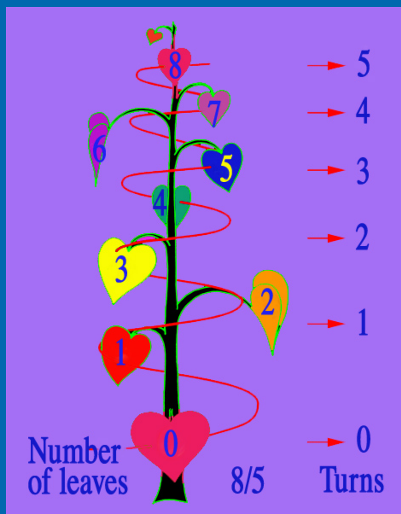
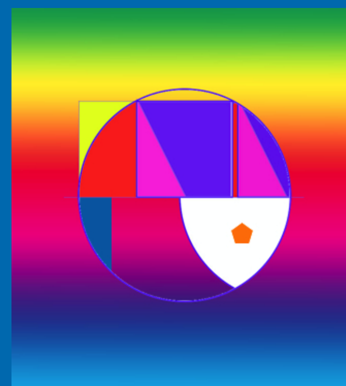
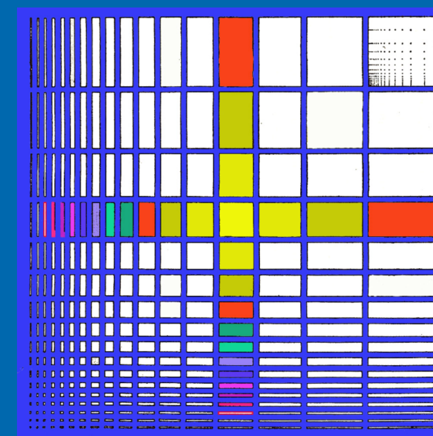
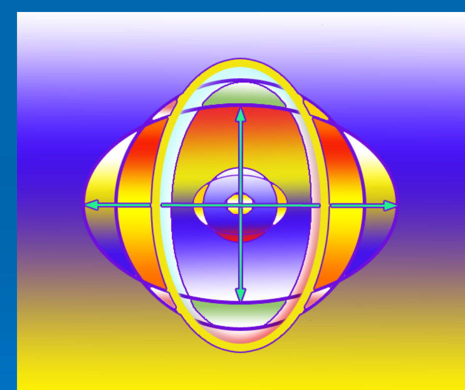
SG104.Ia PHI: the Golden Ratio & the Fibonacci Series

Online Module SG 104 / Intro IV









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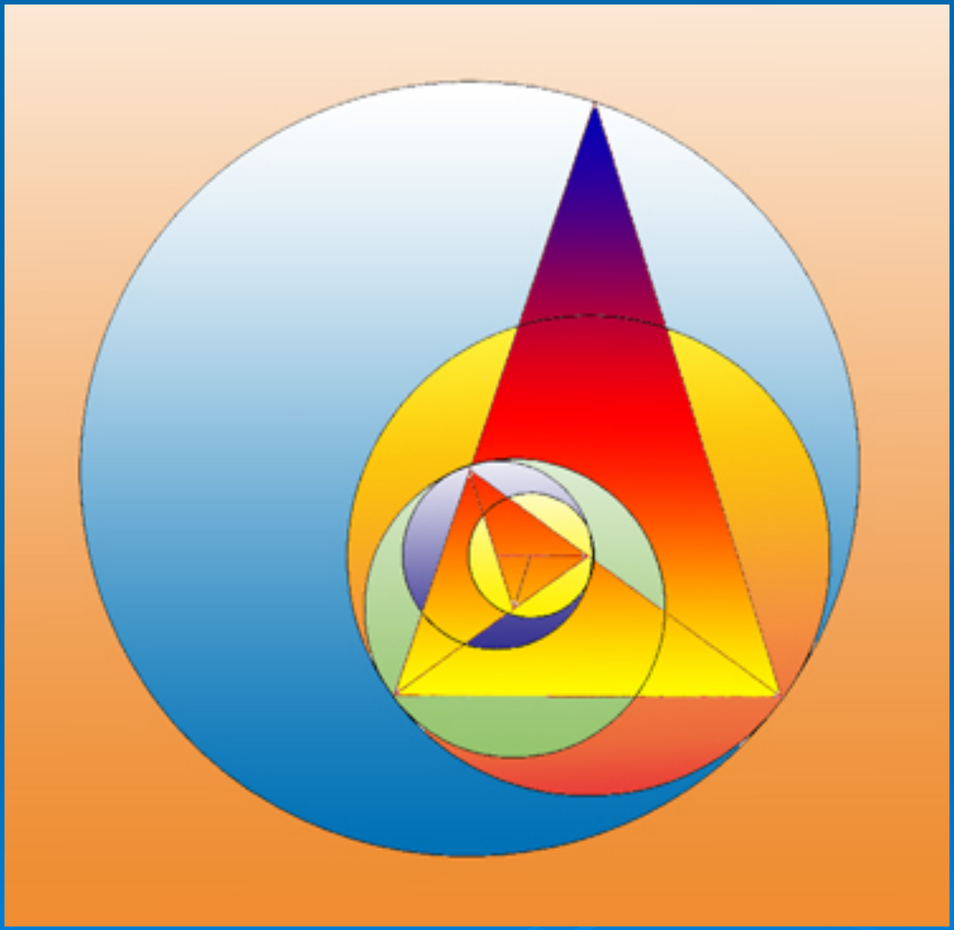
SG104.Ib PHI: Golden Ratio & Fibonacci - Contents

SG104.Ic PHI: Golden Ratio & Fibonacci



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SG104.Id PHI: Golden Ratio & Fibonacci - Introduction (1)

The Golden Proportion PHI (F) is the hidden treasure of Sacred Geometry as well as the secret power harmonizing the cosmos.

It is an irrational & transcendental mathematical constant, manifesting as a RATIO going to infinity, that has a unique property in this universe: its values are in both an additive & multiplicative relationship. In Brief: no matter how far the multiples & sub-multiples of Phi cascade up and down the scales of magnitude (from microcosm to macrocosm, through mesocosm i.e. us the humans), they will always recognize each other, mutually embrace and enter a dance of harmonic resonance, thus creating friendly & secure highways for energy transfers. Because of this unique property, the Phi Proportion is the perfect and most efficient MEDIATOR between waves.

Phi, the Golden Ratio

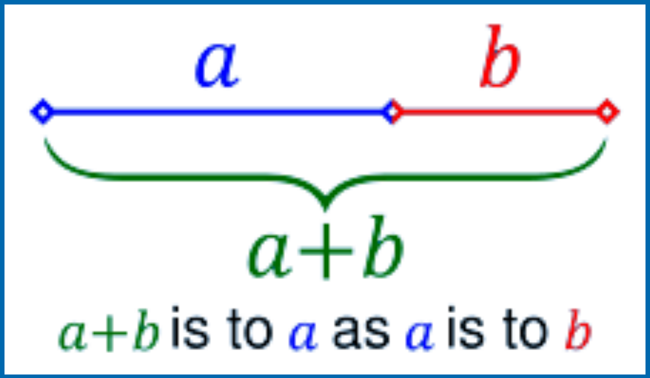
**Think of waves as people.
When using the Phi power
of harmonic relationship,
waves can happily "*grow & multiply*"
without losing their personality.**

**Even the "*children*" (fractal parts)
will always keep an inner resonance,
a holographic link with their "parents"
(the relative whole). This is now called
non-local interconnectedness.**

SG104.Ie PHI: Golden Ratio & Fibonacci - Introduction (2)

So, remember this: Phi has a unique power to create Harmony because of its special mathematical (and spiritual) ability to unite the different parts of a whole so that each part preserves its own identity and yet blends into the larger whole.

Cutting edge science points to the Golden Ratio as providing the Holy Grail of physics: the Unified Field.
Phi may be the geometric constant changing electro-magnetism into gravity - compression of charge becoming acceleration of charge.



In this module SG104, we will journey through the elegant geometry & simple mathematics of Phi and its generating tribe: the Fibonacci Series.

The art, beauty and full-being understanding of Sacred Geometry reside in the practice of construction with compass and square [◆SG101.5].

Here we will play with various ways to create the Phi Ratio in basic shapes & forms and thus prepare ourselves to encounter the more elaborate geometries of the Golden Rectangle, the Pentagon/Pentagram and other classical golden geometries… all the way to the 3D Platonic & Archimedean Solids.

Welcome to the Golden Universe!

↑ Leonardo Fibonacci's teachers

SG104. Chapter 1.

The Fibonacci Series

Here we meet again Leonardo-the-First and the Fibonacci Numbers.

(The other Leonardo is Leonardo da Vinci)

… and the Golden Ratio will appear…

1-1-2-3-5-8-13-21-34-55-89-144-233-377...

SG104.1.1.1 Leonardo of Pisa aka Fibonacci (1)

"When my father, who had been appointed by his country as public notary in the customs at Bugia acting for the Pisan merchants going there, was in charge, he summoned me to him while I was still a child, and having an eye to usefulness and future convenience, desired me to stay there and receive instruction in the school of accounting.

There, when I had been introduced to the art of the Indians' nine symbols, a remarkable teaching, knowledge of the art very soon pleased me above all else and I came to study with whomever was learned in the art, from Egypt, Syria, Greece, Sicily and Provence, and practiced their various methods."

Leonardo Fibonacci
in ***Liber abaci***

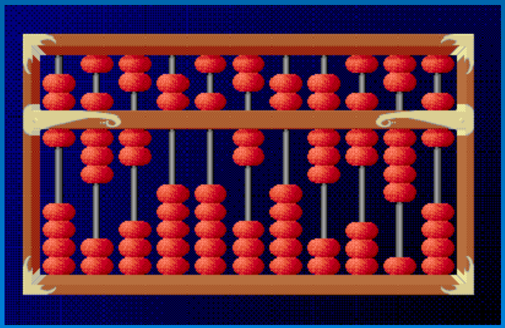
Statue of Fibonacci.
***Fortezza Camp Santo*,**
Pisa, Italy.

Born in Pisa, Italy, Fibonacci (c. 1170 - 1240) was educated in Bugia, North Africa (today's *Bejaia*, Algeria) where his father was an envoy of the Pisa merchants. There, young Fibonacci was exposed to the Arabic numerals and the Hindu counting system using the place-value concept of "zero". Fibonacci was quick to grasp the accounting efficiency and cultural significance of the Hindu-Arabic numerals and set out to share this new knowledge with the rest of Europe, slowly waking up from the medieval slumbering.

SG104.1.1.2 Leonardo of Pisa aka Fibonacci (2)

The Abacus

Up to Leonardo's times, calculations were performed using Roman Numerals coupled with the *abacus*, an ancient and ingenious device providing place-value (the assignment of a value to every number, depending on it place in a row). The place-value was lacking in the Roman Numerals system, which was also extremely clumsy.



Returning to Pisa, Fibonacci wrote and published, in 1202, his epoch-making *Liber Abaci* or Book of Calculation. "Publishing", in the pre-printing days, meant having several copies carefully made by hand.

The updated edition of 1228 is the copy that came down to us. The first section of *Liber Abaci* introduces the Hindu-Arabic numerals and their use as integers and fractions. The second section explains various techniques to apply these new numerals to practical problems of accounting and commercial transactions. The third section is devoted to a variety of mathematical problems. Famous among these problems is the "*Rabbit Problem*".

Merchants were suspicious of these Hindu-Arabic numerals. And historians have commented that "*it took the same 300 years for these numerals to catch on as it did for the completion of the Leaning Pisa Tower*" - which was started during Fibonacci's youth.

"The nine Indian figures are:
9 8 7 6 5 4 3 2 1.
With these nine figures,
and with the sign '0'
which the Arabs call Zephyr,
any number may be written
as is demonstrated below."

(Opening sentence of Liber abaci)

SG104.1.1.3 Leonardo of Pisa aka Fibonacci (3)

Liber Abaci was the sum of the mathematical knowledge of the time and, as it was eagerly received and distributed throughout Europe, it had a profound influence on European thought & culture.

A number of new mathematical terms in use today were first introduced in *Liber Abaci*, such as: "*factus ex multiplicatione*" (the factors of a number), "numerator" and "denominator" (Fibonacci was the first to introduce the horizontal fraction bar).

