about 70.5° . The dihedral angle is the angle between the faces of the solid. We can make a polytope by fitting three regular tetrahedra around an edge, and another by fitting four around an edge. We can also fit five as $70.5 \times 5 = 352.5$ degrees and we'll have a little over seven degrees left to fold up into the 4D.

By figuring out the dihedral angles of the other regular polytopes, we find that there is a polytope which is formed by fitting three cubes at an edge, one with three octahedra at an edge, and one with three dodecahedra at an edge. The dihedral angle of an icosahedron is too big to be able to fit three at an edge.

SG107.7.1.2 Hyper Platonics (2)

By applying an analogical reasoning, mathematicians figured out the 6 Polytopes.

For instance, to make a 4D Tetrahedron (5-cell Polytope or 4-simplex), take a tetrahedron, add a vertex (in that elusive 4D) and then connect it to all the vertices in the tetrahedron.

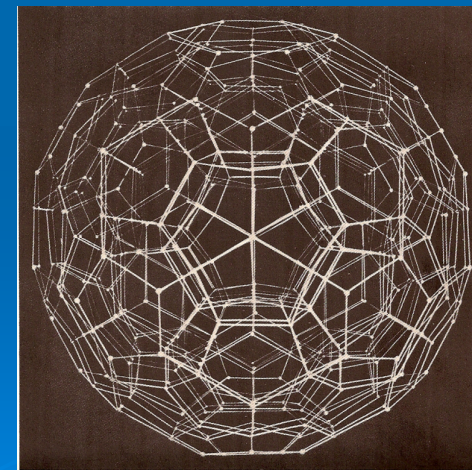
We can then just count how many of each element this new 4D volume has. It has five vertices (4 from the tetrahedron, plus the one we added), 10 edges (6 from the original tetrahedron, plus one more to connect the new vertex to each of the 4 existing vertices), 10 faces (the starting 4 plus one built from each edge of the original edges of the tetrahedron) and, finally, 5 tetrahedra (the original, plus one built on each face of the original tetrahedron.)

Similar arguments can be made for each of the other five regular polytopes. The table on the next page (*Wikipedia*) summarizes the attributes of the 6 regular 4D polytopes.



“Polytope is the general term of the sequence: point, segment, polygon, polyhedron,”

H.S.M. Coxeter. Regular Polytopes. 1963.


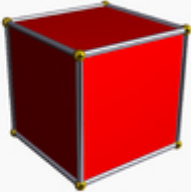


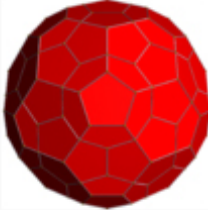
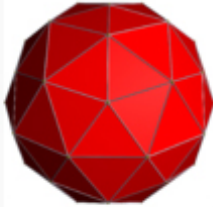


↑ The Hyper-dodecahedron in Coxeter’s book.


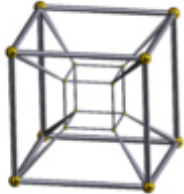

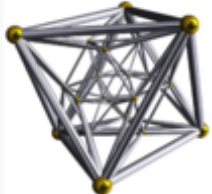
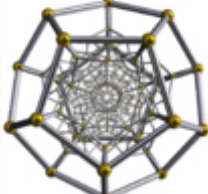
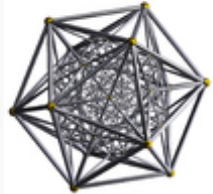
Names	Family	Schläfli symbol	Vertices	Edges	Faces	Cells	Vertex figures	Dual polytope	Symmetry group	
Pentachoron 5-cell pentatope hyperpyramid hypertetrahedron 4-simplex	simplex (n-simplex)	{3,3,3}	5	10	10 triangles	5 tetrahedra	tetrahedra	(self-dual)	A_4	120
Tesseract octachoron 8-cell hypercube 4-cube	hypercube (n-cube)	{4,3,3}	16	32	24 squares	8 cubes	tetrahedra	16-cell	B_4	384
Hexadecachoron 16-cell orthoplex hyperoctahedron 4-orthoplex	cross-polytope (n-orthoplex)	{3,3,4}	8	24	32 triangles	16 tetrahedra	octahedra	tesseract	B_4	384
Icositetrachoron 24-cell octaplex polyoctahedron		{3,4,3}	24	96	96 triangles	24 octahedra	cubes	(self-dual)	F_4	1152
Hecatonicosachoron 120-cell dodecaplex hyperdodecahedron polydodecahedron		{5,3,3}	600	1200	720 pentagons	120 dodecahedra	tetrahedra	600-cell	H_4	14400
Hexacosichoron 600-cell tetraplex hypericosahedron polytetrahedron		{3,3,5}	120	720	1200 triangles	600 tetrahedra	icosahedra	120-cell	H_4	14400

SG107.7.1.3 Hyper Platonics (3)






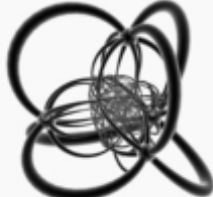
Solid orthographic projections

					
tetrahedral envelope (cell/vertex-centered)	cubic envelope (cell-centered)	octahedral envelope (vertex centered)	cuboctahedral envelope (cell-centered)	truncated rhombic triacontahedron envelope (cell-centered)	Pentakis icosidodecahedral envelope (vertex-centered)

Wireframe Schlegel diagrams (Perspective projection)

					
(Cell-centered)	(Cell-centered)	(Cell-centered)	(Cell-centered)	(Cell-centered)	(Vertex-centered)

Wireframe stereographic projections (Hyperspherical)

					
---	---	---	--	---	---

5-cell

8-cell


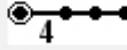

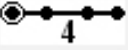
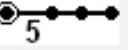

16-cell

24-cell

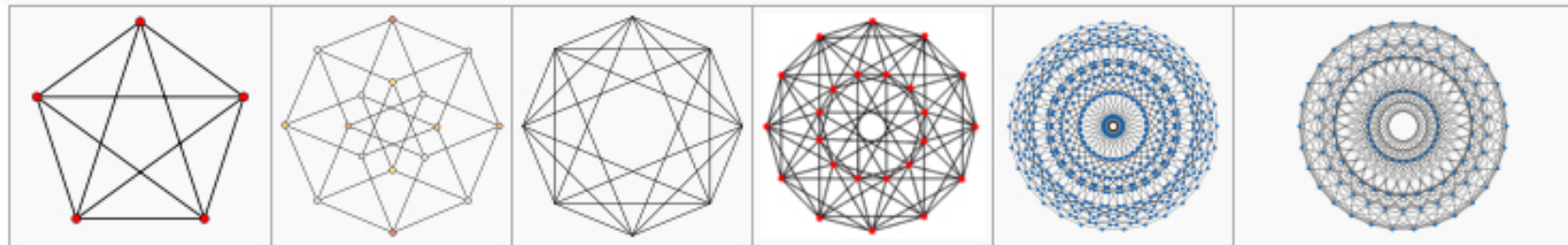
120-cell

600-cell

SG107.7.1.5 Hyper Platonics (5)

5-cell	8-cell	16-cell	24-cell	120-cell	600-cell
{3,3,3}	{4,3,3}	{3,3,4}	{3,4,3}	{5,3,3}	{3,3,5}
					

Wireframe orthographic projections inside Petrie polygons.



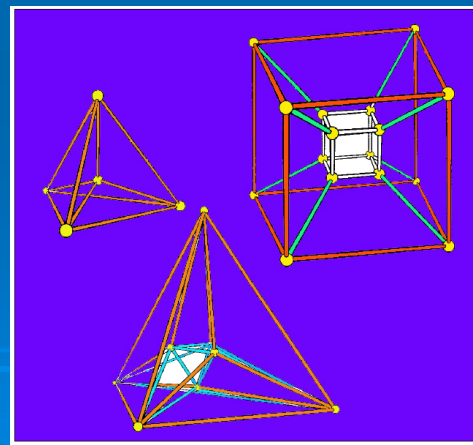
Note: The numbers between {} are Schläfli symbols, a notation of the form (p,q,r) defining polyhedra.

p = number of sides per face

q = number of faces per vertex (cells)

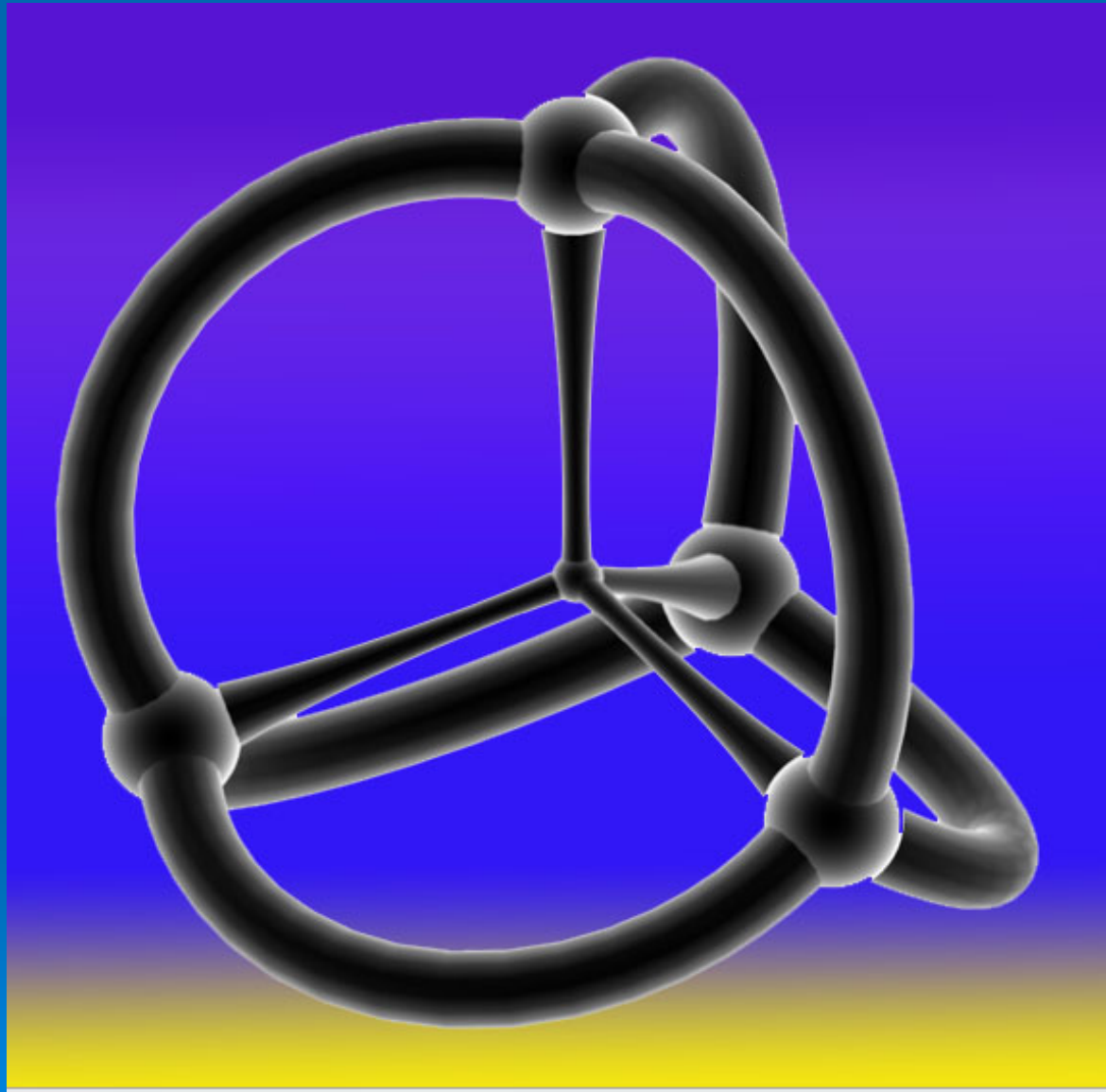
R = number of cells around each edge.

The term “*polytope*” was coined by the mathematician Hoppe, writing in German, and was later and introduced to English by Alicia Boole Stott, the daughter of logician George Boole...



← The 3 first Polytopes

**SG107.7.2.
The 5-Cell
Polytope**



Other names:

**Pentachoron
Pentatope
Hyper-pyramid
Hyper-tetrahedron
4-Simplex**

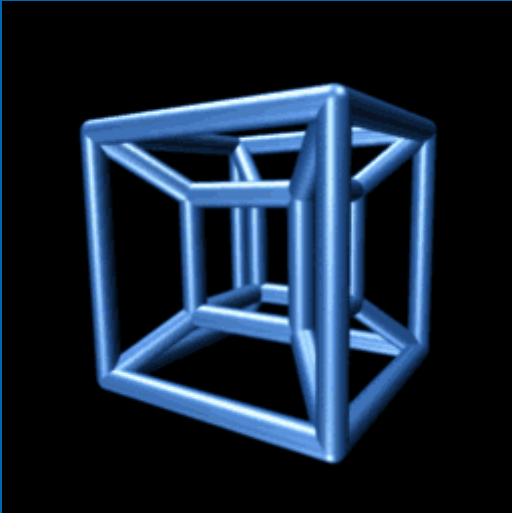


SG107.7.3.1
The 8-Cell
Polytope
(1)

Other names:

Tesseract
Octachoron
Hyper-cube
4-Cube
Tetracube

SG107.7.3.2 8-Cell (2) - Tesseract



↑ View the animation
on Wikipedia:

<http://en.wikipedia.org/wiki/Tesseract>

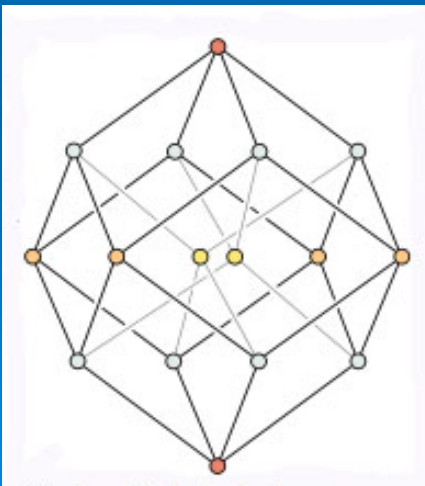
The *tesseract* is the four-dimensional analog of the cube. The tesseract is to the cube as the cube is to the square. Just as the surface of the cube consists of 6 square faces, the hypersurface of the tesseract consists of 8 cubical cells.

According to the Oxford English Dictionary, the word *tesseract* was coined and first used in 1888 by Charles Howard Hinton in his book *A New Era of Thought*, from the Greek “τέσσερεις ακτίνες” (“four rays”), referring to the four lines from each vertex to other vertices.

Since each vertex of a tesseract is adjacent to four edges, the vertex figure of the tesseract is a regular tetrahedron. The dual polytope of the tesseract is called the hexadecachoron, or 16-cell.

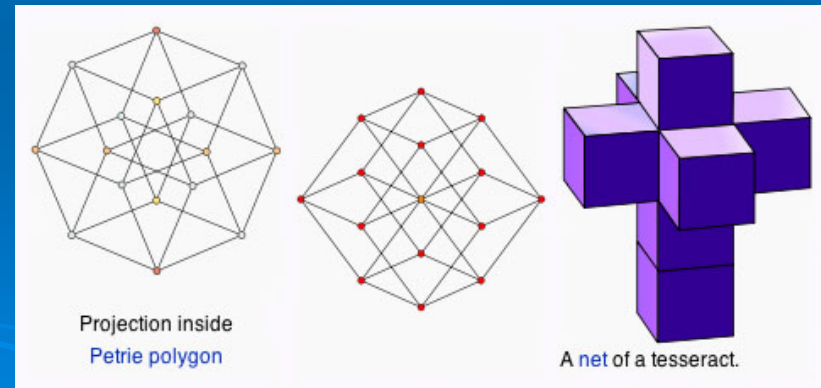
The tesseract can be unfolded into eight cubes, just as the cube can be unfolded into six squares (view animation). An unfolding of a polytope is called a net. There are 261 distinct nets of the tesseract.