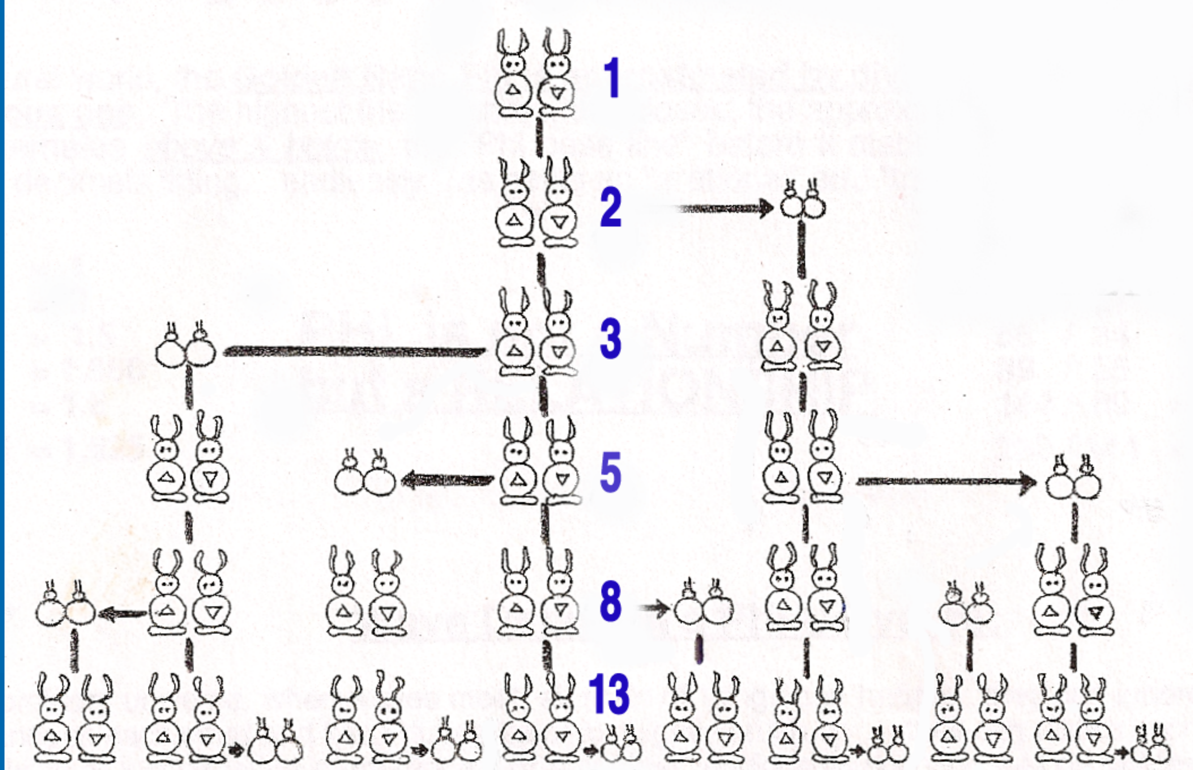
Frederick II
Frederick was a ruler very much ahead of his time, being an avid patron of science and the arts. *(Wikipedia)*

Liber Abaci brought Fibonacci extensive fame and recognition. In the early 1220's, he was invited by Emperor Frederick II of Hauhenstaufen (nicknamed "*Stupor Mundi*" or "Wonder of the World") to appear at the imperial court and was presented with difficult mathematical problems by court mathematician Johannes of Palermo. Fibonacci offered ingenious solutions to all these problems and described them in subsequent books: *Flos* (Flower) and *Liber Quadratorum* (Book of Squares).

Fibonacci direct use and contributions to the Golden Ratio appear in his short book on geometry *Practica Geometriae* (1223), where he presents new calculations & constructions for penta/decagonal figures and the Platonic Solids - directly involving the Golden Ratio Phi. However, Fibonacci's place in history is secured by the innocuous - looking "Rabbit Problem". Mario Livio comments in his book *The Golden Ratio*:

"Fibonacci's role in the history of the Golden Ratio is truly fascinating. On one hand, in problems in which he consciously used the Golden Ratio, he is responsible for a significant but not spectacular progress. On the other, by simply formulating a problem that on the face of it has no relation whatsoever to the Golden Ratio, he expanded the scope of the Golden Ratio and its applications dramatically."

SG104.1.2 The “Rabbit Problem”



Half a year later, there are 13 pairs of rabbits...
and multiplying to the next Fibonacci

A year later, January 1st, there are 233 pairs...
1 1 2 3 5 8 13 21 34 55 89 144 233

The third section (chapter XII) of *Liber Abaci* is where Fibonacci offers the now famous “Rabbit Problem” about the mathematics of rabbit genealogy.

“A certain man put a
pair of rabbits in
a place surrounded
on all sides by a wall.
How many pairs
of rabbits can be
produced from that
pair in a year,
if it is supposed that
every month each pair
begets a new pair
which from
the second month on
becomes productive?”

In this (idealized) situation, the solution is that the number of rabbits follows the “*Fibonacci Series*”.

SG104.1.3.1 The Fibonacci Series (1)

What's so special about this "*Fibonacci Series*"?

1-1-2-3-5-8-13-21-34-55-89-144-233-377...

In the Fibonacci Series,

➔ Each term is the sum of the two preceding terms:
Example: 34 = 21 + 13

➔ Each term divided by the previous one
gives a progressively better approximation for PHI,
the Golden Ratio:
Examples: 144 / 89 = 1.617977…
and 233 / 144 = 1.618055
(accurate numerical value for Φ to 5 decimals)

SG104.1.3.2 The Fibonacci Series (2)

F1	1
F2	1
F3	2
F4	3
F5	5
F6	8
F7	13
F8	21
F9	34
F10	55
F11	89
F12	144
F13	233
F14	377
F15	610 •
F16	987
F17	1597
F18	2584
F19	4181
F20	6765
F21	10946 •
F22	17711
F23	28657
F24	46368
F25	75025

← On the left is a table of the first 25 Fibonacci Number: F1 to F25.

Notice the pattern:

- **Every 3rd Fibonacci number is a multiple of 2**
- **Every 5th Fibonacci number is a multiple of 5**
- **Every 7th Fibonacci number is a multiple of 13** ***And so on…***

On the right → is a table of the first 25 Fibonacci numbers, each one divided by the previous one.

Notice how rapidly the result is approaching the limit of Phi:

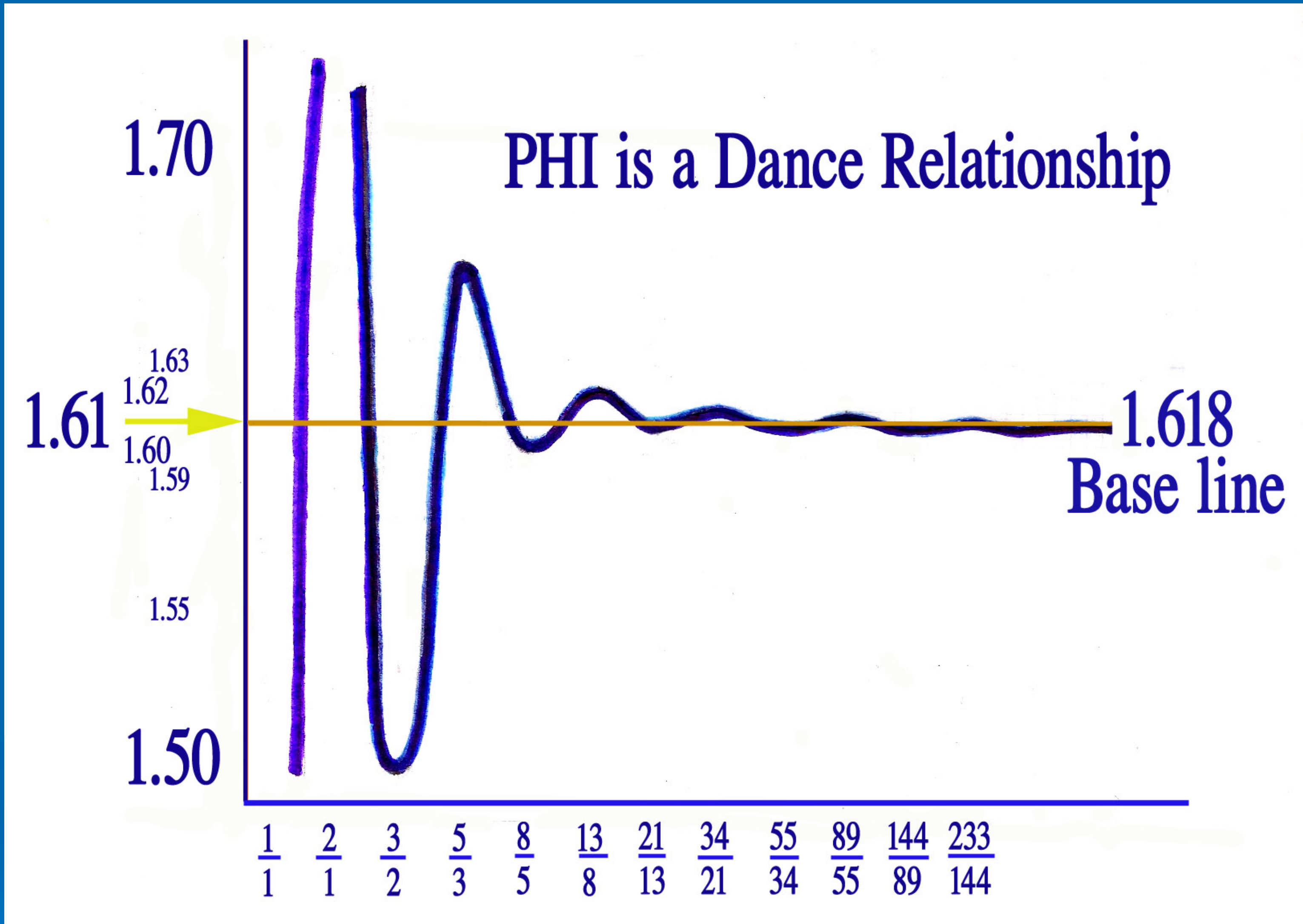
- **At the 12th step, we already have Phi accurate to 4 decimals:**

1.6180

- **At the 23rd step, we have an accuracy to the 9th decimal:**

1.618033988

1/1	1.000000000
2/1	2.000000000
3/2	1.500000000
5/3	1.666666666
8/5	1.600000000
13/8	1.625000000
21/13	1.615384615
34/21	1.619047619
55/34	1.617647058
89/55	1.618181818
144/89	1.617977528
233/144	1.618055555
377/233	1.618025751
610/377	1.618037135
987/610	1.618032786
1,597/987	1.618034447
2,584/1,597	1.618033813
4,181/2,587	1.618034055
6,765/4,181	1.618033963
10,946/6,765	1.618033998
17,711/10,946	1.618033985
28,657/17,711	1.618033990
46,368/28,657	1.618033988
75,025/46,368	1.618033988



SG104.1.3.3 The Fibonacci Series (3)

In the natural world, the Golden ratio Phi is approximated by dividing any Fibonacci number by the previous one. The highest the numbers, the closest the approximation.
Notice that the resulting quotient alternates/fluctuates *above & below* the Phi Base Line, dynamically seeking harmonic balance and rapidly zeroing in / reaching the limit of Phi = 1.618… followed by an infinite number of decimals, as befits a 'transcendental' number.

SG104.1.4.1 Brief History of the Fibonacci Series (1)

The Fibonacci Sequence was well known in ancient India, where it was applied to the metrical sciences (prosody), long before it was known in Europe. Developments have been attributed to Pingala (200 BC), Virahanka (6th century AD), Gopāla (ca. 1135 AD), and Hemachandra (ca. 150 AD).

↑ Acharya Hemachandra, the Indian Fibonacci. (www.jainworld.com)

Pingala's (200 BCE) work contains the basic ideas of the Fibonacci numbers (called *maatraameru*).

Virahanka (6th century) was an Indian Prosodicist who analyzed the combinations of short & long syllables to obtain the meters of sacred songs. In his study of the number of possible ways to form these meters, he found the Fibonacci Series four centuries before Fibonacci himself.

Gopala (12th century) was an Indian mathematician, who studied the Fibonacci numbers in 1135.