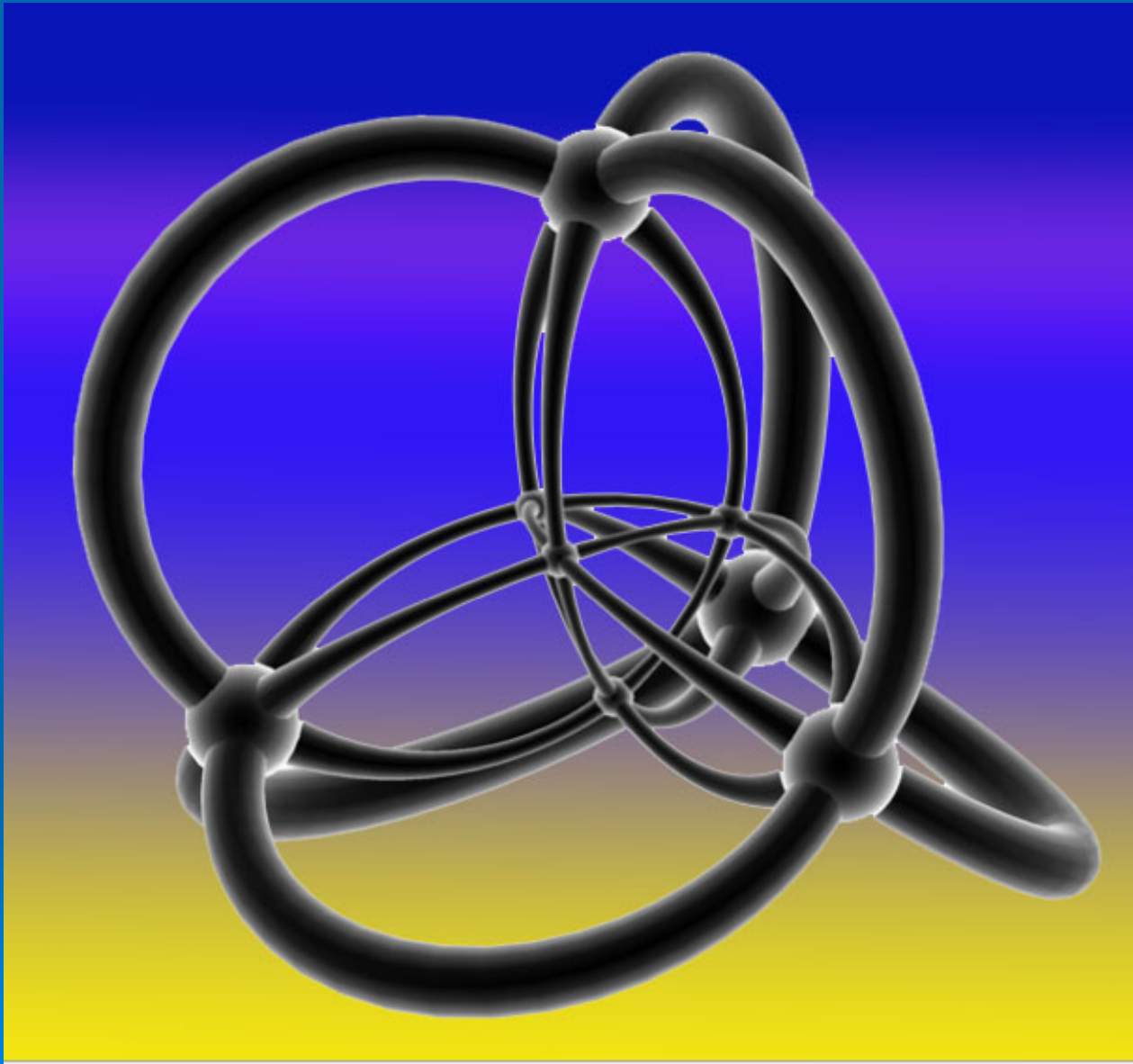
Wikipedia: Tesseract



← The rhombic dodecahedron forms the convex hull of the tesseract's vertex-first shadow.

The number of vertices in the layers of this projection is 1 4 6 4 1 - the fourth row in Pascal's triangle.



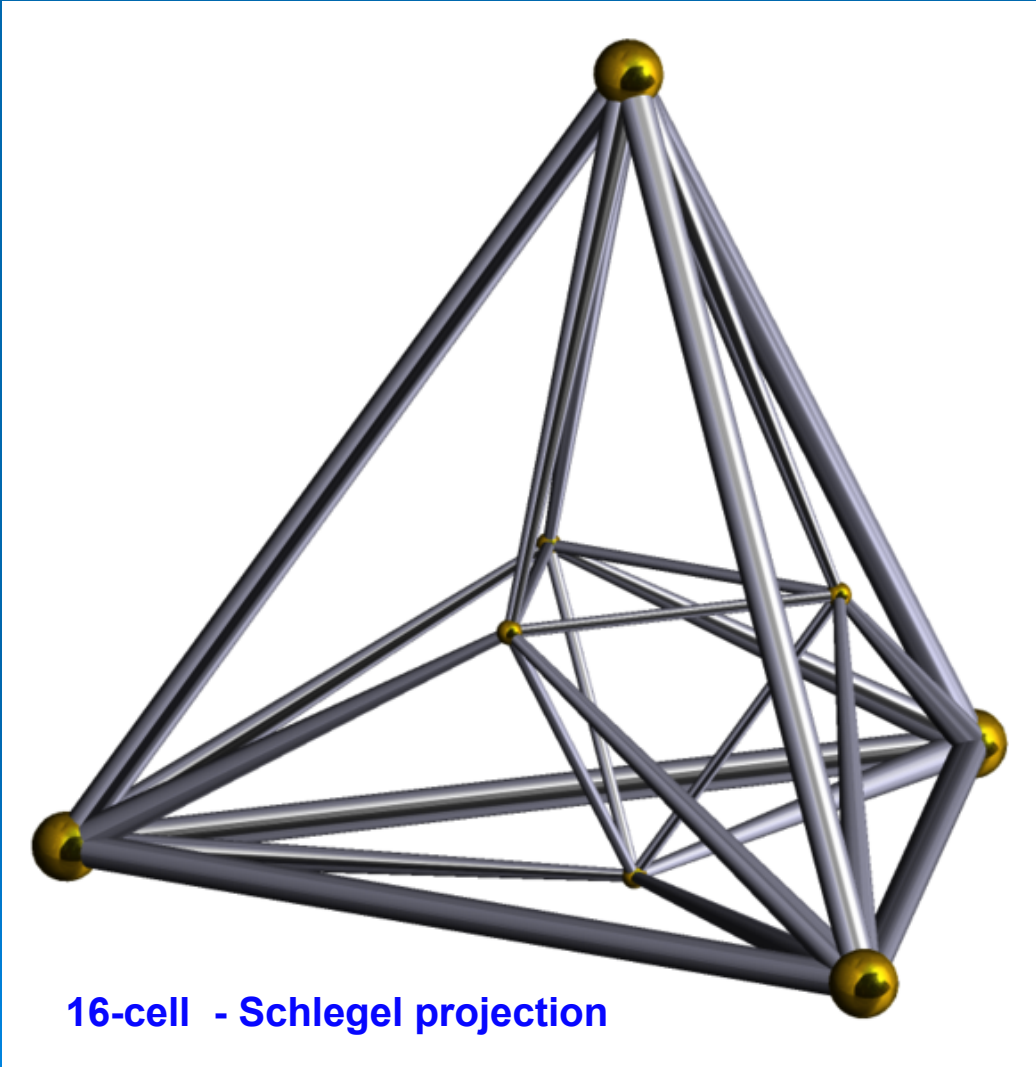


**SG107.7.4.1
The 16-Cell
Polytope
(1)**

Other names:

**Orthoplex
Hyper-Octahedron
4-Orthoplex
Hexadecachoron**

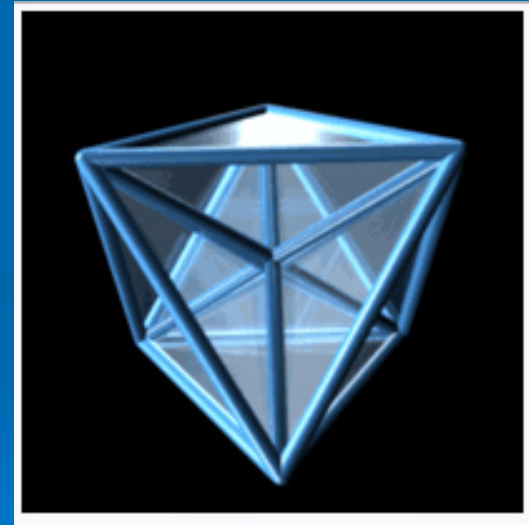
SG107.7.4.2 The 16-Cell Polytope (2)



16-cell - Schlegel projection

The 16-cell polytope or *hexadecachoron* is a member of the family of cross-polytopes, which exist in all dimensions. As such, its dual polychoron is the Tesseract (the 4-dimensional hypercube).

It is bounded by 16 cells, all of which are regular tetrahedra. It has 32 triangular faces, 24 edges, and 8 vertices. The 24 edges bound 6 squares lying in the 6 coordinate planes.



↑ View the animation
on Wikipedia:

<http://en.wikipedia.org/wiki/16-cell>

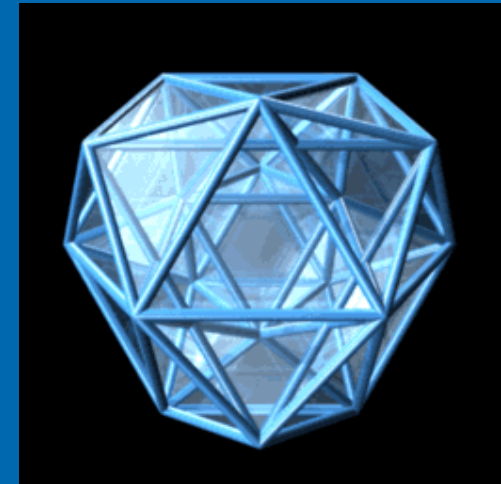
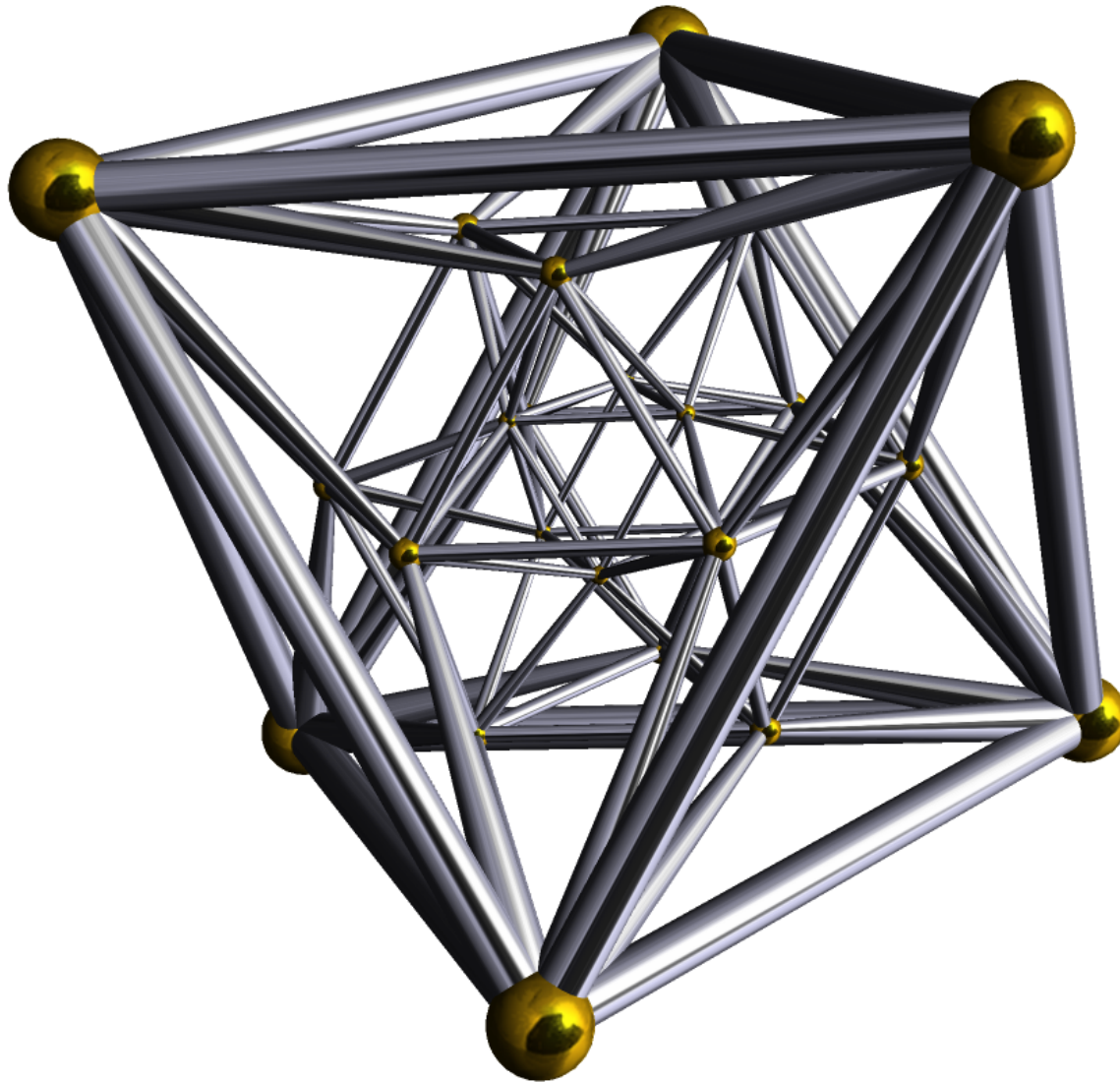


**SG107.7.5.1
The 24-Cell
Polytope
(1)**

Other names:

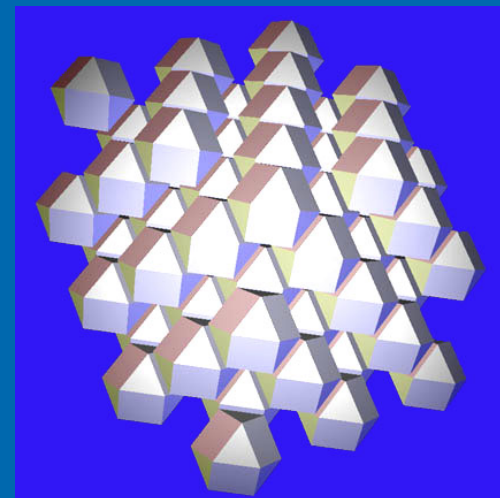
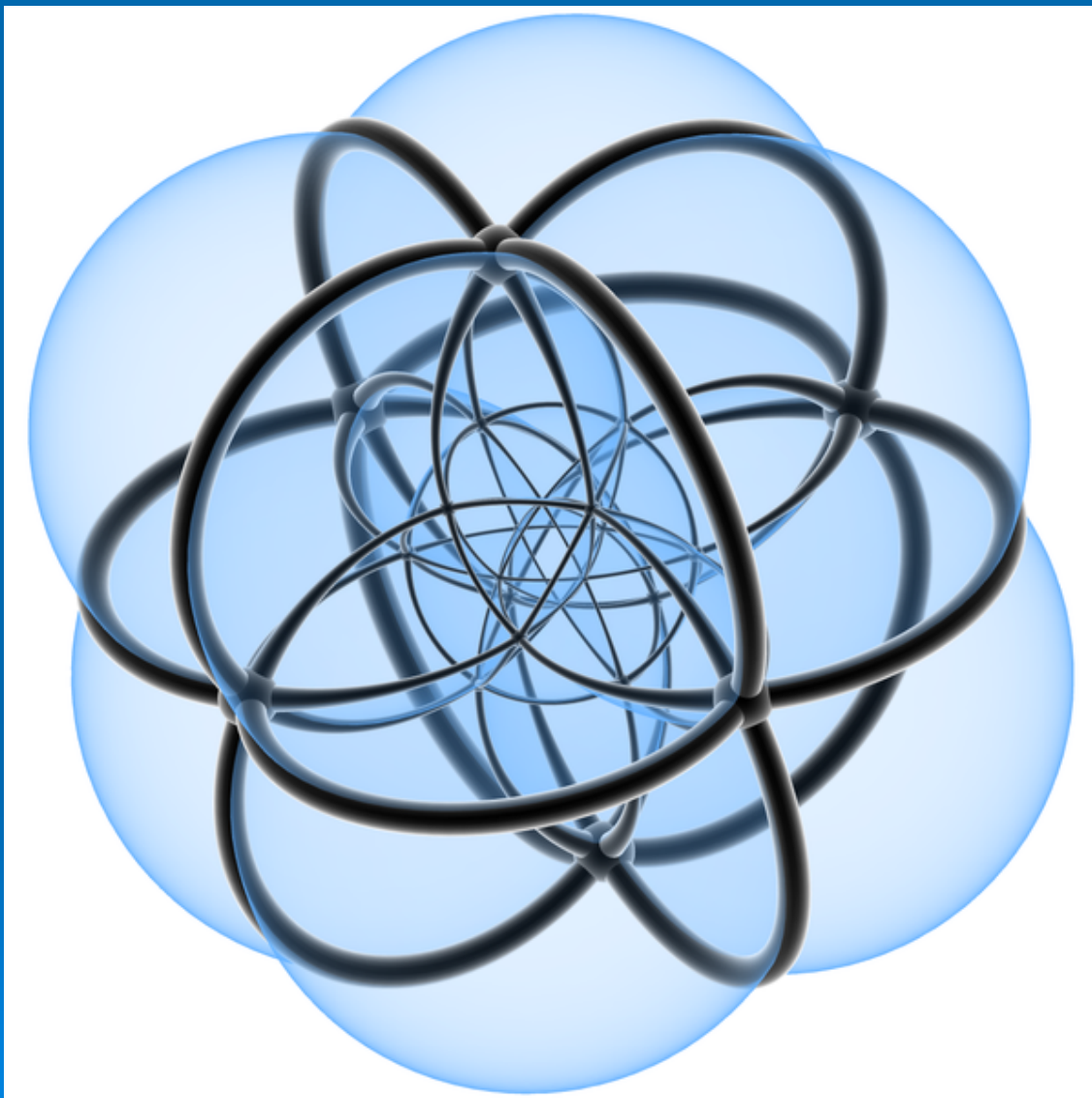
**Icositetrachoron
Octaplex
Polyoctahedron**

SG107.7.5.2
The 24-Cell
Polytope (2)