Hemachandra (1089–1172), following the earlier Gopala, presented what is now called the Fibonacci sequence around 1150, about fifty years before Fibonacci (1202). He was considering the number of cadences of length n, and showed that these could be formed by adding a short syllable to a cadence of length $(n-1)$, or a long syllable to one of $(n-2)$. This recursion relation $F(n) = F(n-1) + F(n-2)$ is what defines the Fibonacci sequence.

In India, the Fibonacci Series is referred to as the Pjngala-Hemachandra Sequence.

Some researchers think that Hemachandra may have been singing his mathematics.

SG104.1.4.2 History of the Fibonacci Series (2) in the West

The smiling irony of history is that Leonardo da Pisa aka Fibonacci only re-discovered his namesake Series and also most probably had little understanding of what he had uncovered with his study of rabbits.

As a matter of fact, the nickname Fibonacci (in Latin: *Filius Bonacci*, the son of the Bonacci family or "*son of good nature"*) was apparently introduced by an historian of mathematics in… 1838. In real life, Leonardo referred to himself as Leonardo Bigollo or Leonardi Bigolli Pisani (in Latin: *Leonardus Pisanus*) which means, in the Tuscan & Venetian dialects respectively, the "*traveler*" and "*man of little importance*".

So, when, some 300 years later, Johannes Kepler (1571-1630) wrote about the "Fibonacci" series in a 1611 publication, the name 'Fibonacci Series' was not yet born. Kepler wrote:

"As 5 is to 8, so is 8 to 13, so is 13 to 21 almost."

As noted in *The Fabulous Fibonacci Numbers: "Centuries passed and the numbers still went unnoticed. In the 1830's, C. F. Schimper and A. Braun observed that the numbers appeared as the count of spirals of bracts on a pinecone. In the mid 1800's, the Fibonacci numbers began to capture the fascination of mathematicians."*

It took another 200 years after Kepler for the current name "*Fibonacci Numbers*" to be given to the series by French mathematician Francois Edouard Anatole Lucas (1842-1891) who also discovered another recursive series closely related to the Fibonacci Series and called the Lucas Series [◆SG104.5]

Another French mathematician Jacques Philippe Marie Binet (1786-1856) had already developed a formula for finding any 'Fibonacci' number from its position in the series. [◆Next page]

1962 saw the creation of the Fibonacci Association and its official publication the *Fibonacci Quarterly*, followed, in 1964, by the First International Conference on Fibonacci Numbers. And today, the Fibonacci Numbers and their harmonic limit Phi show up in all areas of nature and culture and hold the fascination of mathematicians, scientists, artists and researchers all over the world [◆Chapters 5 & 6]

SG104.1.5 The Binet Formula

In the mid 19th century, mathematician Jacques Binet (1786 - 1856) rediscovered a formula that, historians say, was already known to Leonard Euler (1707-1783) and also to Abraham de Moivre (1667-1754).
This formula directly delivers the value of any Fibonacci number!

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Let us not be impressed by this formula! Notice that the formula is using the same 3 numbers: 1 , 2 and √ 5 . (1 + √ 5) / 2 is simply… the Golden Number Phi = Big Φ = 1.618… And (1 - √ 5) / 2 is the reciprocal of the Golden Number Phi = Small ϕ = 0.618… These are the two solutions (positive & negative) to the equation for the 3-terms proportion $x^2 - x - 1 = 0$ defining the Golden Ratio.
We shall study the Golden Number in the next chapter.

Examples: Fibo number 6 $(F_6) = (\Phi^6 - \phi^6) / \sqrt{5} = (1.618^6 - 0.618^6) / 2.236 = 8$

Fibo number 24 $(F_{24}) = (\Phi^{24} - \phi^{24}) / \sqrt{5} = (1.618^{24} - 0.618^{24}) / 2.236 = 46{,}368$

IF you know the preceding Fibo number, you can simply multiply by Phi (Φ) = 1.618 to obtain the next Fibo number, since the Fibonacci Series is a Phi-based recursive series or "Phi Cascade".

Example: $F_{10} = F_9 \times \Phi = 34 \times 1{,}618 = 55$

AND, for larger Fibo numbers , you can just take F_n to be the closest integer to $\Phi^n / \sqrt{5}$. Examples:
$F_{10} = \Phi^{10} / \sqrt{5} = 122.966 / 2{,}236 = 55.$ $F_{18} = \Phi^{18} / \sqrt{5} = 5{,}777.999 / 2{,}236 = 2{,}584$

Now, you might ask: what is the use of knowing these Fibonacci Numbers?
Just wait and see! Or think right now of a Golden Brick (Spiral) Road…

SG104.1.6.1 Fibonacci Numbers & PHI (1)

(Table from Posamentier & Lerhmann. *The Fabulous Fibonacci Numbers.*)

n	Sum		Difference	
1	$\phi + \frac{1}{\phi}$	$= 1\sqrt{5}$	$\phi - \frac{1}{\phi}$	$= 1$
2	$\phi^2 + \frac{1}{\phi^2}$	$= 3$	$\phi^2 - \frac{1}{\phi^2}$	$= 1\sqrt{5}$
3	$\phi^3 + \frac{1}{\phi^3}$	$= 2\sqrt{5}$	$\phi^3 - \frac{1}{\phi^3}$	$= 4$
4	$\phi^4 + \frac{1}{\phi^4}$	$= 7$	$\phi^4 - \frac{1}{\phi^4}$	$= 3\sqrt{5}$
5	$\phi^5 + \frac{1}{\phi^5}$	$= 5\sqrt{5}$	$\phi^5 - \frac{1}{\phi^5}$	$= 11$
6	$\phi^6 + \frac{1}{\phi^6}$	$= 18$	$\phi^6 - \frac{1}{\phi^6}$	$= 8\sqrt{5}$
7	$\phi^7 + \frac{1}{\phi^7}$	$= 13\sqrt{5}$	$\phi^7 - \frac{1}{\phi^7}$	$= 29$
8	$\phi^8 + \frac{1}{\phi^8}$	$= 47$	$\phi^8 - \frac{1}{\phi^8}$	$= 21\sqrt{5}$
9	$\phi^9 + \frac{1}{\phi^9}$	$= 34\sqrt{5}$	$\phi^9 - \frac{1}{\phi^9}$	$= 76$
10	$\phi^{10} + \frac{1}{\phi^{10}}$	$= 123$	$\phi^{10} - \frac{1}{\phi^{10}}$	$= 55\sqrt{5}$

← Table of the successive powers of the sums and differences of Phi (Φ) and its reciprocal 1 / Φ.

What do you notice (highlighted in blue)?

- **It is our now familiar Fibonacci Series!**

And what is next to the Fibonacci Numbers?

- **45, the "pointer" to the Golden Ratio!**

Now, look at the numbers left in black:
1 - 3 - 4 - 7 - 11 - 18 - 29 - 47 - 76 - 123
This is the Lucas Series, another recursive series obviously closely related to the Fibonacci series.
Same Phi Cascading: 123 / 76 = 1.618

[SG104.5]

SG104.1.6.2 Fibonacci Numbers & PHI (2)

Let us show the direct relationship of PHI and the Fibonacci numbers in a even more direct way.

➔ On the right is a table of the powers of PHI, in terms of Fibonacci multiples of PHI and the regular Fibonacci Series.

Quite a tie-in
and an elegant beauty
in the symmetry.

Later, we will consider the
numerical expansions
of the Powers of PHI,
as well as the PHI Reciprocals
and their respective Roots.

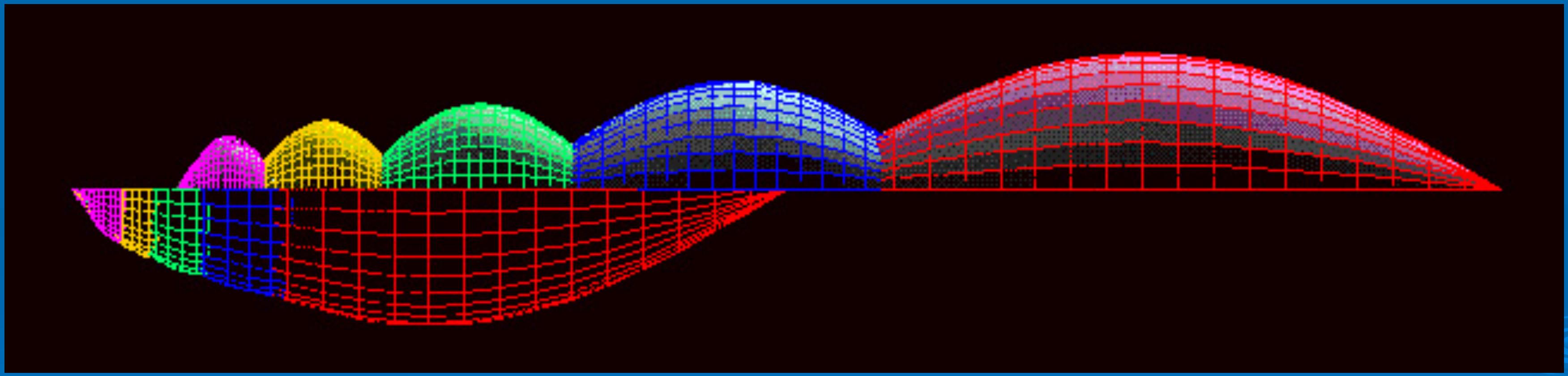
$\varphi = \varphi = \emptyset = \Phi$ in this table

$$\varphi^2 = \varphi + 1$$
$$\varphi^3 = 2\varphi + 1$$
$$\varphi^4 = 3\varphi + 2$$
$$\varphi^5 = 5\varphi + 3$$
$$\varphi^6 = 8\varphi + 5$$
$$\varphi^7 = 13\varphi + 8$$
$$\varphi^8 = 21\varphi + 13$$
$$\varphi^9 = 34\varphi + 21$$
$$\varphi^{10} = 55\varphi + 34$$
$$\varphi^{11} = 89\varphi + 55$$
$$\varphi^{12} = 144\varphi + 89$$
$$\varphi^{13} = 233\varphi + 144, \text{ etc.}$$

SG104.1.6.3 Fibonacci Numbers & PHI (3)

By now, do you get a sense of the deep organic connection between the Fibonacci Numbers and Phi the Golden Ratio, via the medium of $\sqrt{5}$?
They generate each other. They are in mutual harmonic inter-connectedness.

The Fibonacci Series is the expression in space & time of the archetypal Harmonic Ratio Phi. Conversely, the Fibonacci Series converges back to the Golden Ratio as it expands.
So, we are seeing here two levels of the same Law of Harmony: an extra-dimensional principle (Phi) inexpressible with exactness in 3D (hence called "*irrational*" by mathematicians) AND a definite manifestation in space-time ("*made manifest*") as sequences of numbers (Fibonacci & Lucas) whose relationships generate the causal/formative principle Phi.



🡅 Phi surfing the Fibo Wave

Let's now proceed to explore, in various ways, the basic expressions of this intriguing Golden Ratio Phi: geometric, mathematical, visual, kinesthetic, aesthetic, intuitive and ultimately spiritual.
In later modules, we will visit many examples & applications of the Golden Ratio.

SG104.2 Chapter 2. The Golden Ratio PHI

“Avoid extremes - Keep the Golden Mean”
(Cleobulus. 6th century BCE)

SG104.2.1 PHI of Many Names

Our friendly Golden Ratio has been given a variety of names by many people and under many skies:

- **Greek antiquity used the term τομή (tomé = the "cut", the "section"). Hence the use of the Greek letter τ (tau) in the professional mathematical literature. Tau (τ) is now being supplanted by Phi (Φ).**
- **Euclid of Alexandria (325-265 BCE) gave the first written definition of the Golden Ratio as "The Division between Extreme and Mean Ratio" (ακρος και μεσος λογος)**
- **The Romans called it Aurea Sectio (Golden Section).**
- **Luca Pacioli (1445-1517) wrote a whole book to justify calling it the Divine Proportion.**
- **Christopher Clavius (1538-1612), a German Jesuit mathematician and astronomer, and main developer of the Gregorian calendar, called it The Godlike Proportion.**
- **Johannes Kepler (1571-1630) says of the Golden Ratio that it is like a "Precious Jewel".**
- **Johann F. Lorentz used the name the "Continued Division".**
- **Martin Ohm, the brother of Georg Simon Ohm (after whom Ohm's law in electromagnetism is named), is believed to be the first one, in modern times, to use the name "*Golden Cut*" (Der Goldene Schnitt). In the late 19th century, Adolf Zeizing researched the aesthetic appeal of the Golden Ratio under the same name "Golden Cut". Thereafter, the name spread into other languages: Sezione Aurea, la Section d'Or…**
- **Matila Ghyka, in 1931, published an influential treatise called "Le Nombre d'Or" (The Golden Number).**
- **J. Leslie wrote about the "Medial Section". Ralph B. Nunnelley talked about the "Magic Number". And Callum Coats (who introduced the work of Schauberger to the public) calls it the "Transmutation Number".**

Finally, American mathematician Mark Barr gave the Golden Ratio the name of Phi (Φ), after the first Greek letter of *Phidias* (c. 490-430 BCE), the Greek sculptor who helped direct the building of the Parthenon and is said to have applied the Golden Ratio to the "Athena Parthenos" and to the "Zeus" in Olympia.

The name has stuck and has become the global icon of Sacred Geometry.

We will see now why this name-symbol is so perfect.

SG104.2.2.1 The Shape of PHI (1)

🡐 Phi
Capital form (left)
Lower case form (right)
(shaped like a spiral)

The 21st letter of the Greek alphabet PHI now used, by global consensus, as the symbol for the Golden Ratio is a very rich graphic icon, a modern day hieroglyph, a multi-dimensional graph. Indeed, the PHI symbol contains a cascade of meanings, and can be read/perceived as archetypal space, symbolic reference or literal diagram.

- **PHI is the union of the twin cosmic archetypal shapes: the Curve and the Line. Remember the reverence of Sacred Geometers for the Compass (Curve) and the Builder's Square (Line)?**
These two primordial geometries are complementary as Yin & Yang, Yoni & Lingam, Sacred Rock & Holy Sword, Sacred Circle & Vertical Axis, "O" & "I" (as vowel sounds)…
The Phi symbol is a hieratic sign, standing in cosmogonic majesty but it gains by being seen in dynamic motion, as a contemporary Tao sign holding in potential the Golden Spiral seen in the lower case format.

O I

- **Lower case Phi is a dynamic sign, opening the circle to form a spiral.**

- **In world mythologies & creation stories, PHI symbolizes the Cosmic Circle / Sphere of the universe / Egg of Creation being cut open by the Axis of the World. It is the Primordial Ocean-Void being churned by the Prime Creator's Spoon or Lightning/Thunderstorm or Cosmic Serpent… Actually PHI is more like an ellipse than a circle, thus truer to celestial revolution orbitals.**

Φ

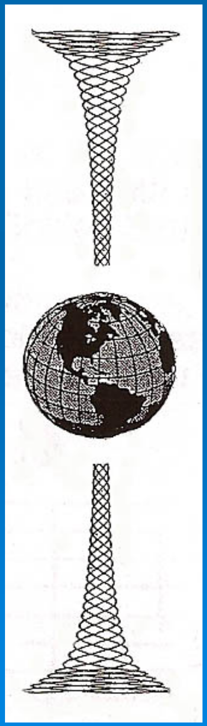
SG104.2.2.2 The Shape of PHI (2)

- **PHI is the graphic combination of ZERO + ONE (0 + 1), the union of the place-value sign "0" and the first integer "1". As "0" and "1", PHI also refers to TEN ("10"), the Sacred Tetraktys of the Pythagoreans.**
In symbolic arithmosophy ("*Wisdom of Numbers*"), ZERO is the Womb of the Mother Goddess / the Original Void / the Quantum Plenum and ONE is the Father-Source Unity pre-existent to all numbers. This cosmic parental pair begets the entire manifested universe.

0 + 1
= 10
⊙

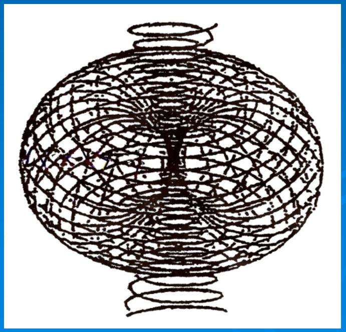
- **PHI also represents the Tree of Creation, with roots above and roots below, the Tree of Knowledge.**

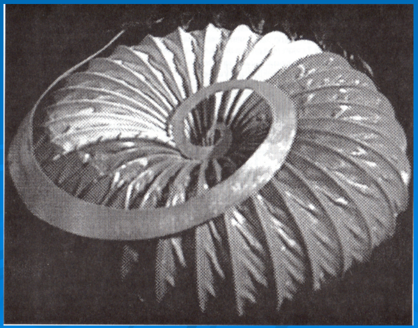
- **Seen from above, PHI looks like the astrological sign for the Sun: a dot within a circle.**

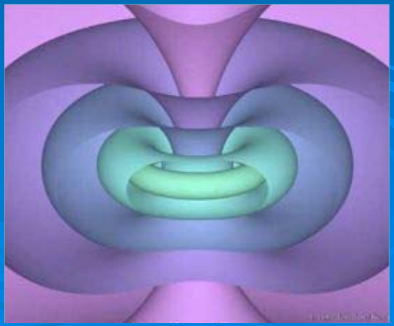


- **As a Zen calligraphy, first the " *Enso circle*" is traced in a counter-clockwise direction to draw out a dynamic boundary, which is then cut through by a downward stroke of the incarnating intention.**

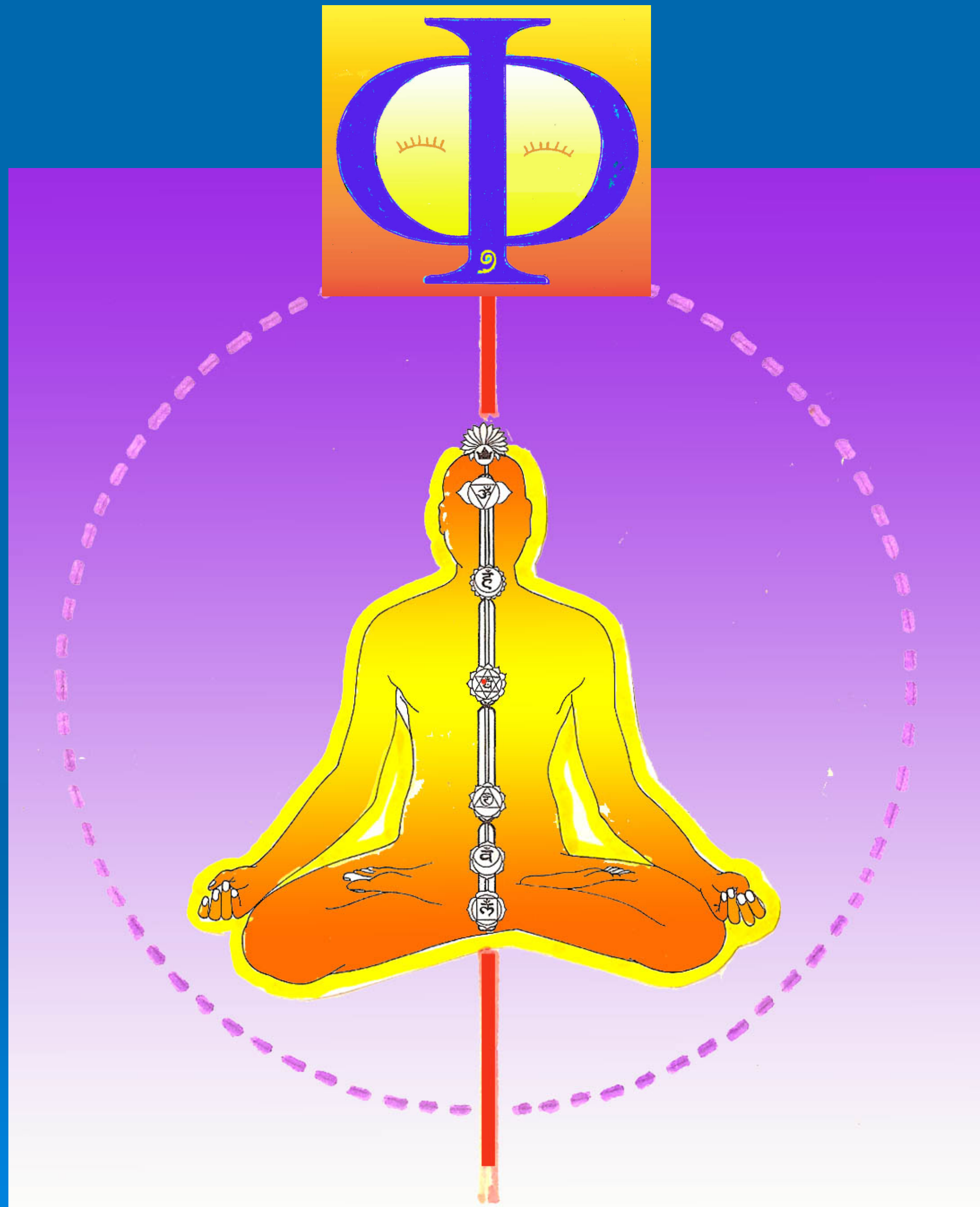
- **As a 3D shape, PHI is related to the shape of the Earth (an obloid) or a galaxy.**
PHI is the 2D cross-section of a VORTEX, the typical dynamics of all energy interactions in the universe, from atoms to hearts to human energy fields to planets to galaxies. Vortices transmit energy inter-dimensionally, in the most efficient way, based on the Phi Ratio.







SG104.2.2.3 The Shape of PHI (3)



SG104.2.3 Ratios & Proportions

On their quest for Oneness, the ancients distinguished several degrees of relatedness or steps of harmonic coherence, which they expressed by the concepts of *ratios & proportions*.

A RATIO is the comparison of two different items (sizes, quantities, qualities, concepts…) such as:

a : b ("a is related to b")

In Latin, "Ratio", the root word for our "reason", also means measure.

A Ratio is the measure of a difference.

A PROPORTION is a relationship between two ratios such as:

a : b :: c : d ("a is to b as c is to d"). This is called a *discontinuous proportion of 4 terms*.

Example: 2 : 4 :: 3 : 6 ("2 is to 4 as 3 is to 6")

There are several kinds of Proportions:

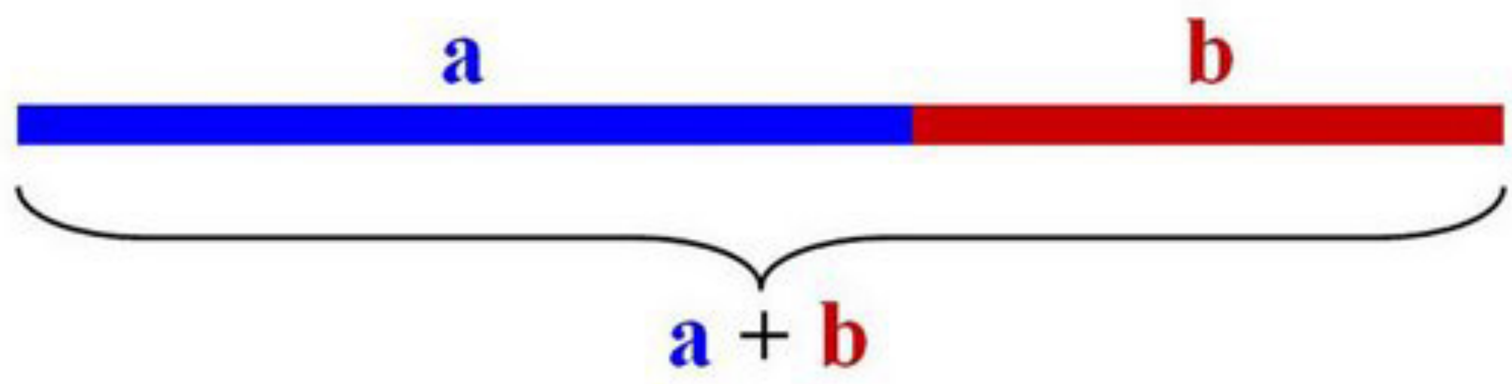
1. Discontinuous Proportion of 4 terms. See above.
2. Continuous Proportion of 3 terms: **a : b :: b : c**. In this proportion, the perceiver (b) forms the equivalency between observed differences or concordances (a and c), as we experience the world due to registering variations of the wave frequencies patterns pervading our sensory awareness.
3. Continuous Proportion of 2 terms. There is one, and only one, proportional division possible with two terms and getting us even closer to the sense of Unity - the Golden Proportion: **a : b :: b : (a + b)**

"The Smaller term (a) is to the Larger (b) as the Larger (b) is to the Sum of the Small and the Large (a + b)"

This is a 3 terms proportion constructed from 2 terms, a symbol of the mystical Trinity or Three that are Two that are One.