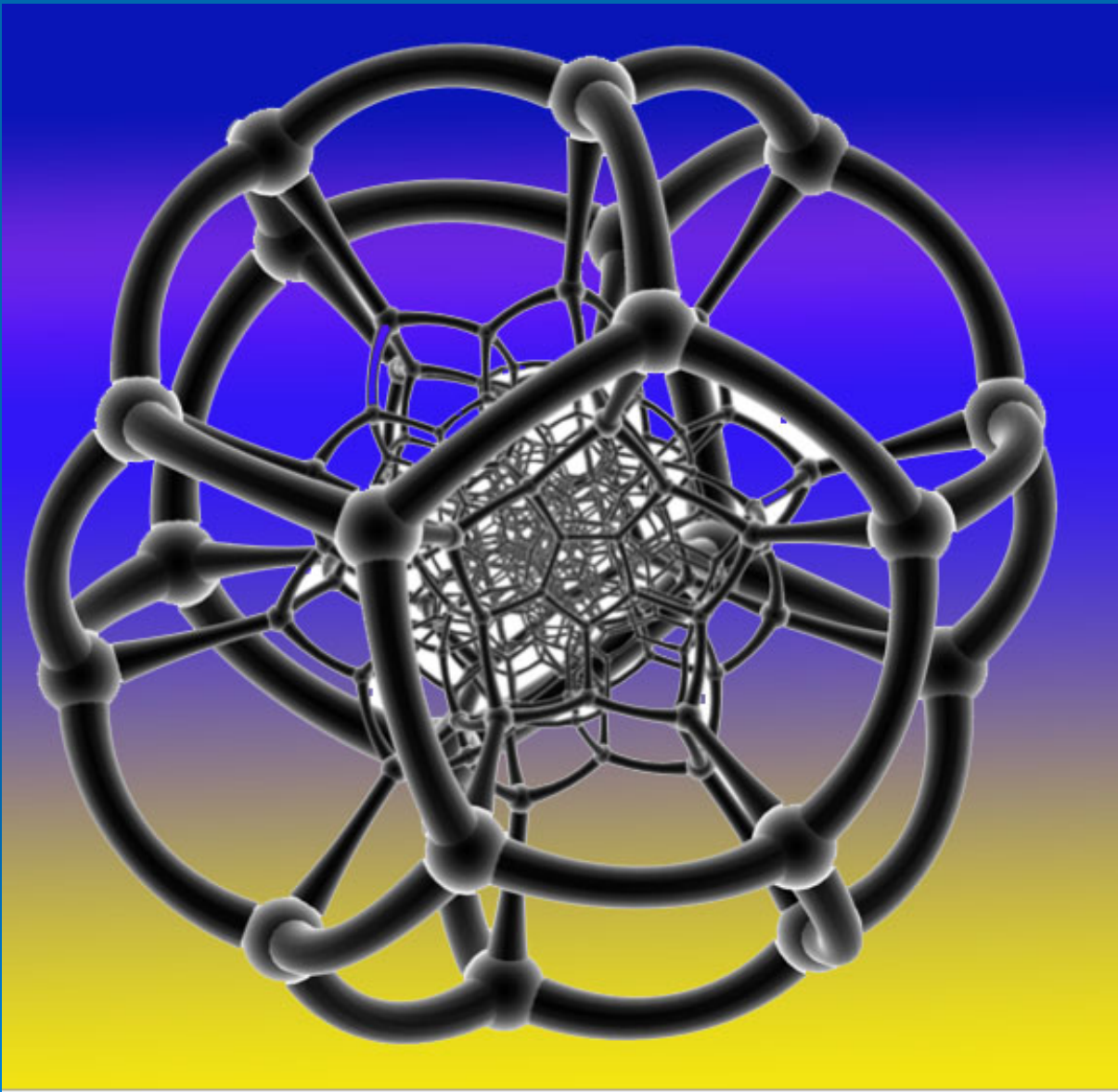
← ↑ 24-cell Schlegel
Projections

SG107.7.5.3 The 24-Cell Polytope (3)



↑ 24-cell Cross section

← Stereo Projection of the 24-cell polytope.

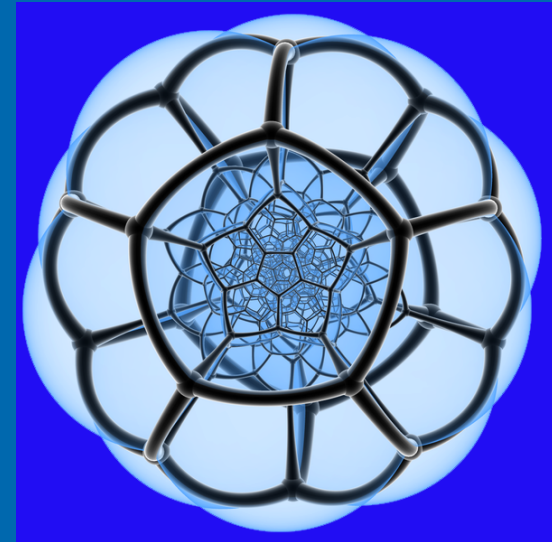
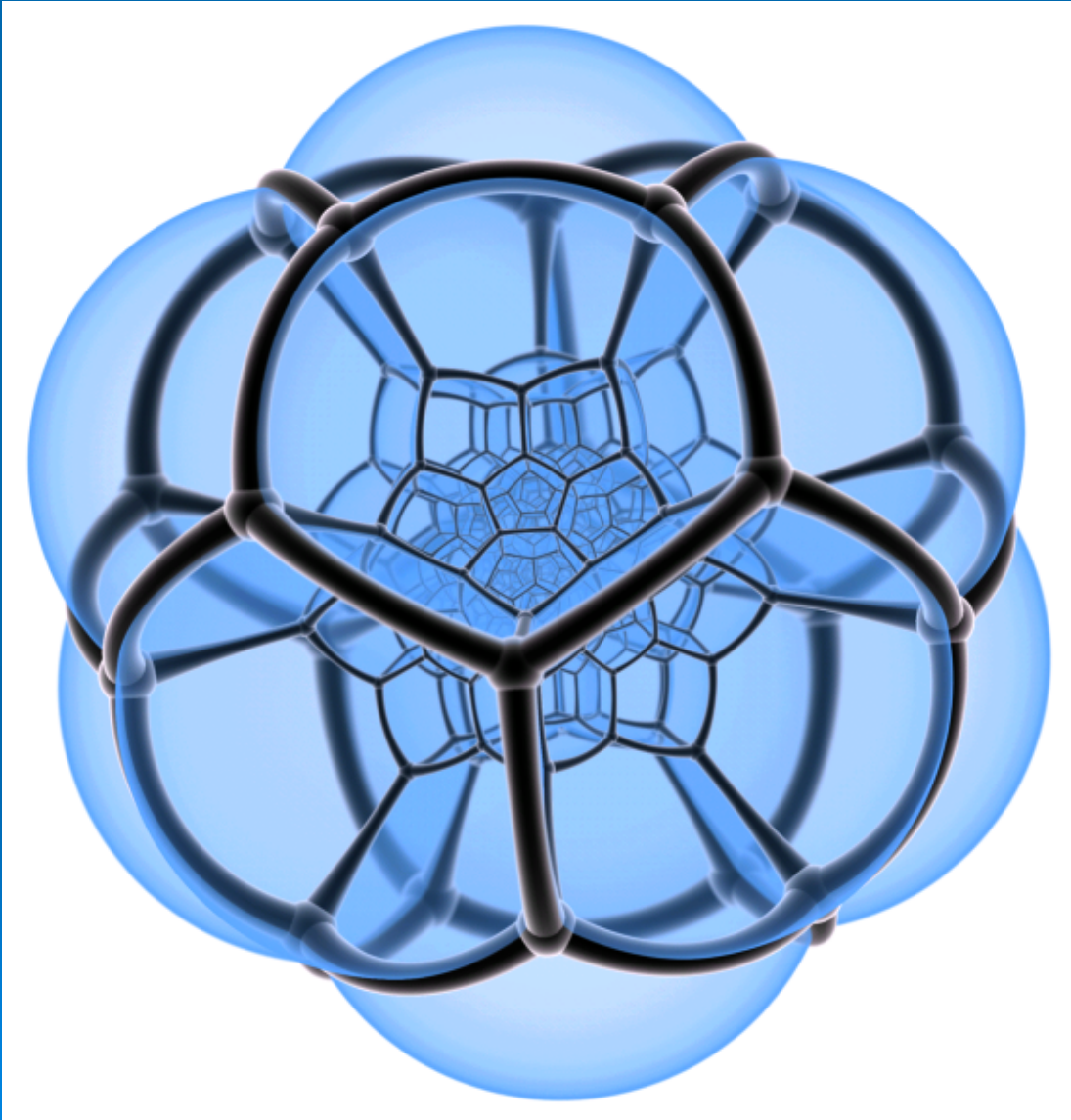


**SG107.7.6.1
The 120-Cell
Polytope (1)**

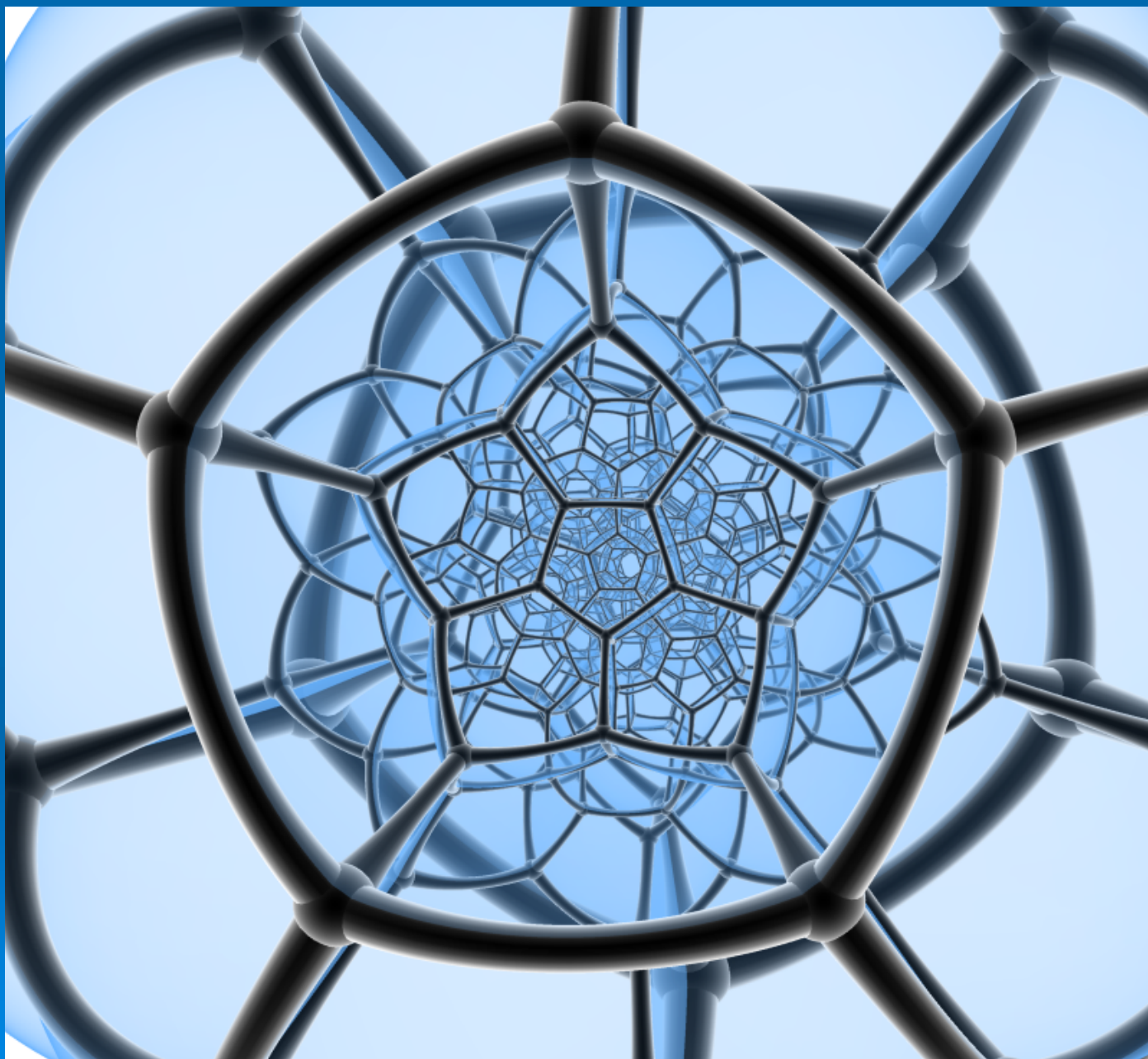
Other names:

**Hecatonicocachoron
Dodecaplex
Hyper-Dodecahedron
Polydodecahedron**

SG107.7.6.2 The 120-Cell Polytope (2)



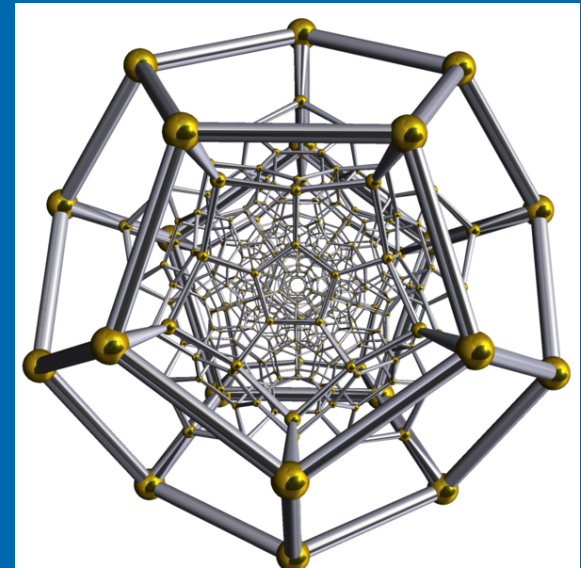
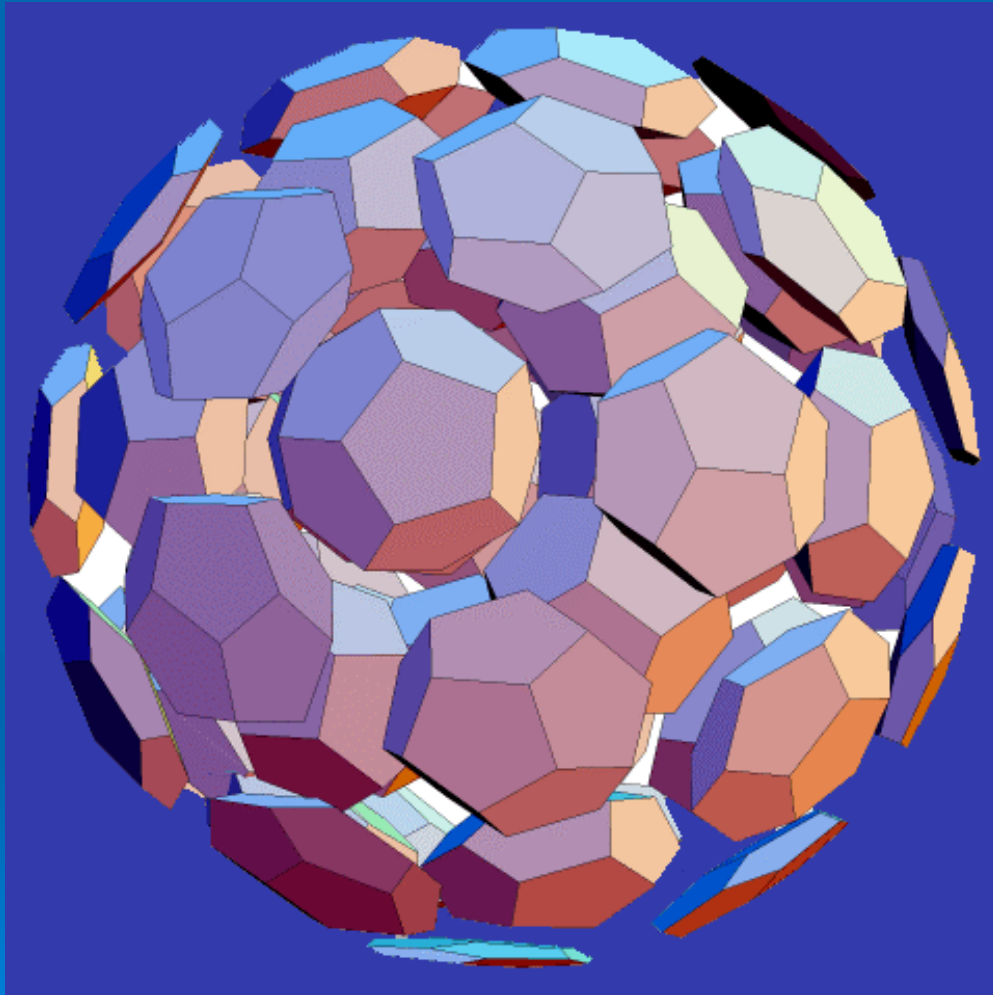
↔ Stereographic projections of the 120-cell polytope onto a 4D sphere.



SG107.7.6.3
The 120-
Cell
Polytope
(3)

← Zoom on
the stereo
projection,
showing a
tunneling
fractal
effect
of
penta-
symmetry

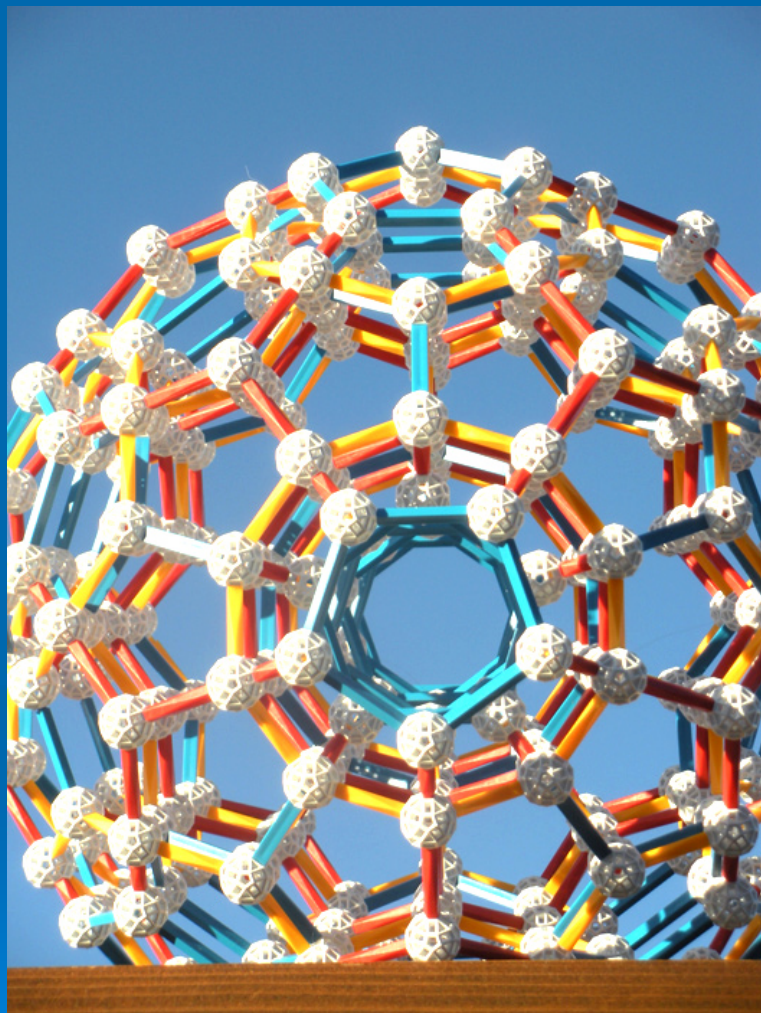
SG107.7.6.4 The 120-Cell Polytope (4)



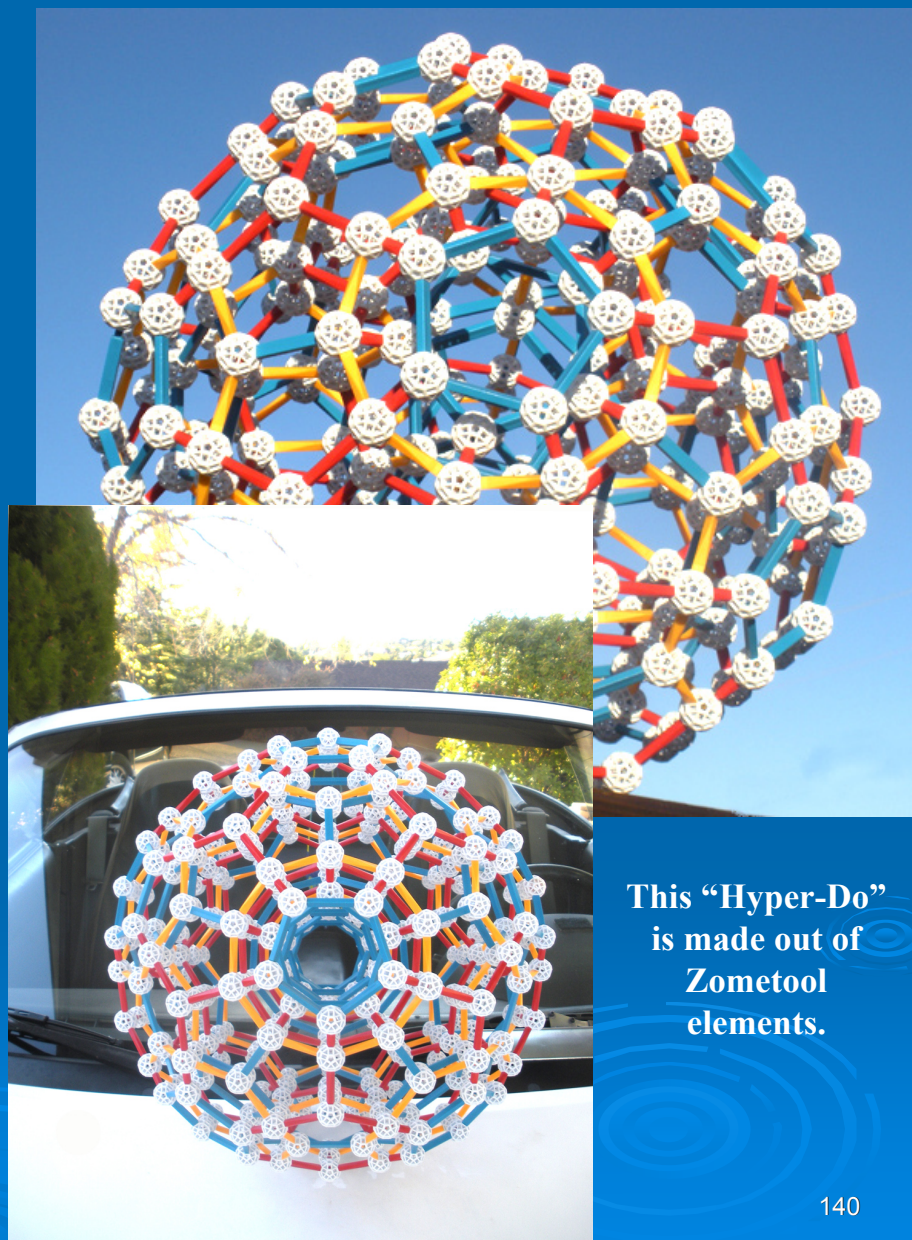
↑ Schlegel diagram of the 120-cell
It looks like the top view of
some “Hyper DNA”

← “Exploded” view of the 120-cell
showing some of its 120 dodecahedra

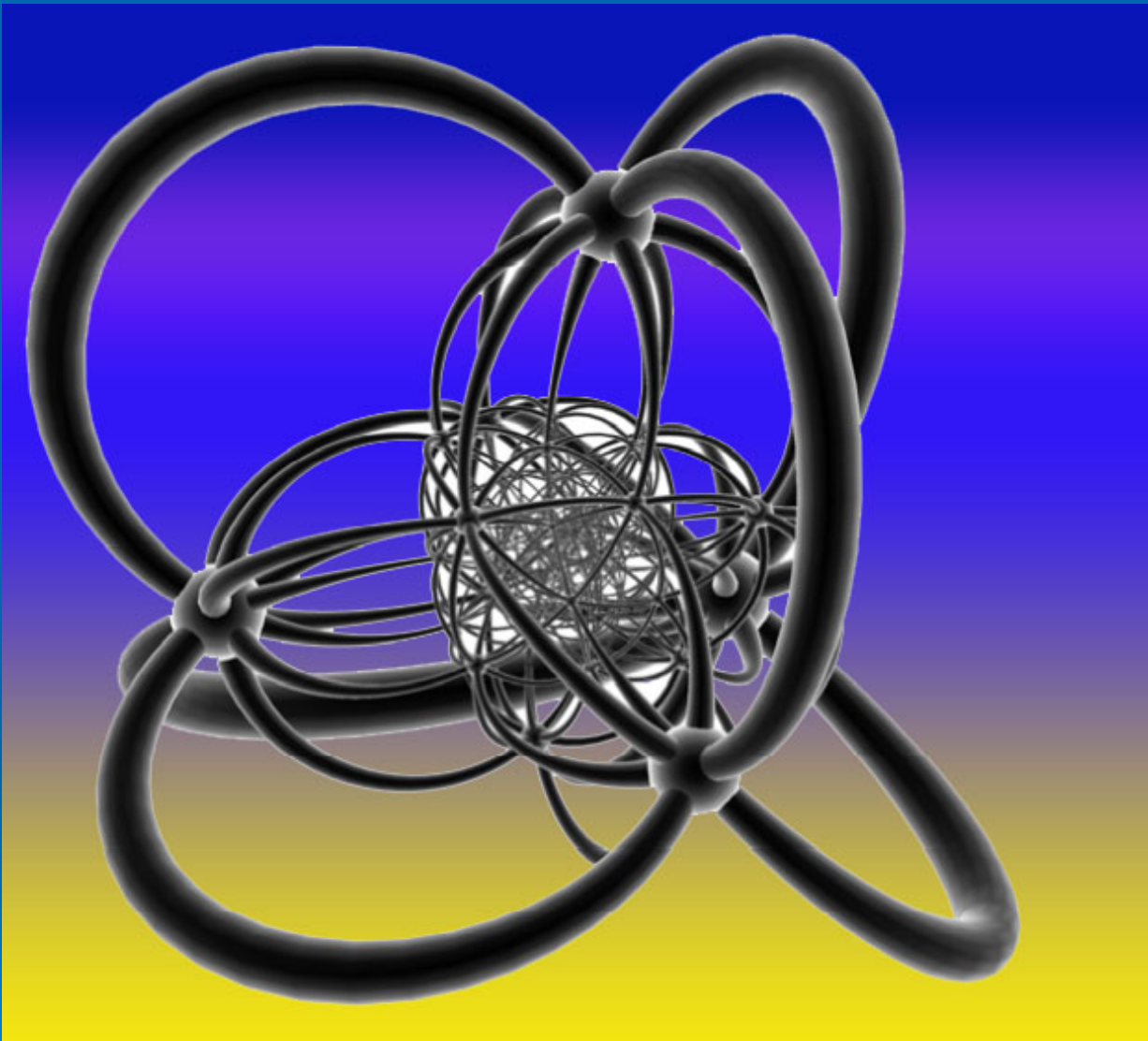
SG107.7.6.5 The 120-Cell Polytope (5)



Views through ↕→
the “Tunnel of Love”.



This “Hyper-Do”
is made out of
Zometool
elements.

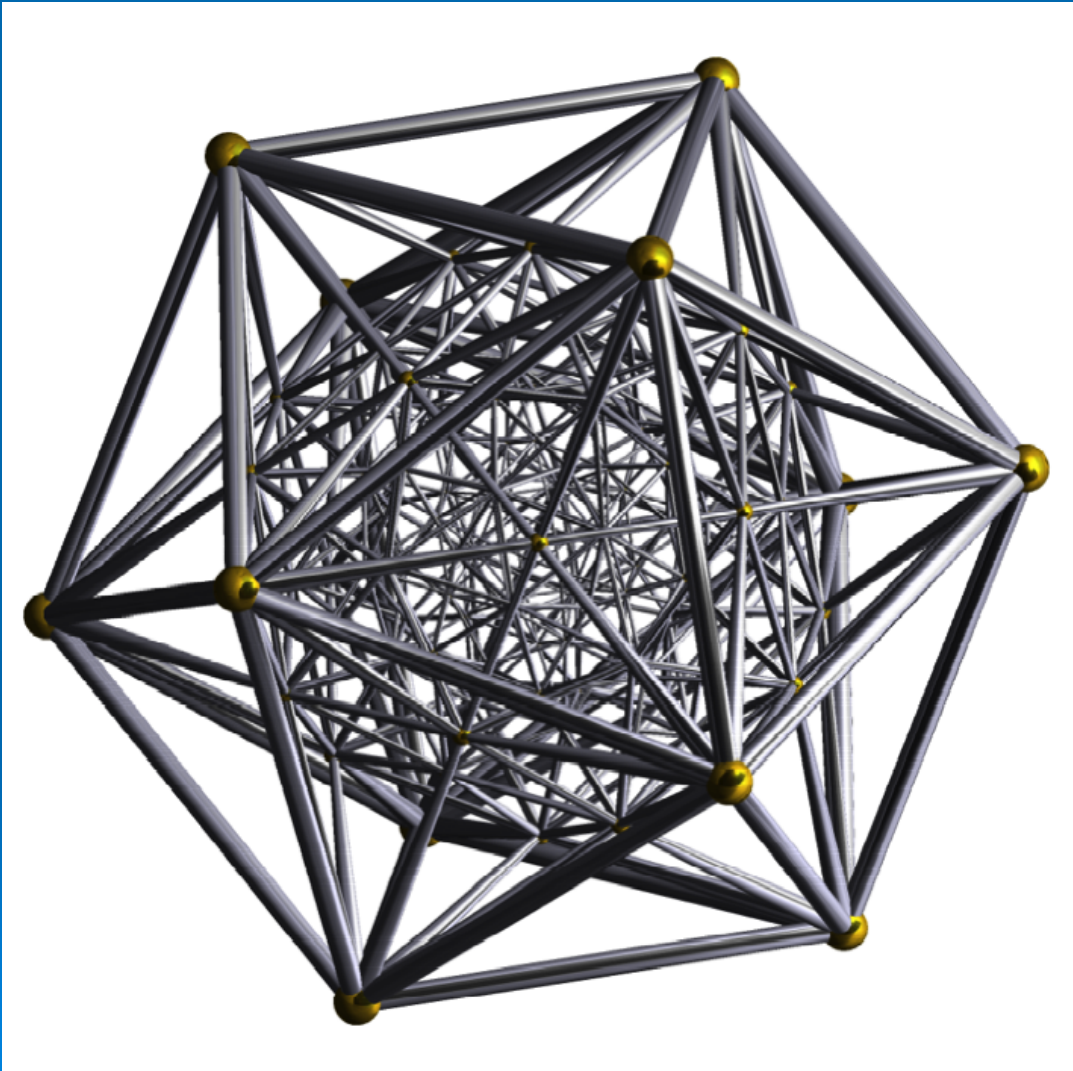


SG107.7.7.1
The 600-Cell
Polytope

Other names:

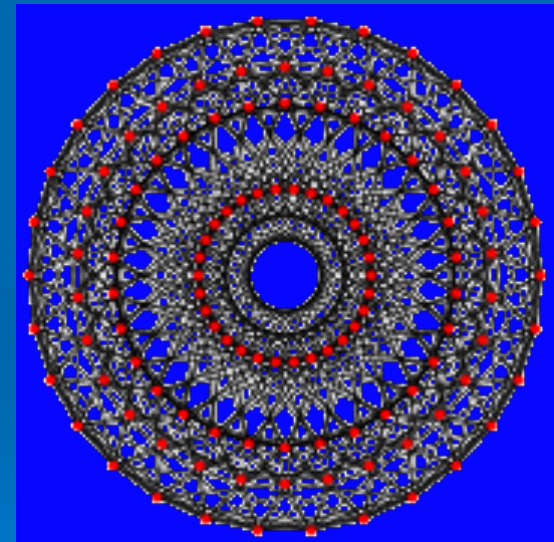
Tetraplex
Hyper-icosahedron
Polytetra

SG107.7.7.2 The 600-Cell Polytope (2)



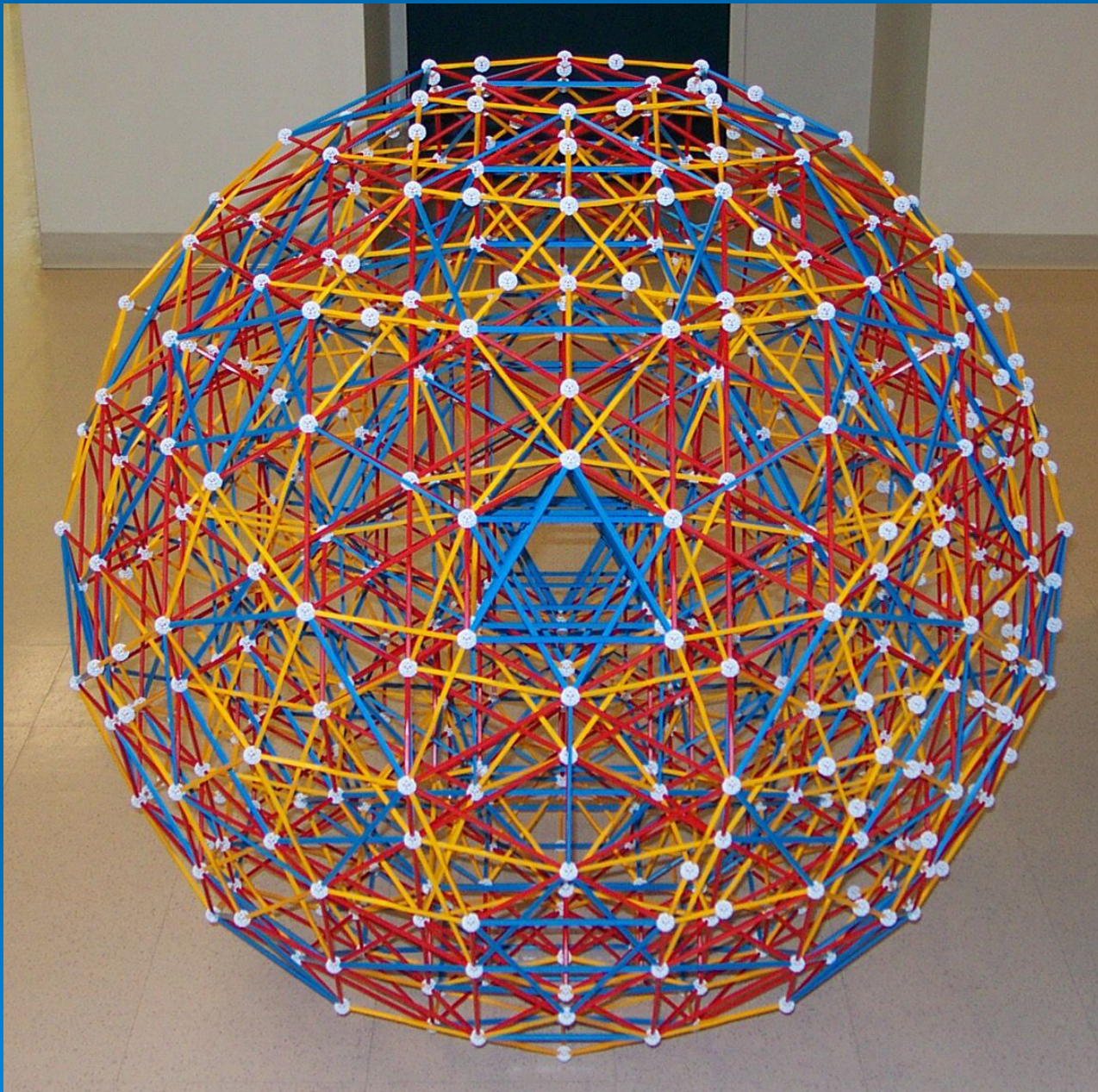
The 600-cell boundary is composed of 600 tetrahedral cells with 20 meeting at each vertex. Together they form 1200 triangular faces, 720 edges, and 120 vertices.