This is the ultimate goal of the proportional contemplation offered by Sacred Geometry.

SG104.2.4.1 Meet The Golden Ratio PHI (1)



$$\boxed{\frac{a+b}{a} = \frac{a}{b}}$$

The line is "cut unevenly" at the Golden Section such as the Whole a + b (Φ = 1.618…) is to the Longer a (= 1) as the Longer a is to the Shorter b (= 1/Φ = .618…), thus co-creating a cascade of harmony.

SG104.2.4.2 Meet The Golden Ratio PHI (2)

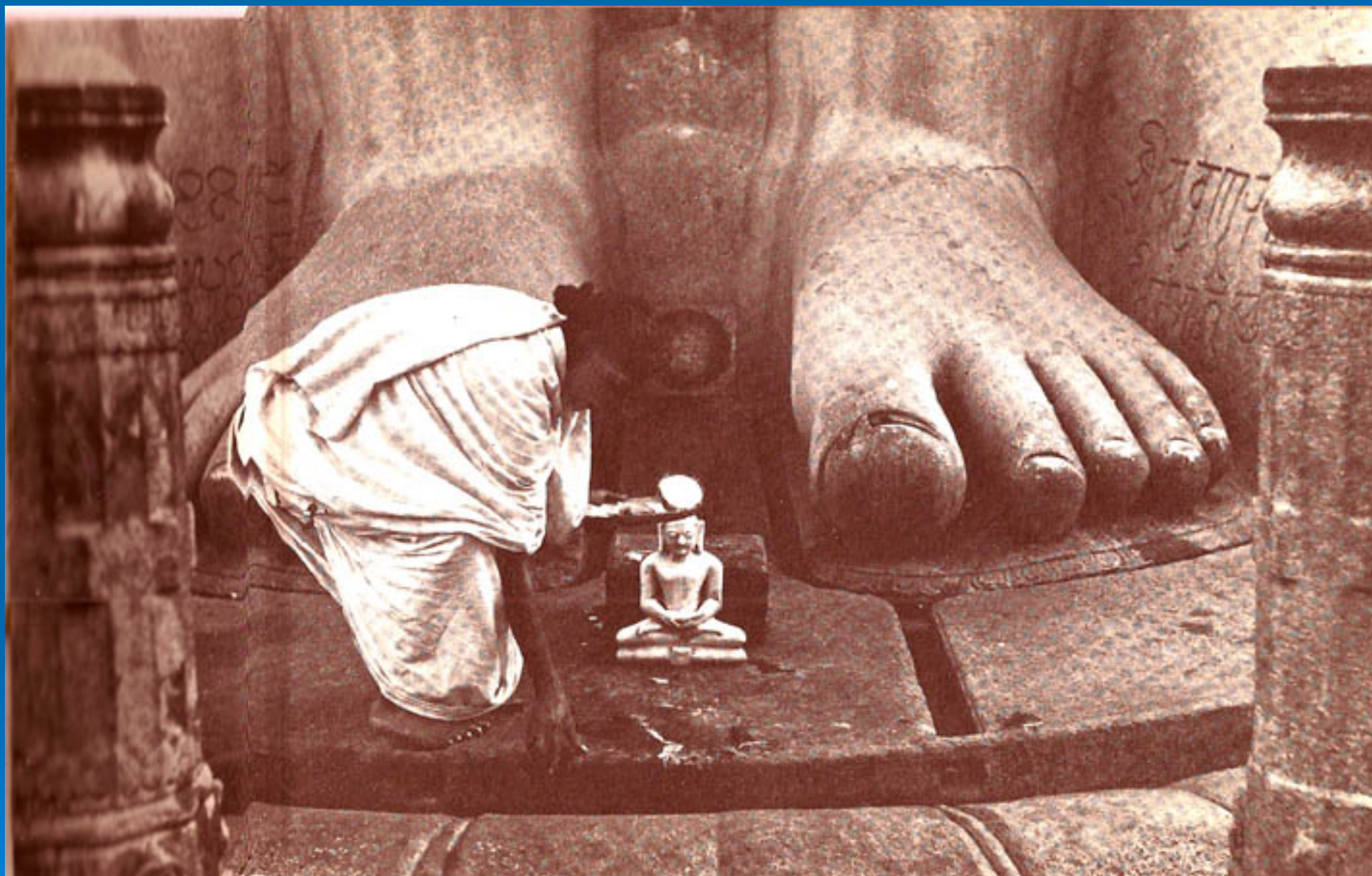


Image: Louis Frederic. *Tresors de l'Art des Indes*. Belgium. 1965.

The Small, the Large and the Whole…

SG104.2.4.3 Meet The Golden Ratio PHI (3)

PHI is NOT a number!!! PHI is a PROPORTION resulting from dividing a line into two unequal (but harmonically complementary) parts.

Numerically, PHI = 1.618033988…

PHI is an incommensurable number, an "irrational" number. It cannot be expressed as a simple whole integer. Yet its exact mathematical values show up in many Sacred Geometry constructions, such as the "sacred" Pentagram with a side to base ratio of PHI.

The unique property of PHI is that its values ($1, \Phi, \Phi^2, \Phi^3 \ldots$) are in both a multiplicative and additive relationship:

$$1 : \Phi :: \Phi : \Phi^2 \text{ (geometric multiplication)}$$

$$1 + \Phi = \Phi^2 \text{ (arithmetic addition)}$$

This property allows PHI to be the perfect mediating factor for organic growth - as opposed to inorganic nature normally using simple addition.

With PHI, waves can both add and multiply without losing their identity or "original nature". A unique achievement.

"The Golden Proportion is the most intimate relationship that proportional existence - the universe - can have with unity. For this reason, the ancients called it Golden."

(Robert Lawlor. Sacred Geometry, 1982.)

SG104.2.5.1 Maths of PHI (1) Uniqueness of PHI

PHI is unique in the universe: its values are *simultaneously* in an additive AND multiplicative relationship

$$\Phi + 1 = \Phi \times \Phi = \Phi^2$$

and also in a subtractive AND divisional relationship:

$$\Phi - 1 = 1 / \Phi$$

This is how nature creates growth as accretion and diminution.
Only the powers & roots of Phi satisfy this request of cascading harmony, and, in practice, the Fibonacci & Lucas Series provide a close approximation.

PHI (Ø)& Pi (π)

$Ø = 1.4\pi/e$

$\pi = 4/\sqrt{Ø}$

$\pi = 6/5\ Ø^2$

$1.2\ Ø^2 = 3.1416$

$Ø \times \pi = 5.08$

$Ø / \pi = .515$

Each PHI term is simultaneously the *sum* of the preceding two AND the *product* of the previous term multiplied by PHI

Example: $\Phi^4 = \Phi^2 + \Phi^3 = \Phi^2 \times \Phi^2 = \Phi^3 \times \Phi$

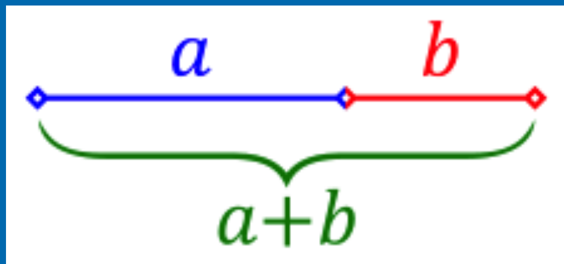
and also the *difference* between the two preceding ones AND the *result of the division* of the previous one by PHI.

Example: $1/\Phi^4 = 1/\Phi^2 - 1/\Phi^3 = 1/\Phi^3 \div \Phi$

$$\Phi = 1 + 1/\Phi \qquad \Phi = \sqrt{1 + \Phi}$$

$$\Phi + 1 / \Phi - 1 = \Phi^3$$

SG104.2.5.2 Maths of PHI (2)



Let us remember the classical definition of the Golden Section (continuous proportion of two terms):

"The Small part (b) is to the Large part (a) as the Large (a) is to the Sum of Small + Large (c = a + b)"

$$b : a :: a : c \quad \text{or} \quad b/a = a/c$$

Mathematically, a (large) = x and b (small) = 1, so we have (Step #1):

$$b/a = a/c \quad \text{or} \quad 1/x = x/x + 1$$

Multiplying both sides by x, we get $X^2 = x + 1$

Which is the quadratic equation $X^2 - x - 1 = 0$

Where $a = 1$, $b = -1$ and $c = -1$

Step #2:

Using the formulas for the two solutions

$x = -b + \sqrt{b^2 - 4ac} / 2a$ and $x = -b - \sqrt{b^2 - 4ac} / 2a$

We get:

$x_1 = (1 + \sqrt{1 + 4}) / 2 = (1 + \sqrt{5}) / 2 = 1.61803398...$

$x_2 = (1 - \sqrt{1 + 4}) / 2 = (1 - \sqrt{5}) / 2 = .61803398...$

X_1 is PHI (Φ) = 1.618…

X_2 is Phi (ϕ) = 1/Φ = .618…

SG104.2.6 PHI Major & Phi Minor

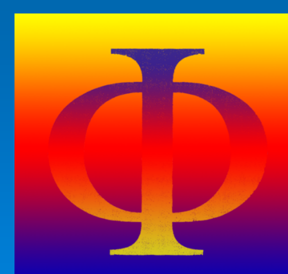
On the left is
"PHI" or Φ
On the right is
"PHI Minor" or 1/Φ.

They are Twin Souls
and play resonant music

Throughout these
Sacred Geometry modules,
we will keep this convention:

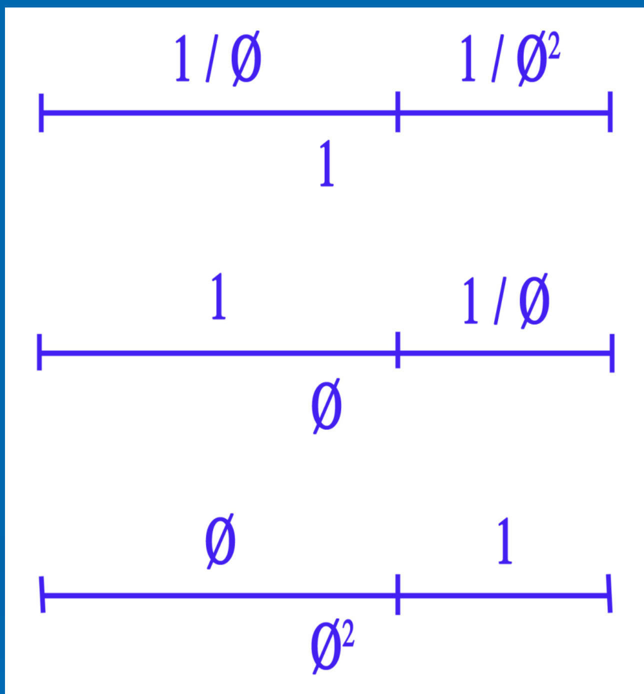
Capital letter = PHI = Φ = 1.618…

Lower case letter = Phi = ϕ φ
= .618…



Unity/Oneness (1) is dancing with PHI. "1" can appear as the Whole, the Small Part or the Large Part. And Phi can appear as Ø, $Ø^2$, 1/Ø, $1/Ø^2$ or any other multiple or sub-multiple that are in PHI proportion.

SG104.2.7.1 Figuring PHI (1)



… $1 / Ø^2$ 1 / Ø 1 Ø $Ø^2$ …

How to practically figure out PHI?

By intuition: the Phi Ratio is inherent to natural perception as we are basically built with it through our DNA. Natural perception only needs to be reawakened so that the PHI Ratio can be intuited accurately. An artist who understand sacred Geometry will quickly develop a pretty good intuitive sense of where to place the PHI proportion and compose a piece accordingly. It "happens by itself". It is inner knowledge.

Using a calculator: If you need to be very accurate, use your calculator. Remember though that the numerical value is a convention because PHI is a relationship and not a finite number. Even though you rely on a calculator and a graduated ruler, try to estimate the correct ratio.

Keep handy the following table, play with the basic equations and notice the reciprocal harmonies:

$1 / Ø^2 = .382$ $1 / Ø = .618$ $Ø = 1.618$ $Ø^2 = 2.618$ $Ø^3 = 4.236$

$$\Phi = 1 + 1 / \Phi = 1 + .618 = 1.618 \quad \Phi + 1 = \Phi \times \Phi = (1.618)^2 = 2.618$$

SG104.2.7.2 Figuring PHI (2)

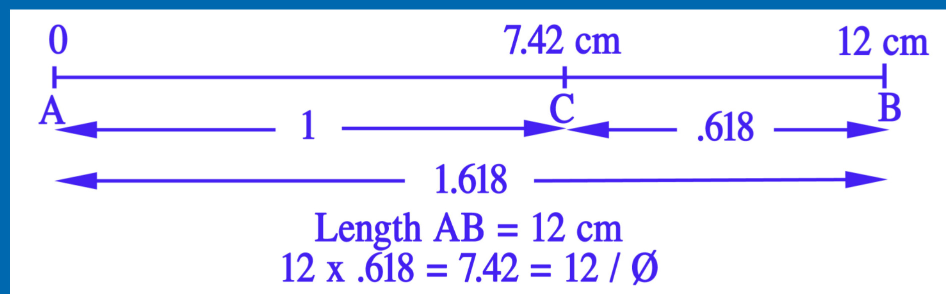
Practically, it is very simple. All you need to remember are the two values:

PHI (Φ) = 1.618

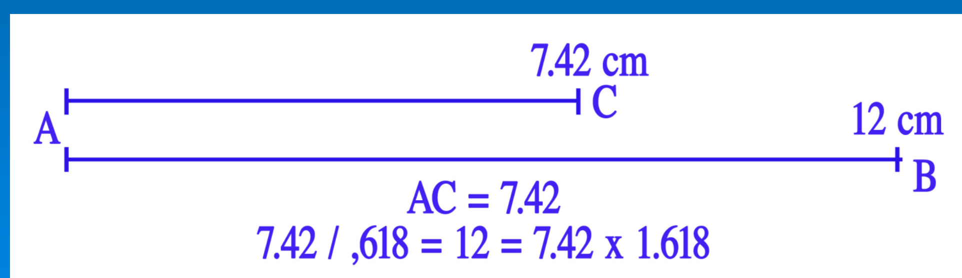
phi (1/Φ) = .618

With these 2 values you can figure out the Golden Ratio up and down the scale, ad infinitum.

Case #1: IF you have a length AB and you want to find it Golden Cut,
MULIPLY it by Phi = .618
or DIVIDE it by PHI = 1.618



Case #2: IF you have a length AC and you want to find its next "Larger Whole" or harmonic PHI node,
DIVIDE it by Phi = .618
or MULTIPLY it by PHI = 1.618



SG104.2.8.1 A Fraction of Oneself (1)

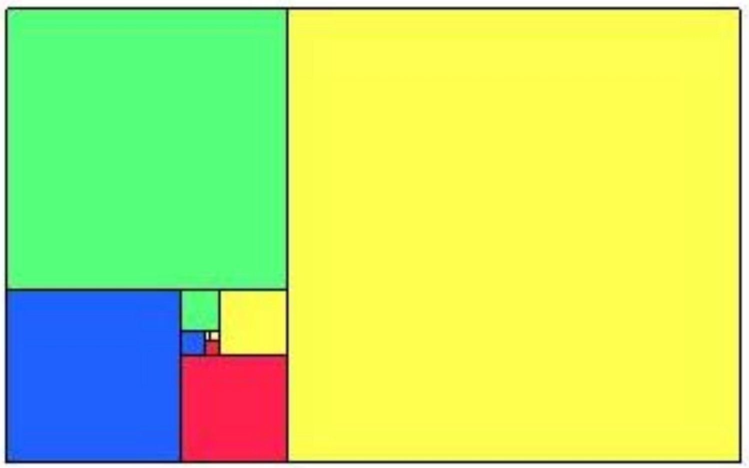
Mathematical formulas for PHI are exclusively composed of the number One. They show an endless chain of parts that both resemble each other and the whole, as in holograms.
PHI is a mathematical hologram or fractal.

$$\varphi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}.$$

↑ The √ formula of nested radicals is a continuous recursive rooting process seeking and navigating all possible rootlets and gathering them to go back to the Primordial Root-Source.

Each formula is Unity relating to itself.
Oneness embedded with itself.
Unity Dancing with Itself.
This resembles the traditional mystic descriptions about the "Divine Looking at Itself"
or the seeker "embracing his/her own Self".

Like a cosmic hologram,
PHI mediates the essential interconnectedness PART-WHOLE.
The part contains the code for the Whole.
The seed contains the whole tree.



decimal expansion of $\phi =$ 1.61803398874989

convergent of continued fraction = 1.61803278688525

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}}}}}}}}}$$

↑ The fraction (fractal) formula is a continuous matrix process encompassing all possible branches & florets and linking them to the same trunk, under the same roof.

SG104.2.8.2 A Fraction of Oneself (2)

$$\phi = 1 + \frac{1}{\phi} = 1 + \cfrac{1}{1 + \cfrac{1}{\phi}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \phi}}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + ..}}}}$$

$$1 + \frac{1}{1} \rightarrow 2$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1}} \rightarrow \frac{3}{2}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}} \rightarrow \frac{5}{3}$$

These partial continued fractions lead up to the infinite continued fraction representation of PHI

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} \rightarrow \frac{8}{5}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}}} \rightarrow \frac{13}{8}$$

Note the Fibonacci Numbers

SG104.2.9 PHI & Five

Phi and the number Five have a special relationship:

The Pentad (Fiveness or Penta-symmetry) reoccurs throughout all geometric constructions of the Golden Ratio, Golden Rectangle/Spiral, Pentagram/Pentagon and the PHI growth process all over nature.

Mathematically, Phi = $\Phi = (\sqrt{5} + 1)/2 = 1.618\ldots$

PHI can also be expressed by various equations involving the number 5:

$$\Phi = \sqrt{(5 + \sqrt{5}) / (5 - \sqrt{5})} = \sqrt{(7.236 / 2.764)} = \sqrt{2.618}$$

$$(\Phi + \phi)^2 = (1.618 + .618)^2 = 5$$

$$\Phi + 1 = 2.618 = .5236 \times 5 = (\sqrt{5} + 3) / 10$$

$$\Phi = 2\cos(\pi/5) \qquad \pi = 5 \arccos(.5\Phi)$$

$$\pi = 6/5\, \Phi^2 = 3.1416 = 1.2\, \Phi^2 \qquad \Phi \times \pi = 5.0830$$

SG104.2.10 PHI in Trigonometry

PHI and Phi show up extensively in trigonometric functions, corresponding to the fact that the length of the diagonal of a regular pentagon is Φ times the length of its side, and similar relations in a pentagram.

$$\Phi = 1 + 2 \sin (\pi/10) = 1 + 2 \sin 18^{o}$$

$$\Phi = 1/2 \csc (\pi/10) = 1/2 \csc 18^{o}$$

$$\Phi = 2 \cos (\pi/5) = 2 \cos 36^{o}$$

$$1/\Phi = \phi = 2 \sin 18^{o}$$

$$1/\Phi = 2 \cos 72^{o}$$

Note: there are many trig forms of these angles as they grow in multiples of 18º, except those that would give zero or multiples of 90º.
Example: $2 \cos 108^{o} = 1/\Phi$

$$-(\Phi/2) = \sin 666^{o}$$

$$\sin (\pi/2 - i \ln \Phi) = 1/2i$$

Look at this one: $\sin .000018^{o} = .000000314159265 4\ldots$ (values of π)

SG104.2.11.1 The Roots & Powers of PHI / Tables

$\Phi = \Phi = 1.618$

$\Phi^2 = \Phi + 1 = 2.618$

$\Phi^3 = 2\Phi + 1 = 4.236$

$\Phi^4 = 3\Phi + 2 = 6.854$

$\Phi^5 = 5\Phi + 3 = 11.09$

$\Phi^6 = 8\Phi + 5 = 17.944$

$\Phi^7 = 13\Phi + 8 = 29.034$

$\Phi^8 = 21\Phi + 13 = 46.978$

$\Phi^9 = 34\Phi + 21 = 76.012$

$\Phi^{10} = 55\Phi + 34 = 122.99$

$\Phi^{11} = 89\Phi + 55 = 199.002$

$\Phi^{12} = 144\Phi + 89 = 321.992$

$\Phi^{13} = 233\Phi + 144 = 520.994$

…

Notes:

1. These are numerical values with infinite decimals…
2. See the re-occurrence of Fibonacci numbers

$1/\Phi = .618$

$1/\Phi^2 = .382$

$1/\Phi^3 = .236$

$1/\Phi^4 = .145$

$1/\Phi^5 = .090$

$1/\Phi^6 = .055$

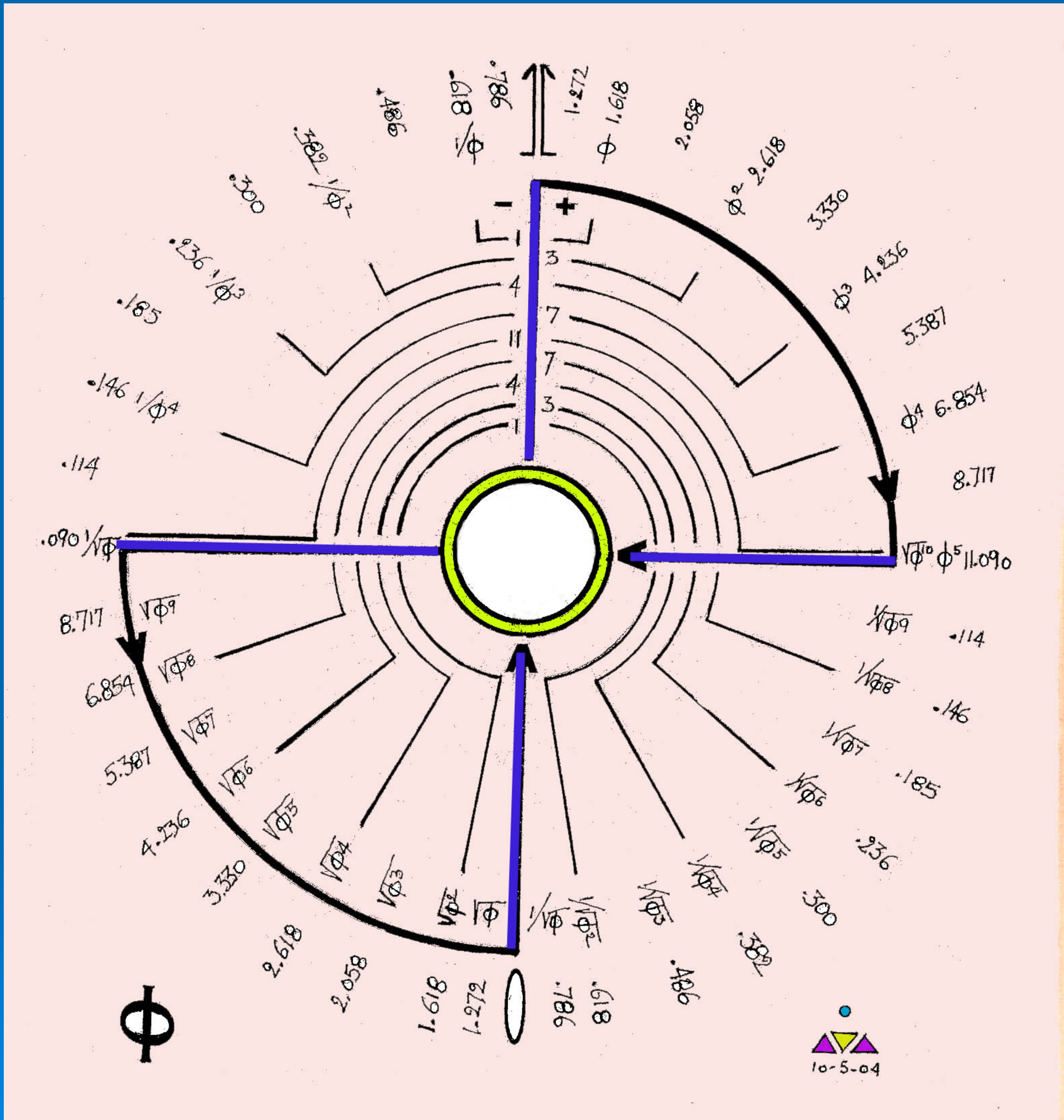
$1/\Phi^7 = .034$

$1/\Phi^8 = .021$

$1/\Phi^9 = .013$

$1/\Phi^{10} = .008$

…



SG104.2.11.2 Roots & Powers / Wheel

This is the Golden Ratio Tree or PHI Spiral Vortex

Could it be that the Golden Ratio Mandala-Tree is the old "Tree of Knowledge" uprooted from humanity's consciousness as the human DNA was scrambled in the course of this earth's history?