It is regarded as the 4-dimensional analog of the icosahedron, since it has five tetrahedra meeting at every edge, just as the icosahedron has five triangles meeting at every vertex.



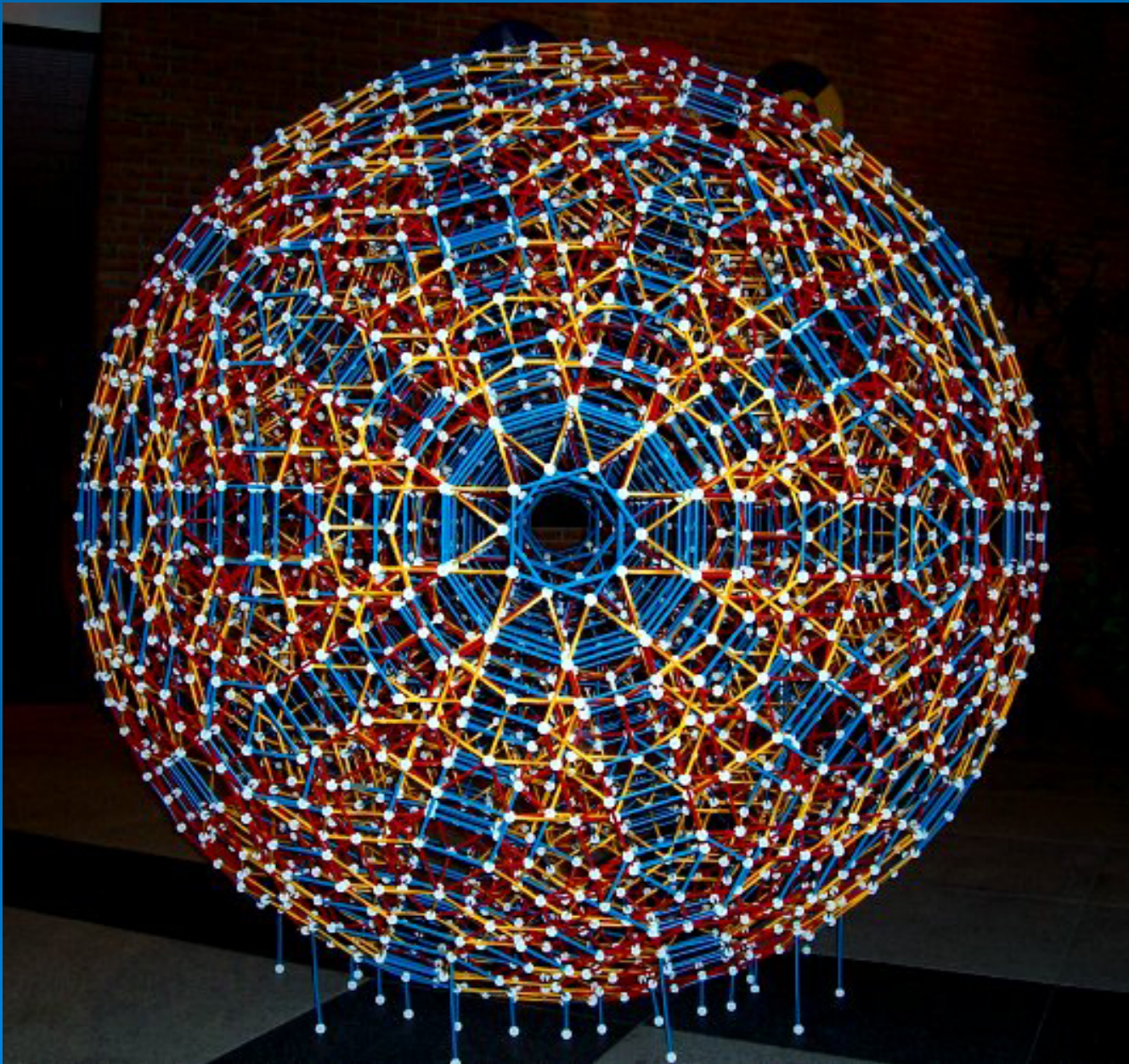
↑ 600-cell graph

← 600-cell Schlegel diagram



SG107.7.7.3
The 600-Cell
Polytope (3)

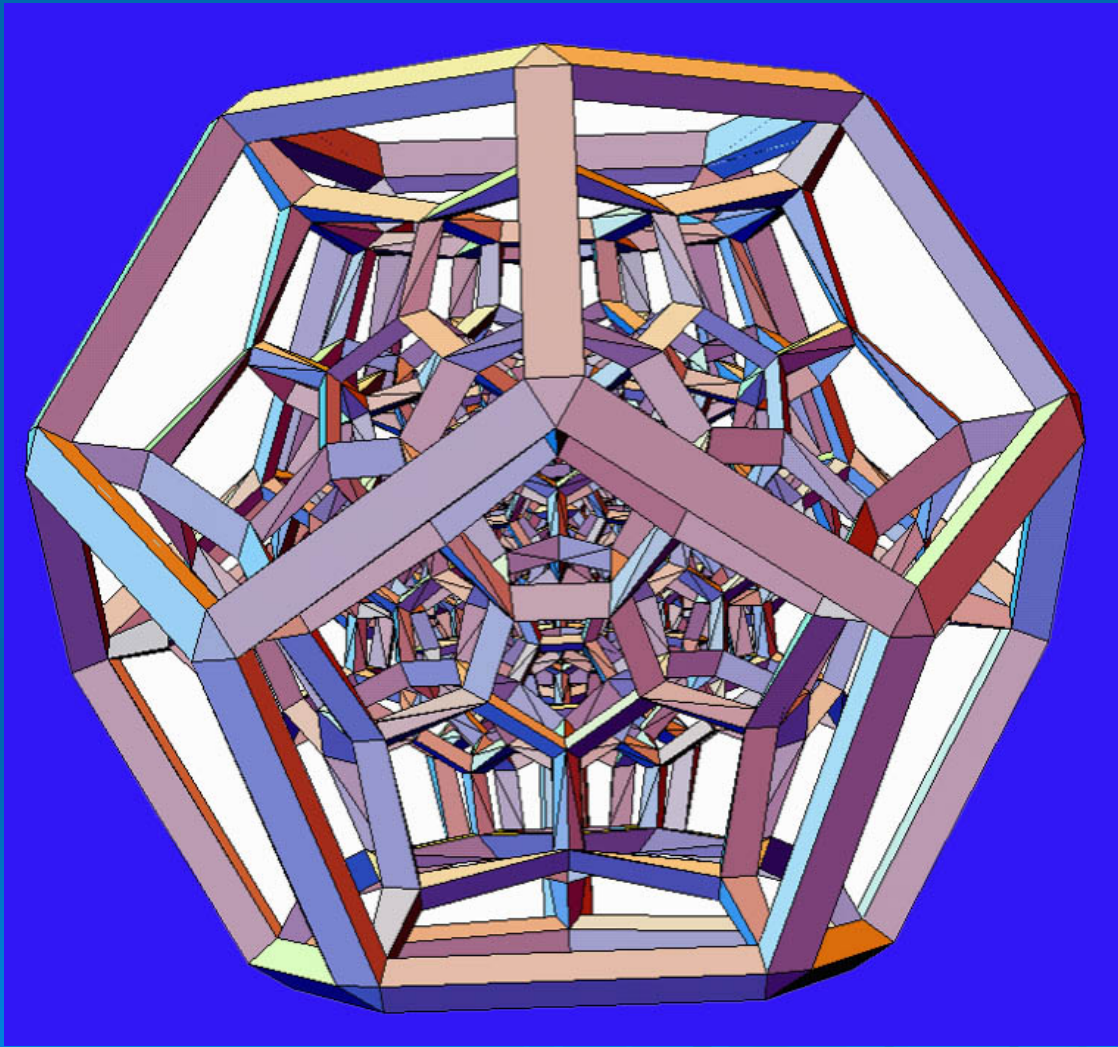
← A Zome-Tool
construction
of the 600-cell



**SG107.7.7.4
The 600-Cell
Polytope
(4)**

**← Runci-truncated
600-cell
built with
Zome-Tools**

SG107.7.8.1 Hyper-Platonics in Art (1)



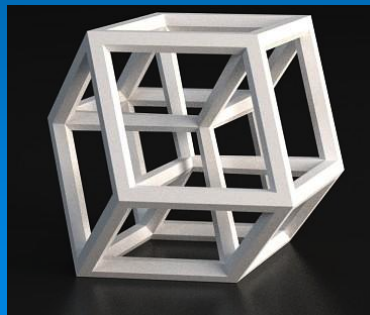
← ↑ 3 renditions of the 120-cell by George Hart

SG107.7.8.2 Hyper-Platonics in Art (2)

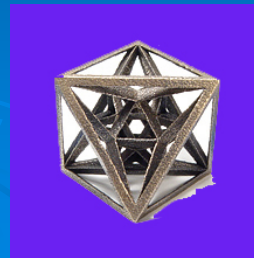
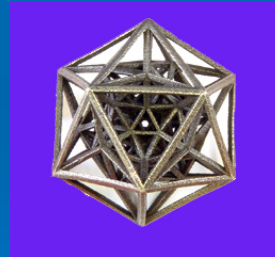
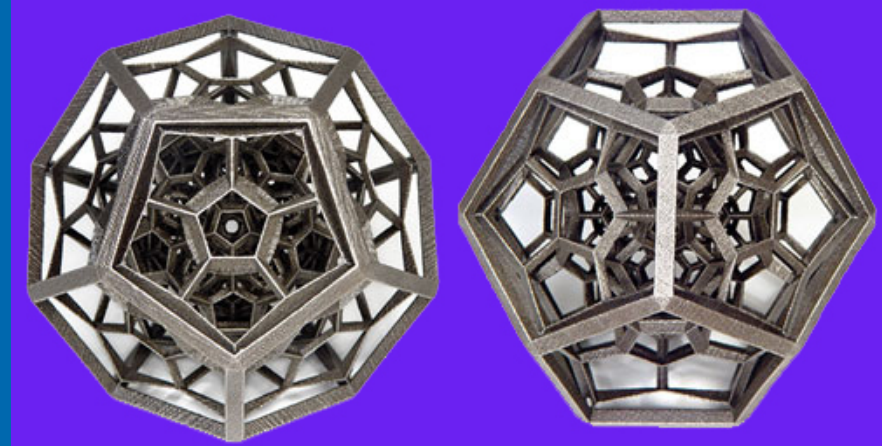


↑ 120-cell
Laser etched on glass

Tesseract →



All sculptures by Bathsheba, an artist exploring math & science in sculpture. She is using CAD 3D modeling and then “direct-metal printing”, a process by which the design is laid down, one layer (.005 inch) at a time, in stainless-steel powder held in place by a laser-activated binder.



www.bathsheba.com



SG107.7.9.1 Hyper-Platonics in Second Life (2)

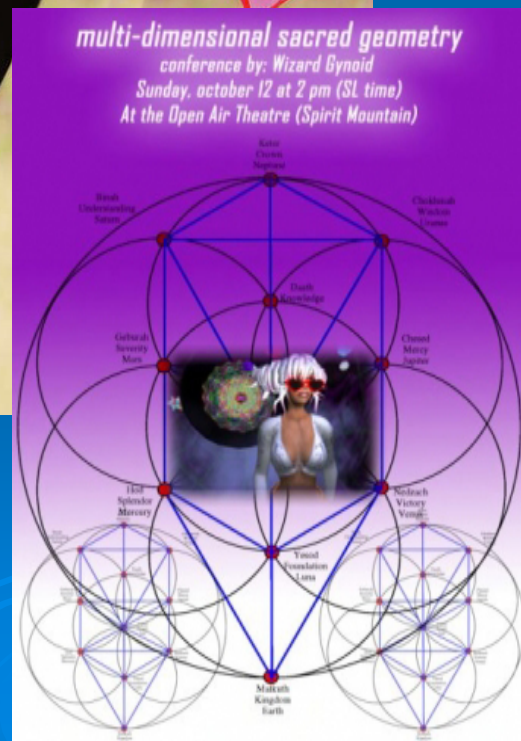


Wizzy Gynoid is a graphic artist taking Sacred Geometry to the cyber world of Second Life.

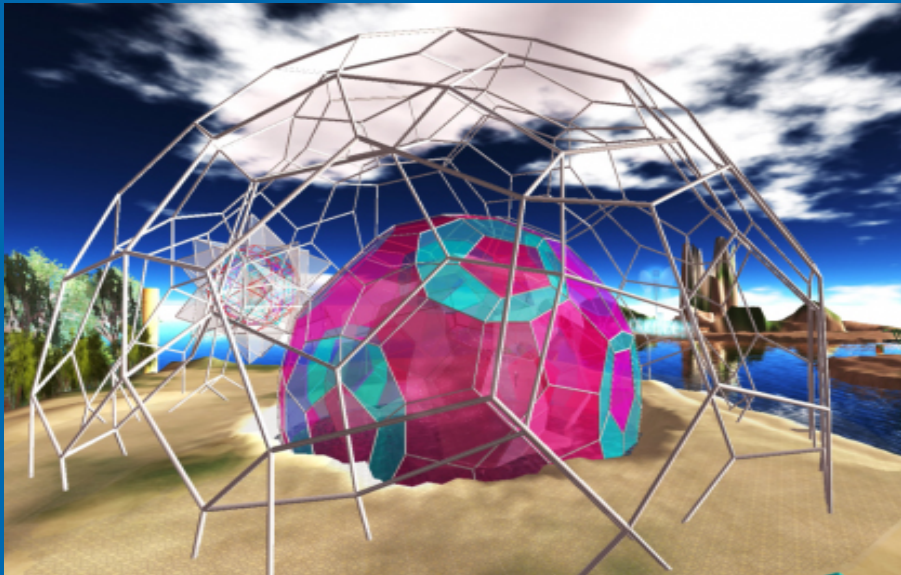
Second Life is a free 3D virtual world imagined and created by its Resident population of millions of real people from around the world.

Each person is represented by an avatar that incarnates their chosen digital persona.

Second Life avatars can walk, "teleport" or even fly to thousands of exciting 3D locations. You can also own land, shop or start a business using the "inworld" currency of \$ Linden. You can even use voice and text chat to communicate with other real people from around the world.



[//.secondlife.com](http://secondlife.com)

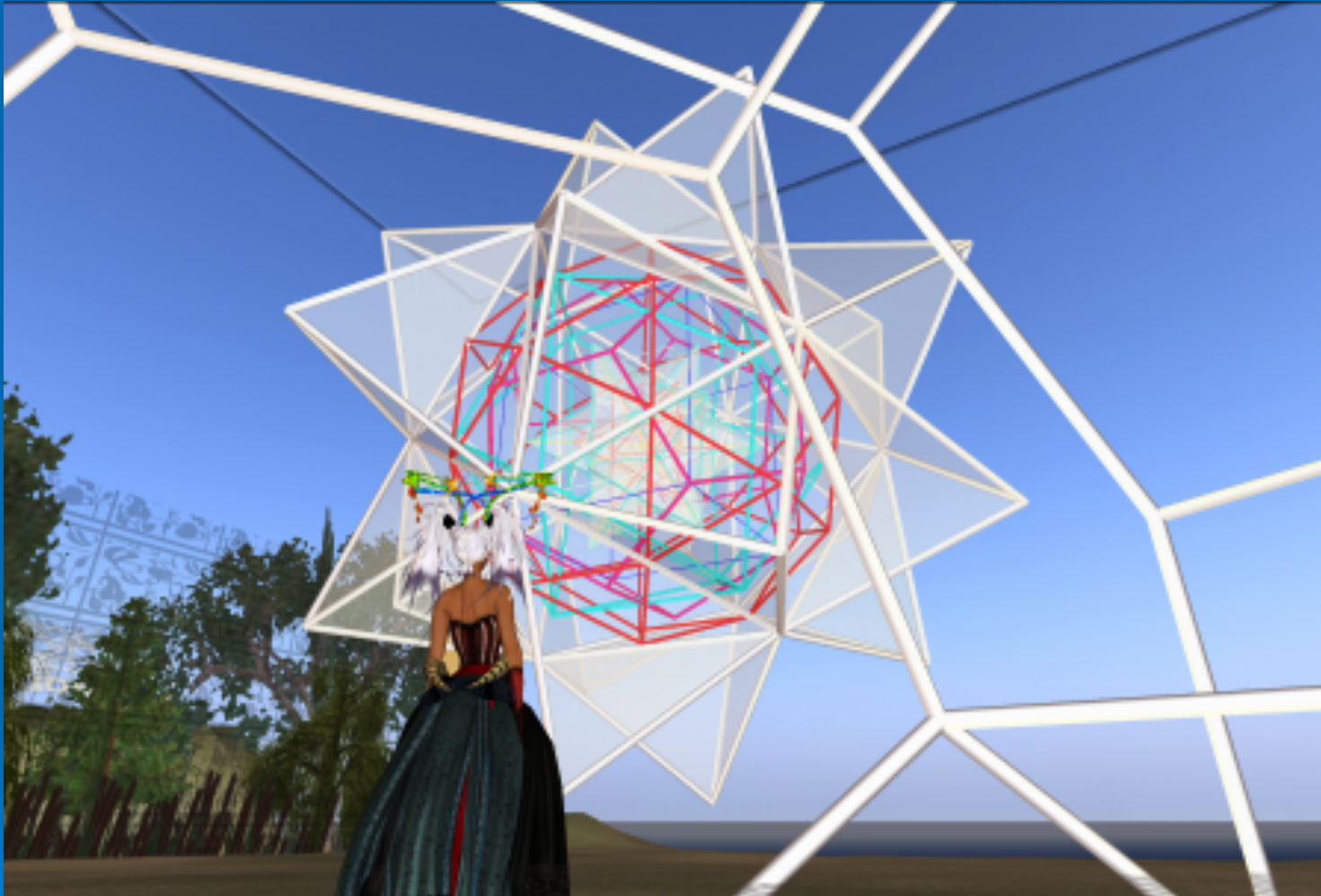


SG107.7.9.2 Hyper-Platonics in Second Life (2)

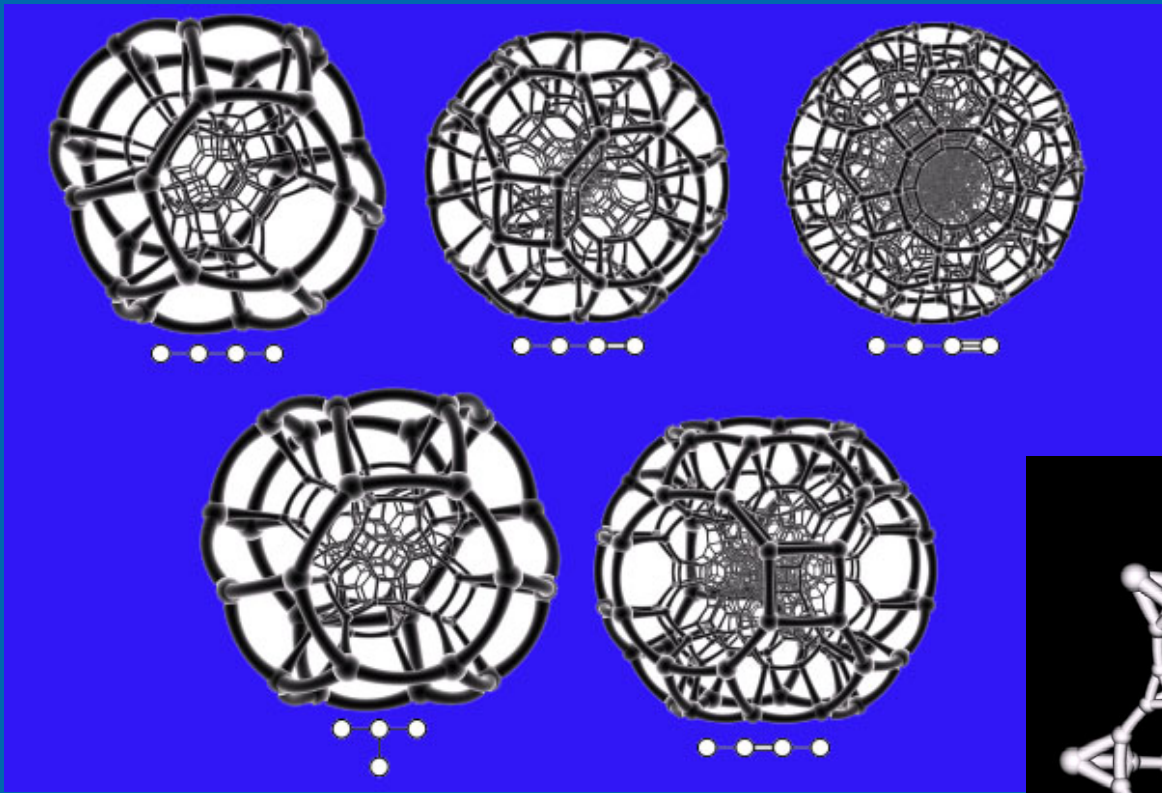
← ↓ Virtual renditions of the 120-cell



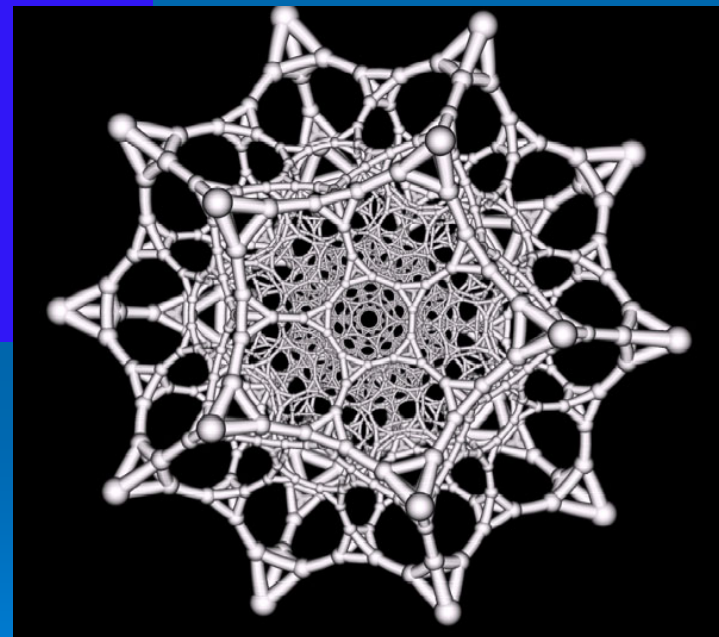
SG107.7.9.1 Hyper-Platonics in Second Life (3)



The “*Super-Duper Star*” in one of Wizzy’s worlds.
The Golden Ratio Φ is encoded throughout the entire structure.



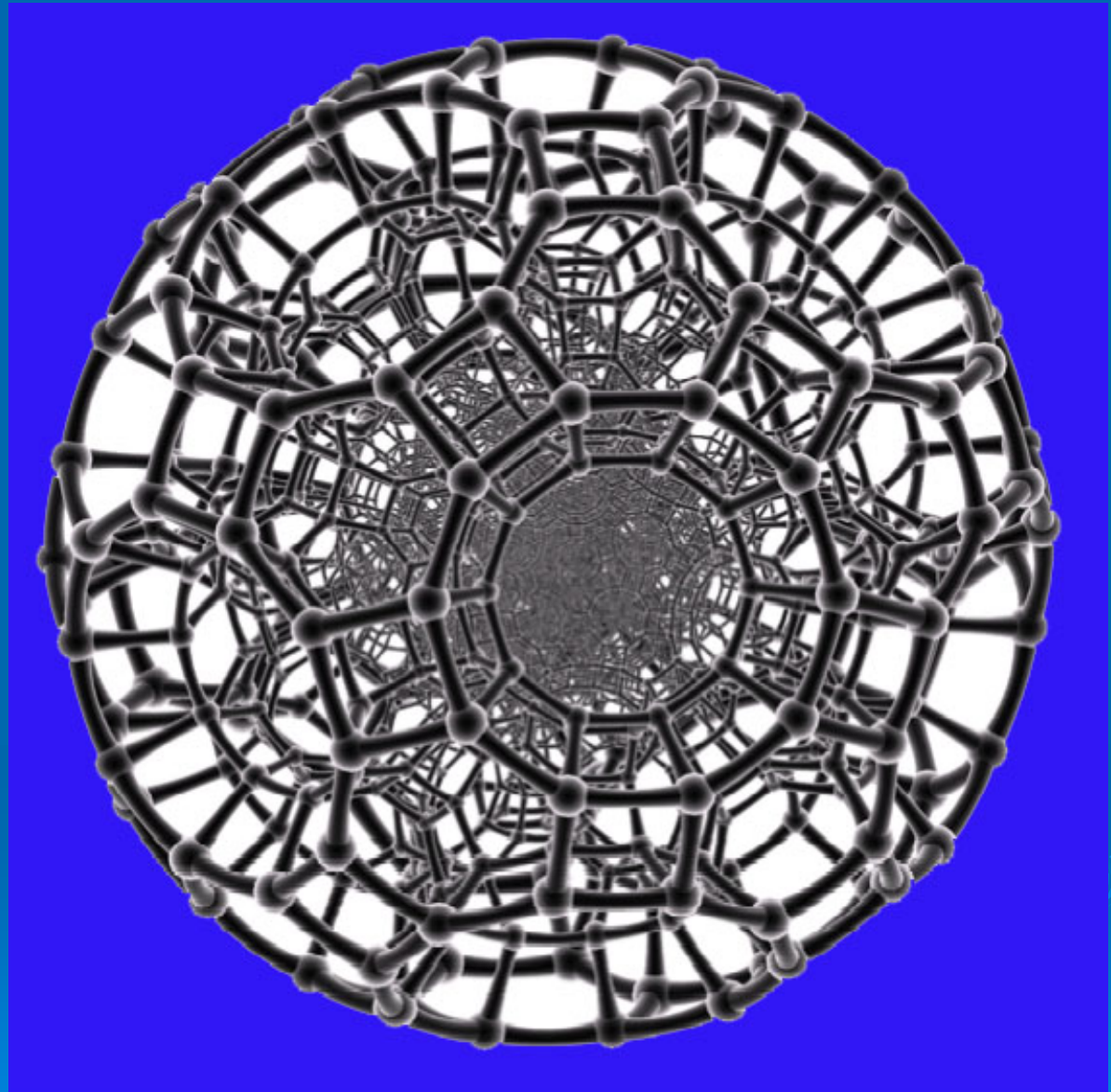
SG107.7.10 Other
Hi-D Polytopes (1)



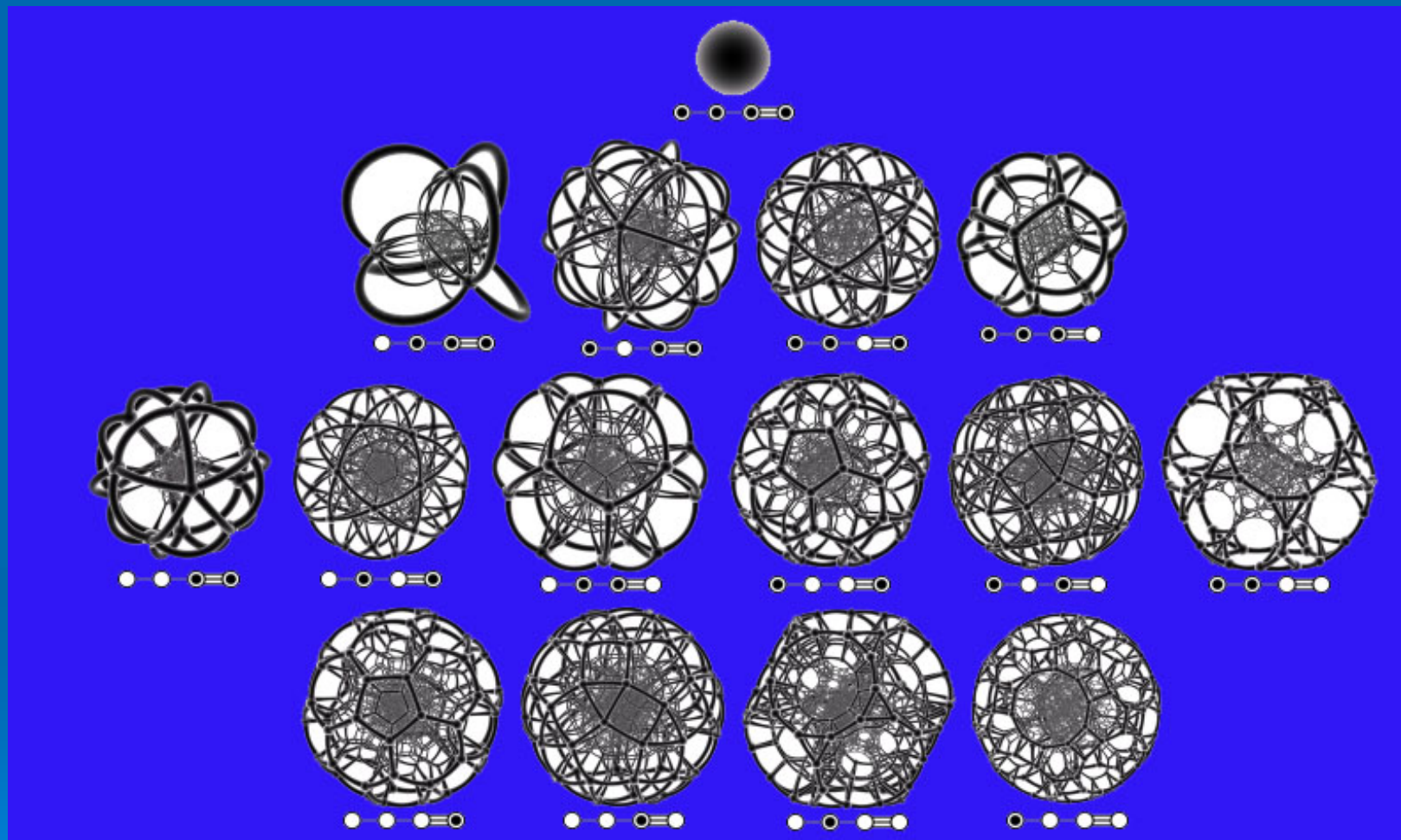
**SG107.7.10.2 Other
Hi-D Polytopes (2)**

A member →
of a finite
Coxeter Group
on 4 generators.

www.math.cmu.edu

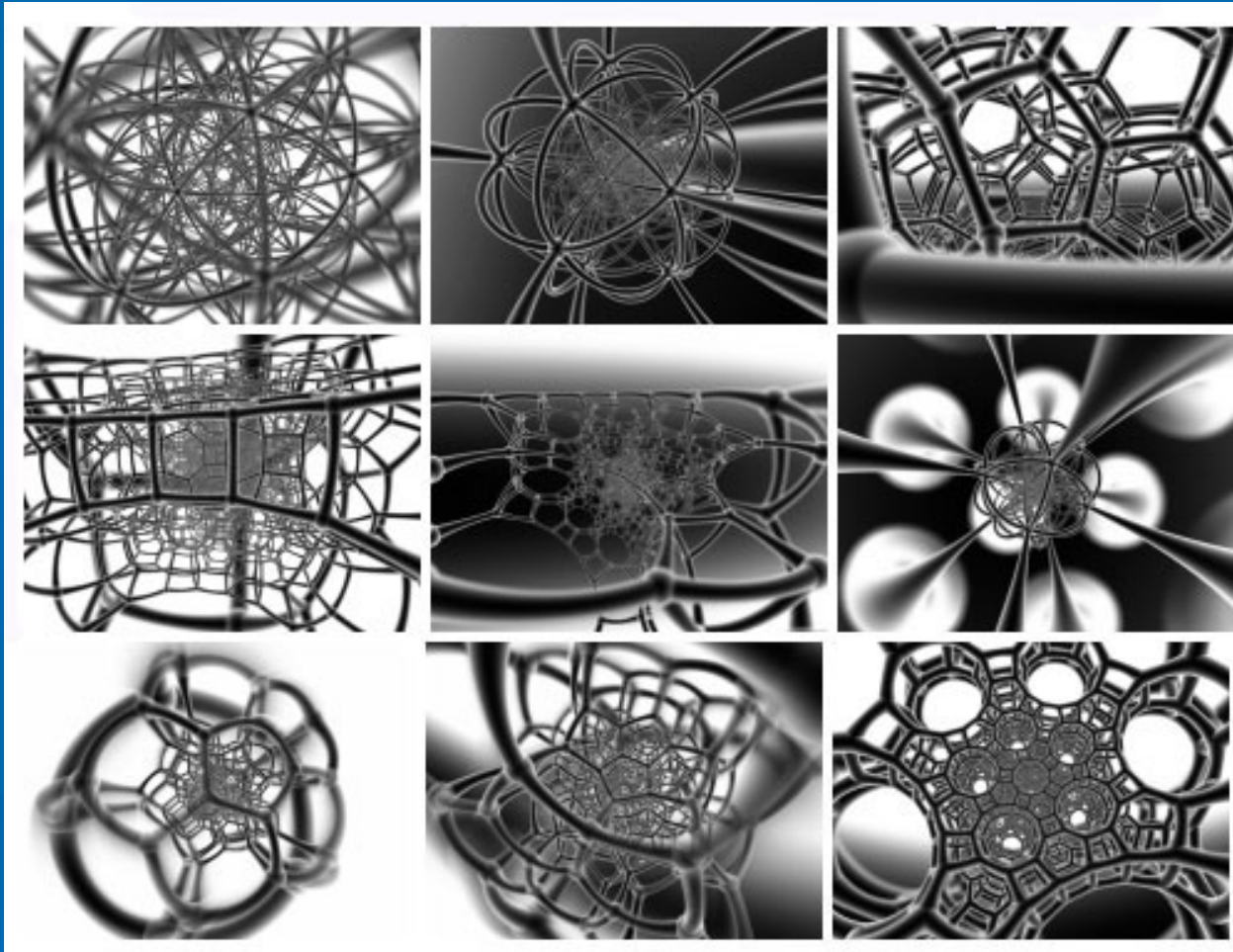


SG107.7.10.3 Other Hi-D Polytopes (3)



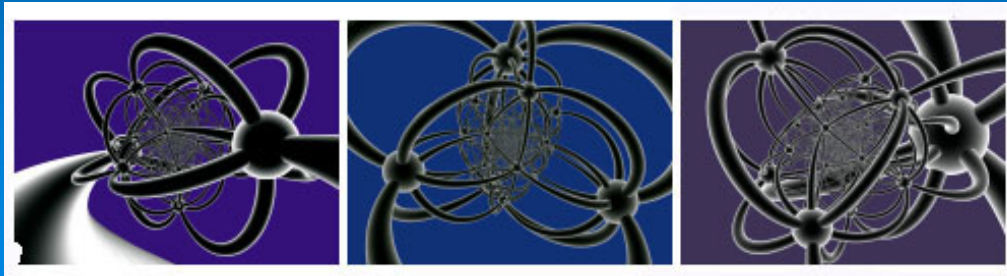
The Lattice of quotients of H_4 (the $\{3,3,4,5\}$ family). www.math.cmu.edu

SG107.7.10.4
Other Hi-D
Polytopes (4)



Wide-aperture zooms.