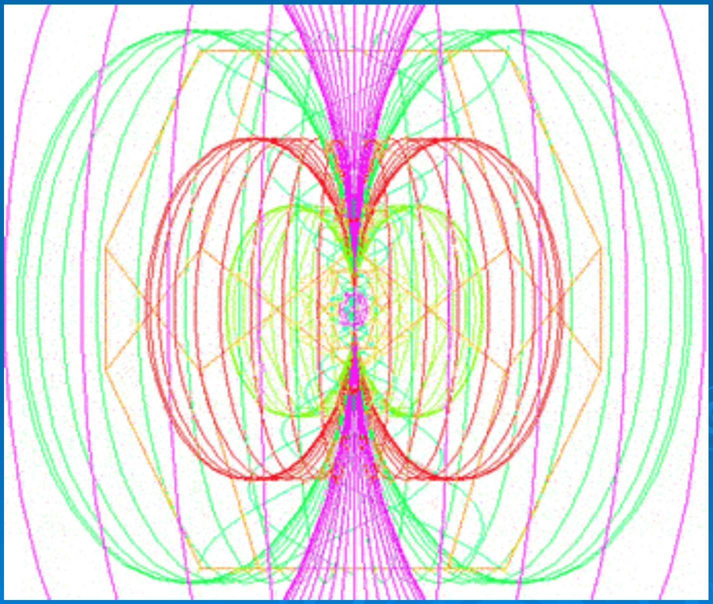
Maypole
Jacob's Ladder
Tree of the World
Inter-Dimensional Vortex

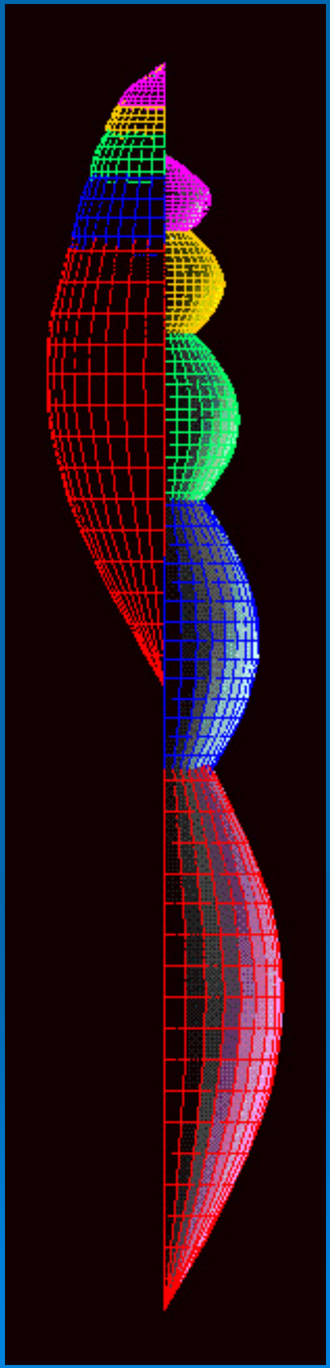
← Mandala Wheel of the Powers & Roots (multiples & submultiples) of the Golden Number $\Phi = 1.618$

SG104.2.12 The Holographic Nature of PHI

This love marriage in heaven between the Small, the Large and the Whole is the simple & beautiful universal answer to the question of the One and the Many: they are One. And we can have it all: be simultaneously the material part in a human body and the infinite Grand Spirit Source, the laborious addition and the exponential multiplication.
In between, we are playing octaves upon octaves of fractal harmonic music. This *Music of the Spheres* is navigating instantly / non-locally the scales of PHI overtones & undertones and can be heard as the Song of the Universe.

Explaining that a structural self-similarity binds the hidden "*implicate order*" to the manifested "*explicate order*", physicist David Bohm states:
"The essential feature of quantum interconnectedness is that the whole universe is enfolded in everything, and that each thing is enfolded in the whole".
In between comes PHI…





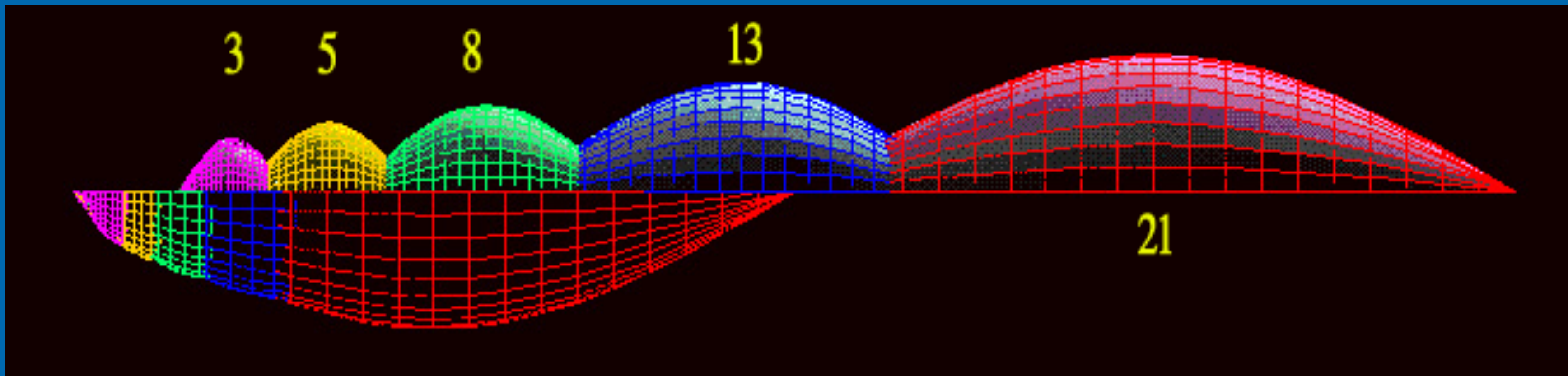
SG104.2.13 PHI Dating Among Waves

In the ordinary universe, when waves meet, all sorts of things can happen: they can ignore each other, they can glance at each other and proceed on their way or gently brush up… they can dislike each other to the point of repulsion or just negative interference… or any other relationship scenario that dissipates energy and brings about small or big chaos… OR they can enter in"positive interference"… and here again there could be several cases. The best case of course is "perfect coupling". When waves merge into each other to form a third unit that resemble and continue them…

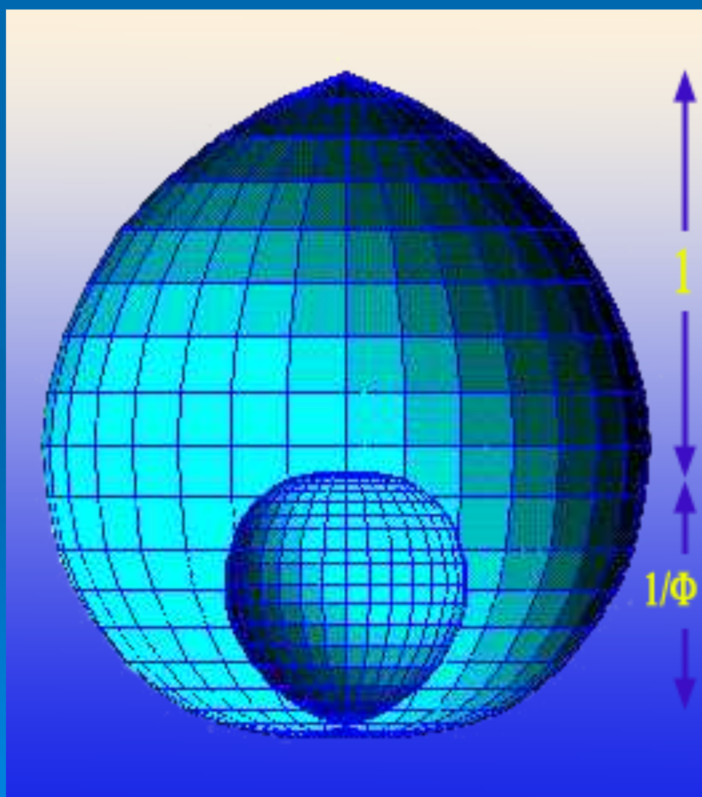
But this can only happen in fairy tales and PHI universes…

In a PHI universe, when waves meet, they are, naturally & instinctively, attracted to both add and multiply, that is engage in two simultaneous levels of relationship… a much richer honeymoon here…
They can (and will) add up their being and possessions to form a new unit and they can self-replicate as themselves and as the new unity… in brief, they can marry, have children and be forever happy… and aware to be happy… The perfect love story is enacted on all levels of the universe: wave meeting wave… and PHI-guiding entire populations of waves…

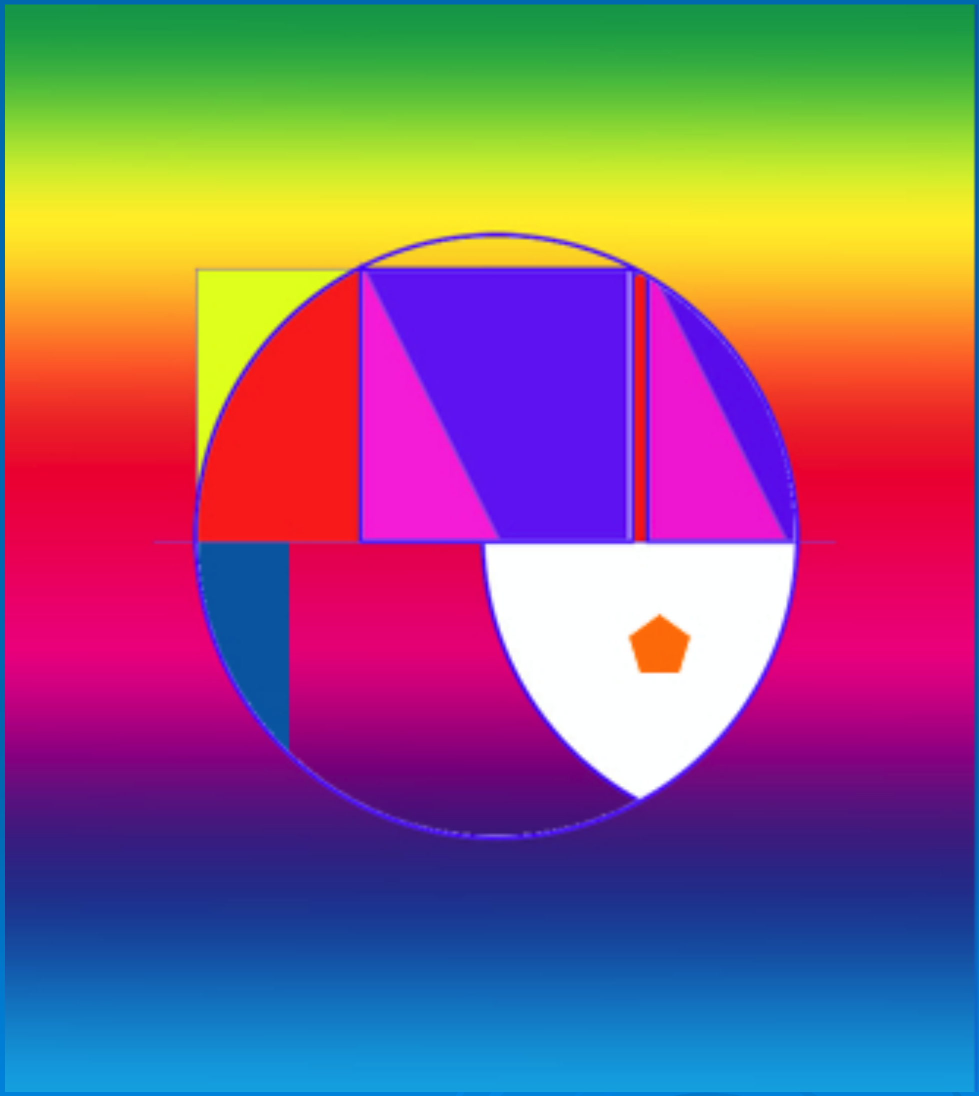
That is why the PHI path is called the Golden Path or yellow-brick road and why a PHI home is called the Golden Nest: it allows waves to do what they, like humans, love to do most: be happy, i.e. add and multiply in the harmony of Oneness…



[◆SG107] SG104.2.14 **PHI in 3D**



SG104.3 Chapter 3. Basic PHI Constructions



This chapter presents traditional PHI constructions based on simple geometry figures: square, triangle, circle…

The construction/tracing steps are explained but not the geometric proofs, as the purpose is to train budding sacred geometers to develop an overall *intuitive sense* of the Golden PROPORTION rather than give them specific linear reasoning.