(The Catalog of Polytopes
www.math.cmu.edu)



SG107.7.11.1 The 8D “E8” (1)

A team of mathematicians have mapped out the inner workings of one of the most complicated structures ever studied: the object known as the *Exceptional Lie group E8*, first discovered in 1887.

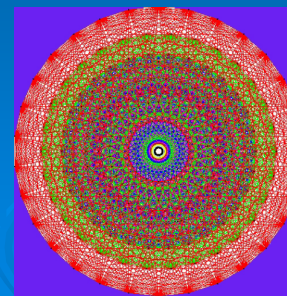
This achievement is significant both as an advance in basic knowledge and because of the many connections between E8 and other areas, including geometry, string theory and quantum gravity. The magnitude of the calculation is staggering: the answer, if written out in tiny print, would cover the area of Manhattan and is larger than the calculation of the human genome.

“At the most basic level, the E8 calculation is an investigation of symmetry. Mathematicians invented the Lie groups to capture the essence of symmetry: underlying any symmetrical object, such as a sphere, is a Lie group.

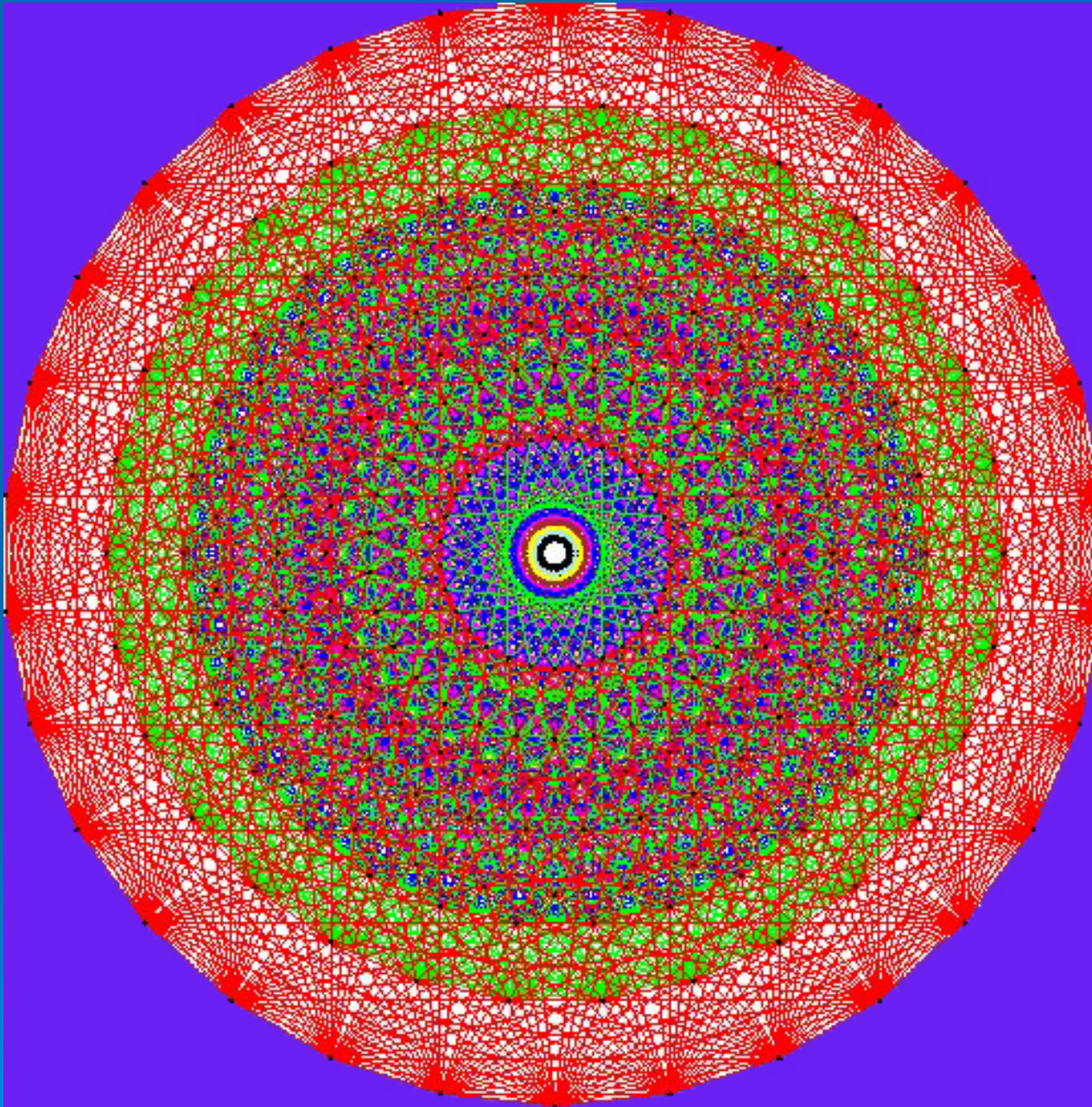
Lie groups come in families. The classical groups $A_1, A_2, A_3, \dots B_1, B_2, B_3, \dots C_1, C_2, C_3, \dots$ and D_1, D_2, D_3, \dots rise like gentle rolling hills towards the horizon. Jutting out of this mathematical landscape are the jagged peaks of the exceptional groups G_2, F_4, E_6, E_7 and, towering above them all, E_8 . E_8 is an extraordinarily complicated group: it is the symmetries of a particular 57-dimensional object, and E_8 itself is 240-dimensional!”

www.aimath.org/E8/

What is of interest to us is that E8 belongs to the symmetry group of the icosahedron and dodecahedron, the two higher Platonic solids.

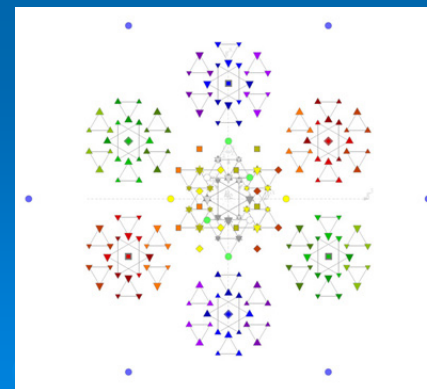
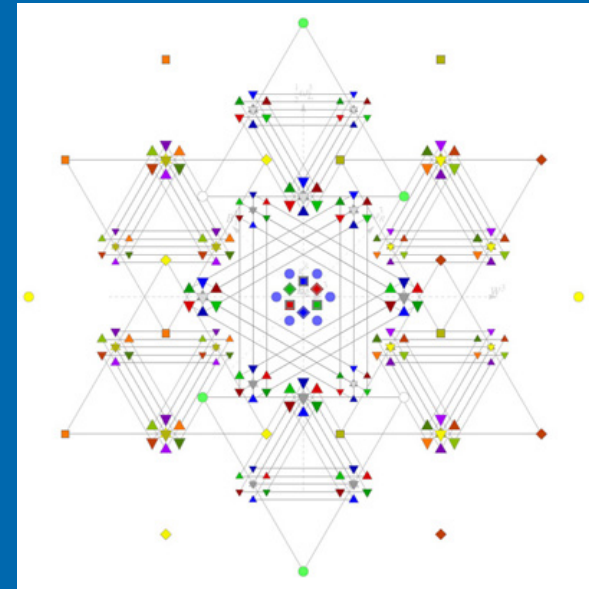
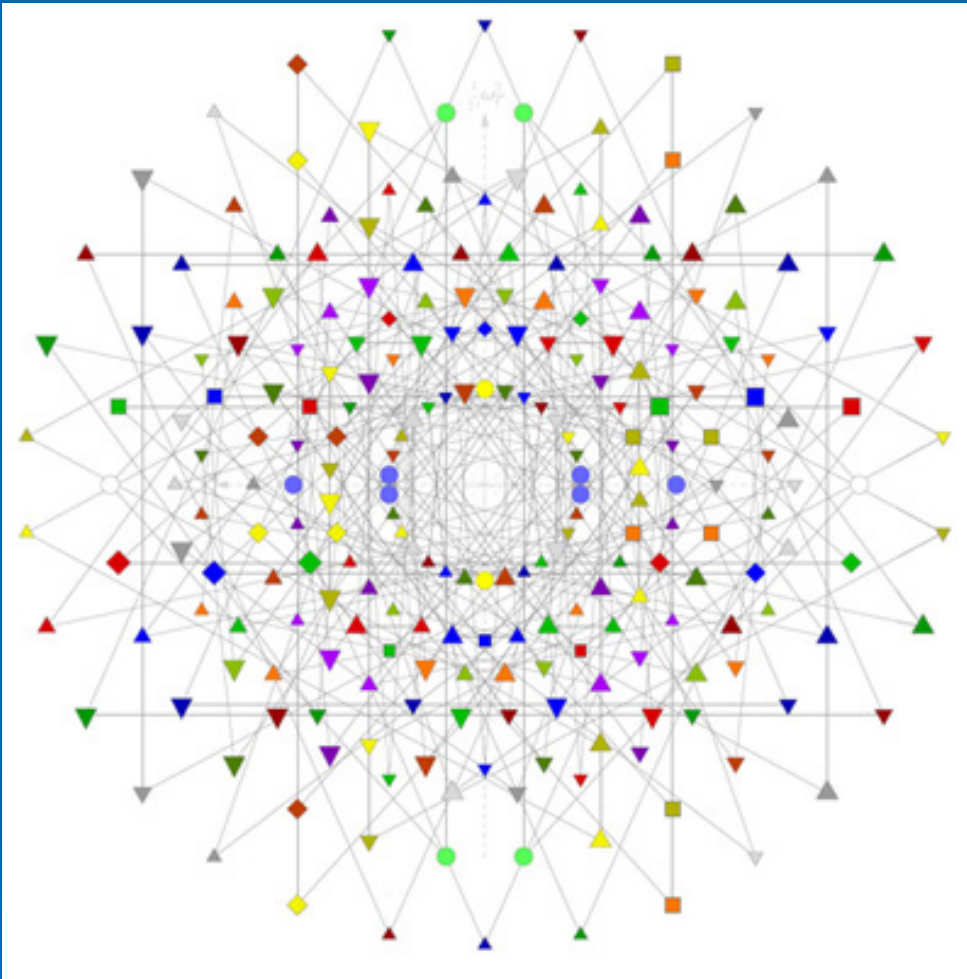


SG107.7.11.2
The 8D "E8" (2)



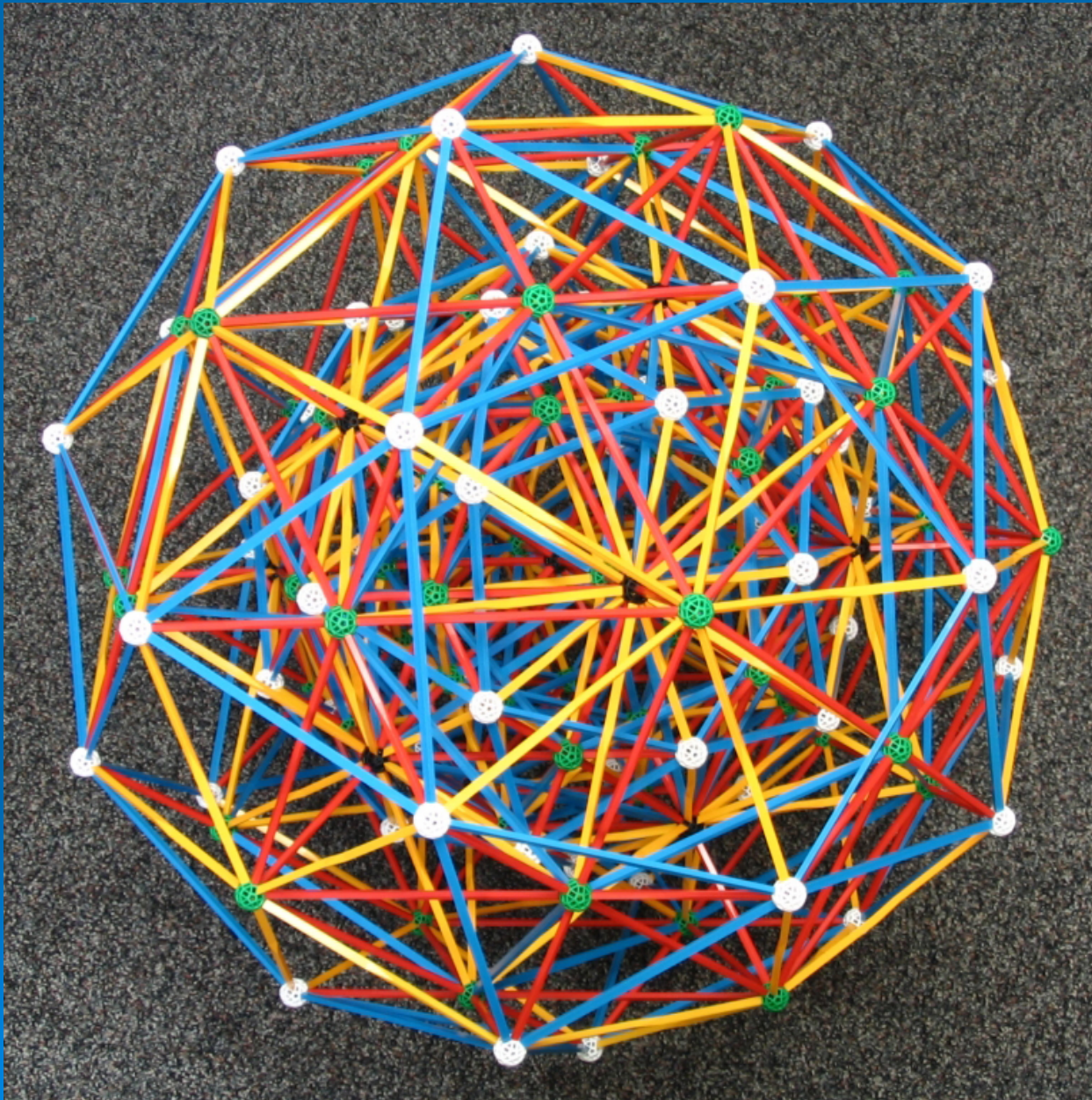
"E8", sometimes called the "*8-dimensional diamond lattice*", delivers the most efficient sphere-packing in 8 dimensions.

SG107.7.7.11.2 The 8D "E8" (3)



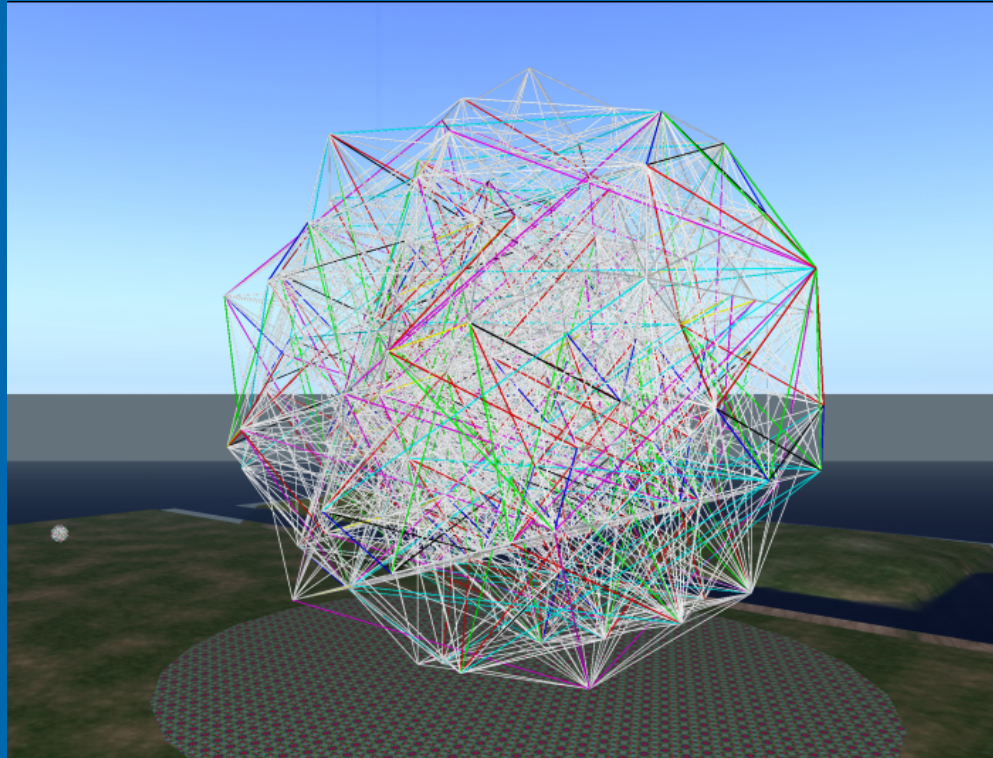
3 layers of the multiple symmetries of E8

The Lie group E8 has a root system that consists of 240 points in 8D. These 240 points are tightly packed together and form a configuration of 696,729,600 symmetries. For comparison, a 3D cube has 48 symmetries.
(Graphics by Garrett Lisi)



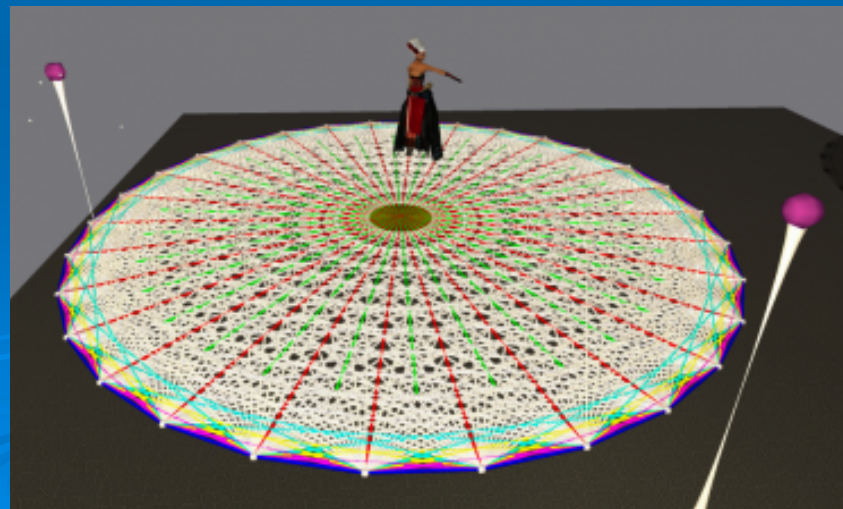
SG107.7.7.11.2
The 8D "E8" (4)

← A Zome-Tool
representation
of E8



SG107.7.7.11.3
E8 in Second
Life

←↓ Two Second Life renditions of the
multi-dimensional object E8.
(By Wizzy Gynoid)

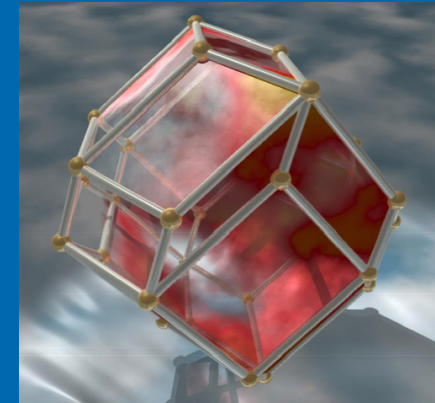


The hypercubes are one of the few families of regular polytopes that are represented in any number of dimensions.

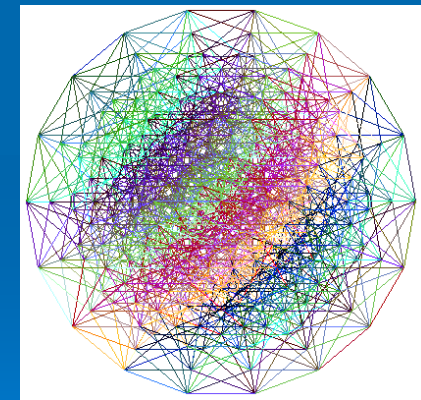
SG107.7.12 Hyper-Cubes

				m
n	Y_n	n-cube	Petrie polygon projection	Names Schläfli symbol Coxeter-Dynkin
0	Y_0	0-cube	.	Point -
1	Y_1	1-cube	—	Line segment $\{ \}$ ●
2	Y_2	2-cube		Square Tetragon $\{4\}$ ●—●
3	Y_3	3-cube		Cube Hexahedron $\{4,3\}$ ●—●—●
4	Y_4	4-cube		Tesseract Octachoron $\{4,3,3\}$ ●—●—●—●
5	Y_5	5-cube		Penteract Decateron $\{4,3,3,3\}$ ●—●—●—●—●

6	Y_6	6-cube		Hexeract Dodecapeton $\{4,3,3,3,3\}$ ●—●—●—●—●—●
7	Y_7	7-cube		Hepteract Tetradeca-7-tope $\{4,3,3,3,3,3\}$ ●—●—●—●—●—●—●
8	Y_8	8-cube		Octeract Hexadeca-8-tope $\{4,3,3,3,3,3,3\}$ ●—●—●—●—●—●—●—●
9	Y_9	9-cube		Enneract Octadeca-9-tope $\{4,3,3,3,3,3,3,3\}$ ●—●—●—●—●—●—●—●—●
10	Y_{10}	10-cube		10-cube icosa-10-tope $\{4,3,3,3,3,3,3,3,3,3\}$ ●—●—●—●—●—●—●—●—●—●



A 5D Hypercube
www.control.eng.cam.ac.uk



A 9D Hypercube
[//emeagwali.com](http://emeagwali.com)

SG107.7.13 The Hendecatope

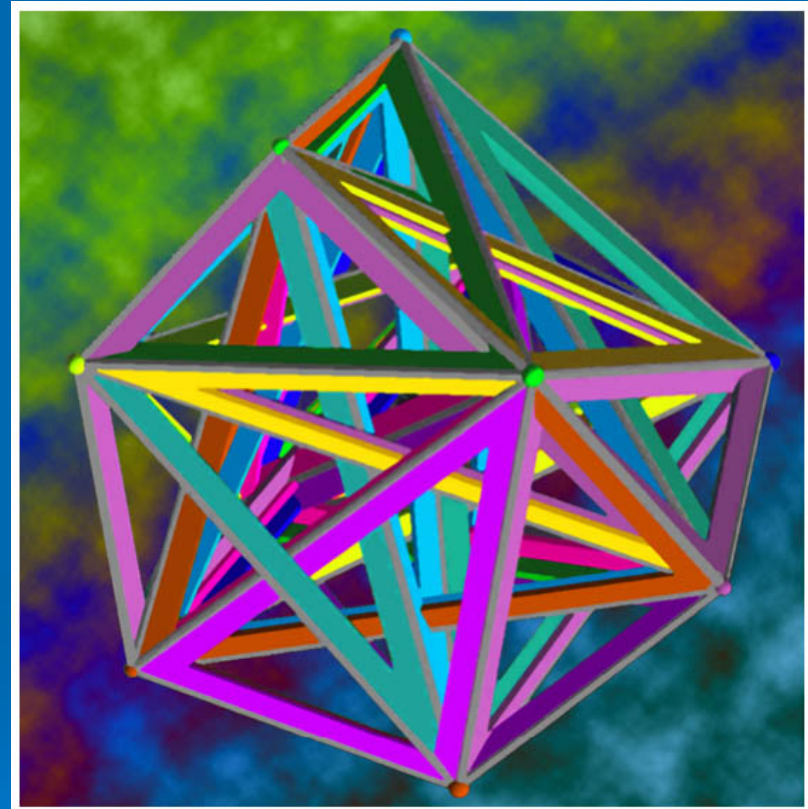
In the April 2007 issue of Discover magazine, Jaron Lanier shares the story of his quest for the *hendecatope* or 11-cell polytope. (*hendeca* = 11).

After reading about the 11-cell, a seemingly “impossible” shape, Lanier wanted to see a model. He then contacted Professor Carlo Sequin (UC Berkeley) who creates programs for robots & lasers to carve out 3D projections of 4D objects.

An 11-cell, a complex assembly of hemicosahedra (hemi-icosahedra), is said to be “abstract” because, if the cells were separated, they could not serve as conventional 3-D objects: they have some odd qualities, such as the fact that their sides can coincide with or pierce each other.

“Amazingly, in 4-D space these forms connect to each other in a perfectly regular symmetry. Furthermore, the form is self-dual, meaning that if you draw lines between the centers of every facet in the 11-cell, you get another 11-cell. If you do that to a cube, you get an octahedron. So, in an important sense, the 11-cell is more elegantly symmetrical than a cube.”

Says Lanier: *“You might think that these 4D shapes are of little more than academic interest, but actually they are incredibly important. They represent some of the most fundamental symmetries in nature... The hendecatope, in its own small way, links together the whole universe.”*



↑ The *hendecatope* or 11-cell polytope

SG107.7.14 Mystical Note on Higher Dimensions

While actually seeing projected representations or animations of “higher dimensional” constructs, as understood by mathematical sciences, can be very mind expanding and can activate various levels of dormant codes within our DNA or energy system, we also need to realize that they are only a mind-based approach to “higher dimensions”. They are not really replacing getting in touch with other “higher” essential aspects of our being: our heart, soul, psychic abilities, spiritual bodies...

Wisdom and mystical traditions have always described other dimensions through the psychic and visionary experiences of peering beyond the veil of 3D: clairvoyance, telepathy, ecstasy, out-of-body experience, spirit revelation, a sense of enlightenment, cosmic oneness... And we may have been graced with such experiences ourselves or we certainly know people who have and who then tried to convey to us a glimpse of the glorious magnificence of “higher dimensions”.

All I am suggesting is that the higher mind and technology-based windows into what higher dimensions might be are wonderful but can be unbalanced and a dead-end if one does not also pursue a spirit pilgrimage of using inner-dimensional techniques of expansion: meditation, prayer, yogas, purification of body-emotions-mind, harmonious life-styles, living foods, focus on spiritual qualities etc... in order to open up inner seeing.



SG107 Ca Homework & Activities

The best way to learn Polyhedra hands-on is to play with some of the many activities offered by Jill Britton on her website: <http://britton.disted.camosun.bc.ca/jbpolyhedra.htm>

These wonderful activities are coordinated with Jill's books for school and home education: *Investigating Patterns: Polyhedra Pastimes* and *Investigating Patterns: Symmetry and Tessellations*.

LINKS & REFERENCES:

GOOGLE keywords & images.

Wikipedia. the Free Encyclopedia. en.wikipedia.org

Don't forget the Wiki Commons: commons.wikimedia.org

George W. Hart Pavilion of Polyhedreality. The *Encyclopedia of Polyhedra* is an amazing collection of over 1000 polyhedra in 3D that can be played with. Visit the many links.

<http://www.georgehart.com/pavilion.html>

Wolfram Math World. "The world's most extensive mathematics resource." Key in your keywords.

[//mathworld.wolfram.com](http://mathworld.wolfram.com)

Wolfram Demonstration Projects. Many animations with polyhedra. Specially the "80 Golden Polyhedra".

<http://demonstrations.wolfram.com/80GoldenRhombhedra/>

Ulrich Mikloweit's Garden of polyhedra paper models. A magnificent collection.

www.polyedergarten.de

Polyhedra Puzzles.

www.terrystickels.com/terryst-gallery.htm

A lot of fascinating Polyhedra/Polytope links: www.ics.uci.edu/~epstein/junkyard/polytope.html

SG107.Cb1 Online SG School Curriculum: Intro & Intermediate

Sacred Geometry Introductory Level: 8 Modules

SG 101	Intro I	Sacred Geometry: Universal Order & Beauty
SG 102	Intro II	History & Traditions of Sacred Geometry
SG 103	Intro III	Sacred Geometry: A Grand Tour
SG 104	Intro IV	PHI: the Golden Ratio & the Fibonacci Series
SG 105	Intro V	The Golden Rectangle & Golden Spiral
SG 106	Intro VI	Pentagons, Pentagrams & the Penta-Modules
SG 107	Intro VII	The Five Platonic & 13 Archimedean Solids
SG 108	Intro VIII	The Vesica Piscis: Cosmic Womb of Creation

Sacred Geometry Intermediate Level: 8 modules

SG 201	Interm I	The Monochord, Music & Cymatics
SG 202	Interm II	The Power of Archetypal Numbers
SG 203A	Interm IIIA	Sacred Geometry Resurgence in Science - Part 1
SG 203B	Interm IIIB	Sacred Geometry Resurgence in Science - Part 2
SG 204	Interm IV	PHI in the Human Body, Biology & DNA
SG 205A	Interm VA	The SG of Nature - Part 1: Plants & Phyllotaxis
SG 205B	Interm VB	The SG of Nature - Part 2: Animals & Minerals
SG 207	Interm VII	SG in Architecture, Sacred Sites & Green Design

SG107.Cb2 Online SG School Curriculum: Advanced

Sacred Geometry Advanced Level: 8 modules

SG 301	Adv I	Golden Cosmos: Planets, Stars & Cosmology
SG 302	Adv II	SG in Art, Culture & Creativity
SG 303	Adv III	Universal Symbols: Primordial Knowledge
SG 304	Adv IV	Labyrinths: a Mini-Pilgrimage to Self
SG 305	Adv V	Mandalas & Yantras: Sacred Vortices
SG 306	Adv VI	Languages & Gematrias: Sacred Communication
SG 307	Adv VII	Sacred Geometry in the Healing Arts
SG 308	Adv VIII	Harmony on Earth: Science & Consciousness of Harmony

Upon completion of each level (Introductory, Intermediate & Advanced), a Certificate of Graduation from the Sedona School of Sacred Geometry will be presented to Certification Students.

Postgraduate seminars on current Sacred Geometry research, discoveries & updates will be offered in harmonic time.

Questions: phi@schoolofsacredgeometry.org

SG107 .Cc StarWheel Blessing



S8#17. "Merkaba-24" . www.starwheels.com



SG107.Cd Contact Info

Sedona School of Sacred Geometry
www.schoolofsacredgeometry.org
phi@schoolofsacredgeometry.org
PO Box 3714, Sedona, AZ 86340

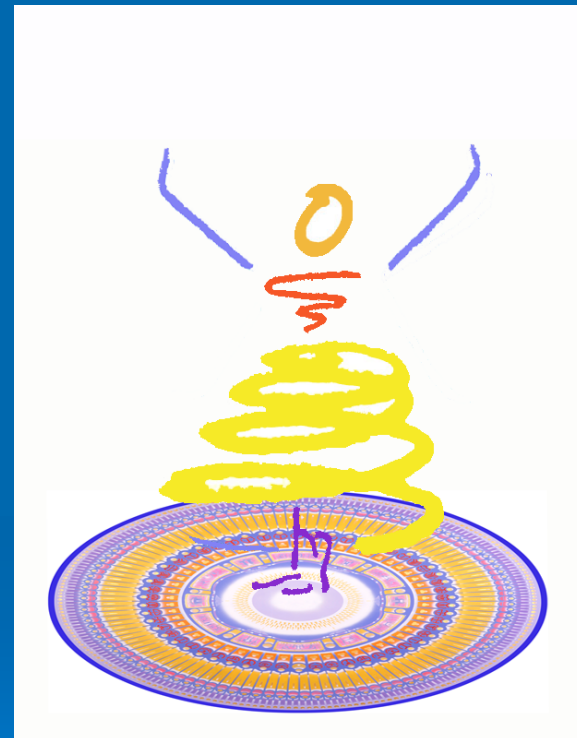
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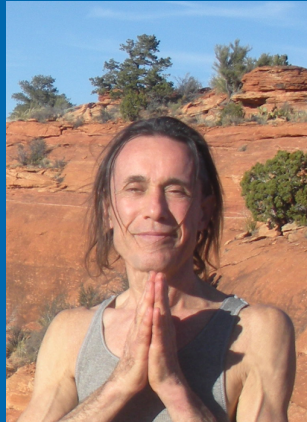
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On Facebook: Aya Sheevaya
FB Group: Sedona School of Sacred Geometry



Φ celebration



SG107.Ce About Aya

A native of France, Aya is a visionary artist and celebration yogi who has dedicated his life to serve humanity and to develop sacred arts education. In his late 20's, Aya realized that his professional life in the French diplomatic service was not fulfilling his heart's desires; he quit everything to go on an extended vision quest. His path took him around the world to visit a variety of sacred sites & cultures and to receive inspiration from many teachers.

In 1985, in Santa Monica, CA, Aya was gifted with a spiritual vision prompting him to create a series of 108 airbrushed neo-mandala paintings: the "StarWheels". The StarWheels, a happy family of vibratory flowers for the Earth, are looking for sacred spaces to be graced with their presence... (www.starwheels.com / www.starwheelmandalas.com)

Moving to Sedona, Arizona, in 1997, Aya has been involved with sacred arts classes & events, mandala creation, Sedona guided tours, labyrinth making and Sacred Geometry teaching. Aya has presented several StarWheel art exhibits, has sponsored community awareness events at the Sedona Library, has developed, in collaboration with Gardens for Humanity, the Peace Garden arboretum at the Sedona Creative Life Center, was a speaker at the Sacred Geometry Conference (Sedona, 2004), co-designed several labyrinth sites (The Lodge at Sedona, Magos' Ranch...), and was on the management team of the Raw Spirit Festival in 2006 - 2008.

Realizing that Sedona was progressively becoming a global spiritual university for many seekers from around the world, Aya founded in 2005 the Sedona School of Sacred Geometry. The school is offering online access to Sacred Geometry PDF modules, with 17 modules completed so far. In the school's website, Aya states: *"We are living at the extraordinary and exciting times of a global transformation to a higher order of human consciousness... Sacred Geometry is the expression and resurrection of our deep innate wisdom, now awakening from a long sleep: seeing again the all-encompassing, fractal-holographic unity of nature, life and spirit... The keyword is HARMONY."* (www.schoolofsacredgeometry.org)

Aya's visionary dream, supported by his non-profit educational organization, the StarWheel Foundation, is the co-creation of an international eco-village "The School of Celebratory Arts" - a green, tropical environment encouraging young people of all nations to develop their creative consciousness and thus contribute to a new, spirited, life-respecting global civilization on Earth. (www.starwheelfoundation.org).

Since 2012, Aya is dancing the body divine, after his re-discovery of Yoga, Partner Yoga and AcroYoga. Aya is currently the AcroYoga.org Jam coordinator for Sedona and a teacher of yoga swing asanas.

Blessings in Anjali!