You are encouraged to pick up your (grid) paper, pencil, compass & ruler and retrace the constructions that appeal to you. Then let the artist-in-you add colors, symmetries and various personal touches…
To this effect, we sometimes include PhiArt designs created by Aya and based on some of these Sacred Geometry constructions.

ENJOY sacred doodling!

PhiArt A1

SG104.3.1.1 PHI - Square (1)

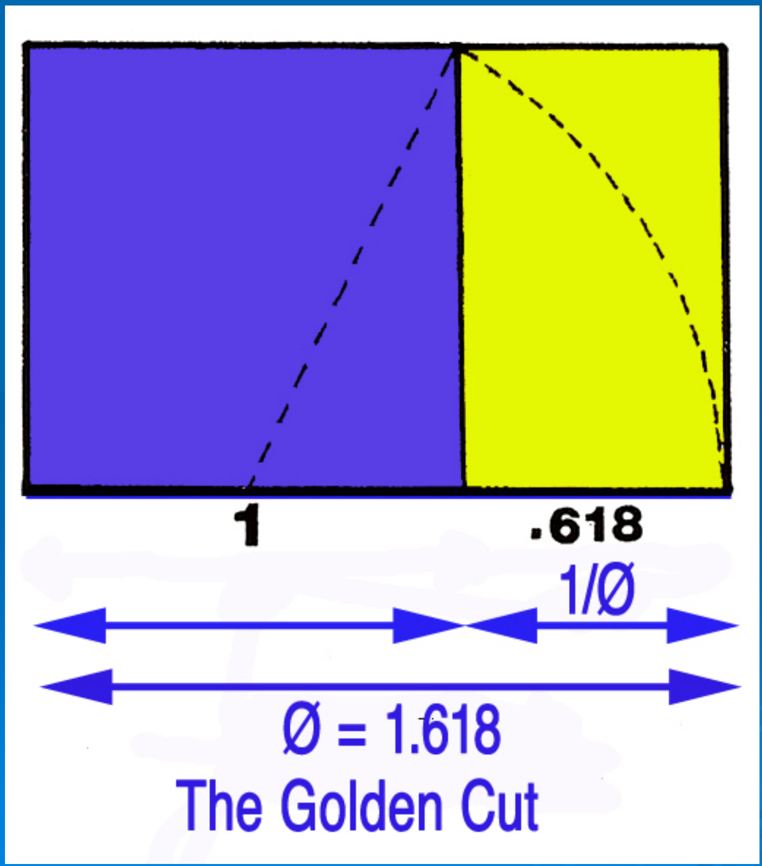
Here is the classic and simplest way to construct the Golden Section or Golden Cut PHI on a given base line:

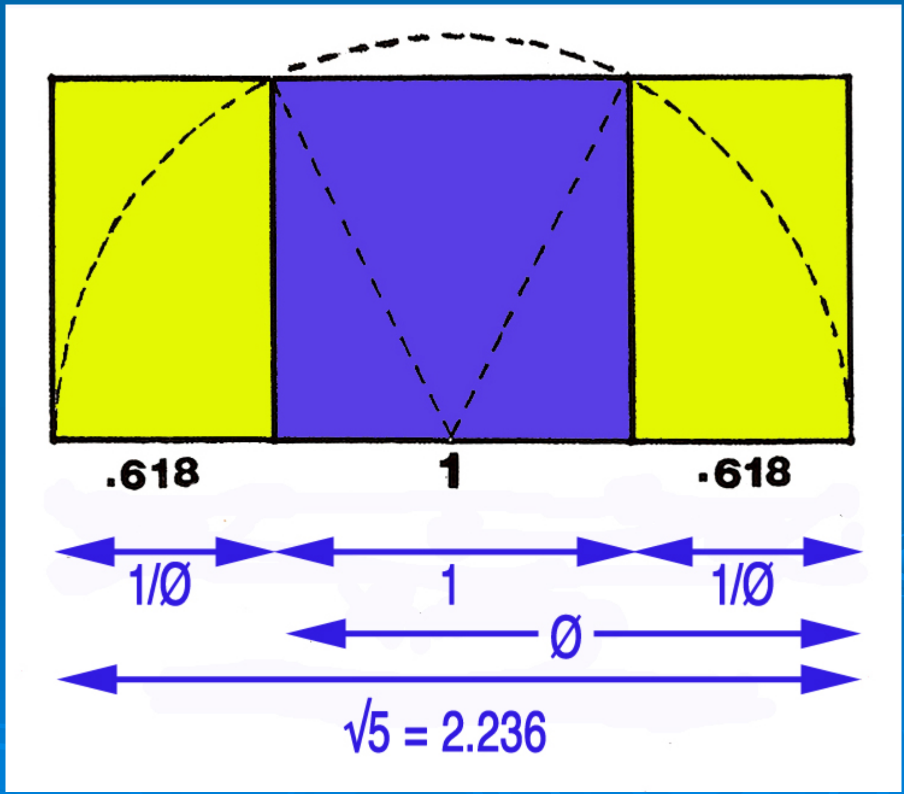
Step #1: Start with a square of sides 1 x 1 (in blue below)

Step #2: From the mid-point on the bottom side of the square extend a diagonal to the top right corner of the square (dotted line)

Step #3: With compass, draw an arc down to the base line. This is the Golden Cut. Yellow length = 1/Φ = .618

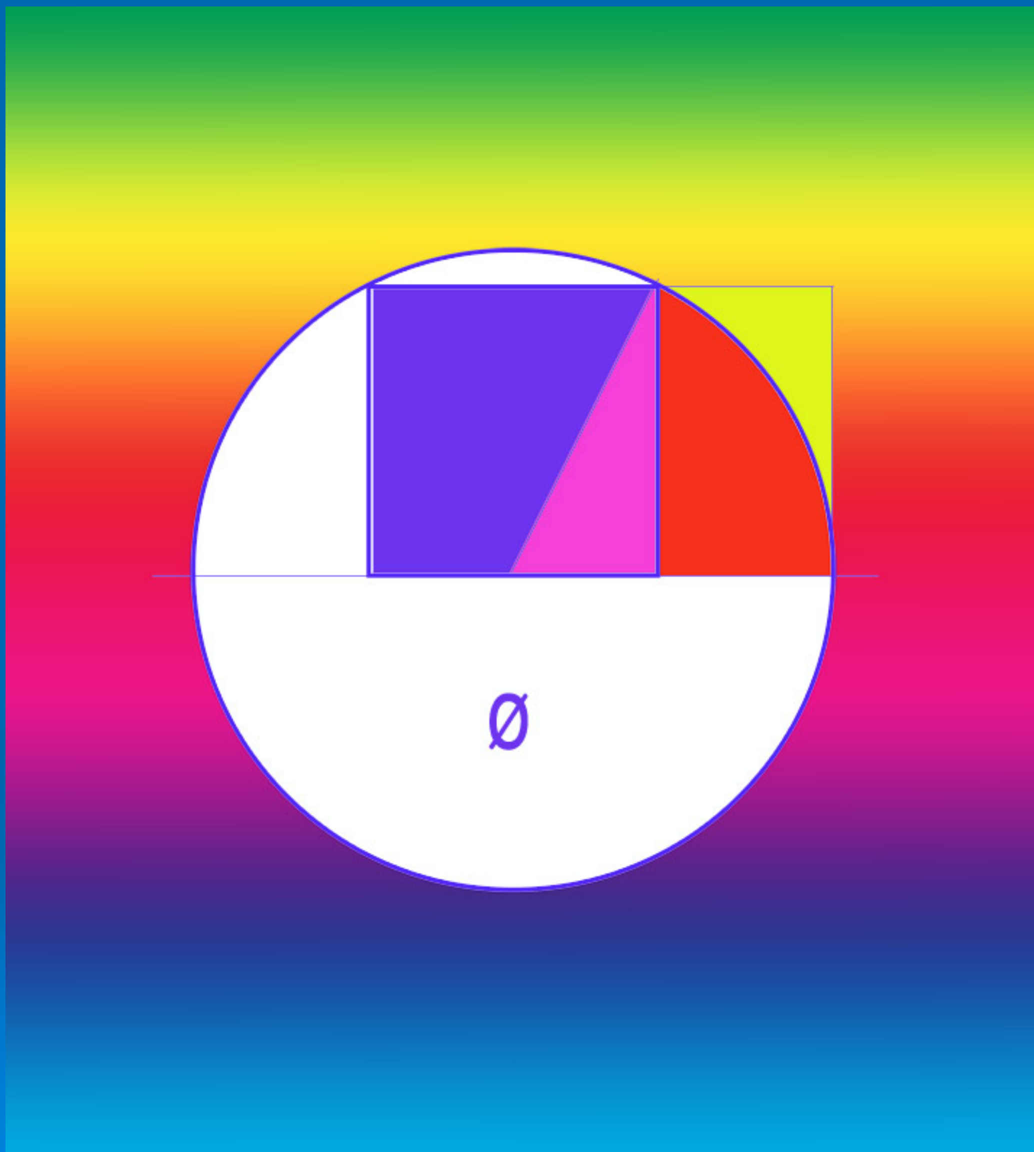
Step #4: Complete the yellow rectangle. Blue square + yellow rectangle = Golden Rectangle (base = Φ = 1.618)



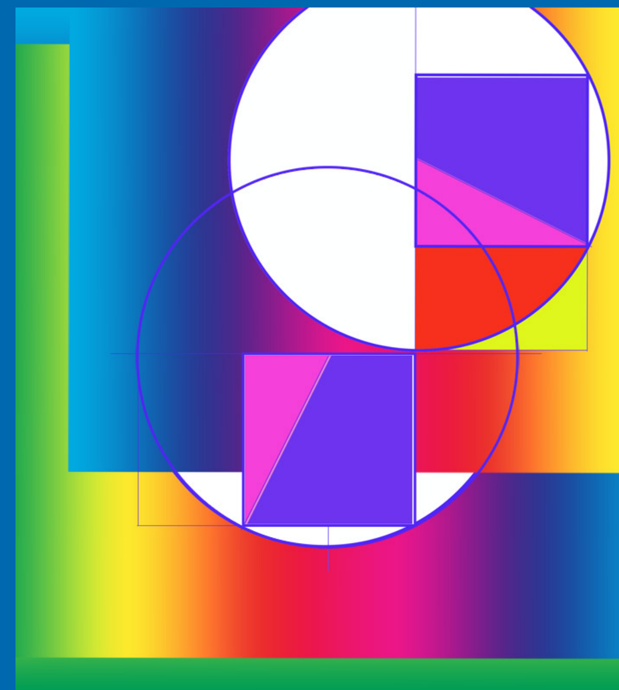


The Golden Cut to the right and to the left

SG104.3.1.2 PHI - Square (2)



PhiArt A2



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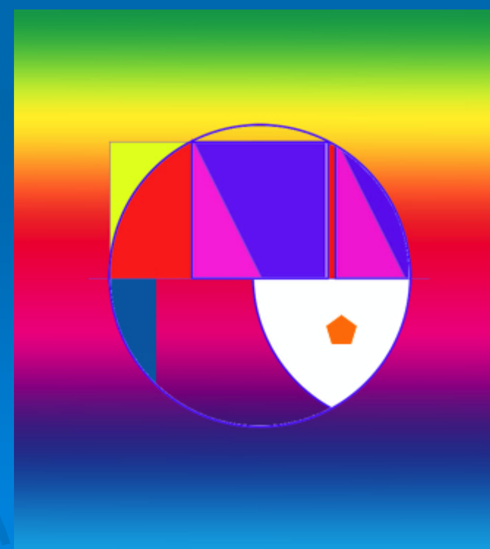
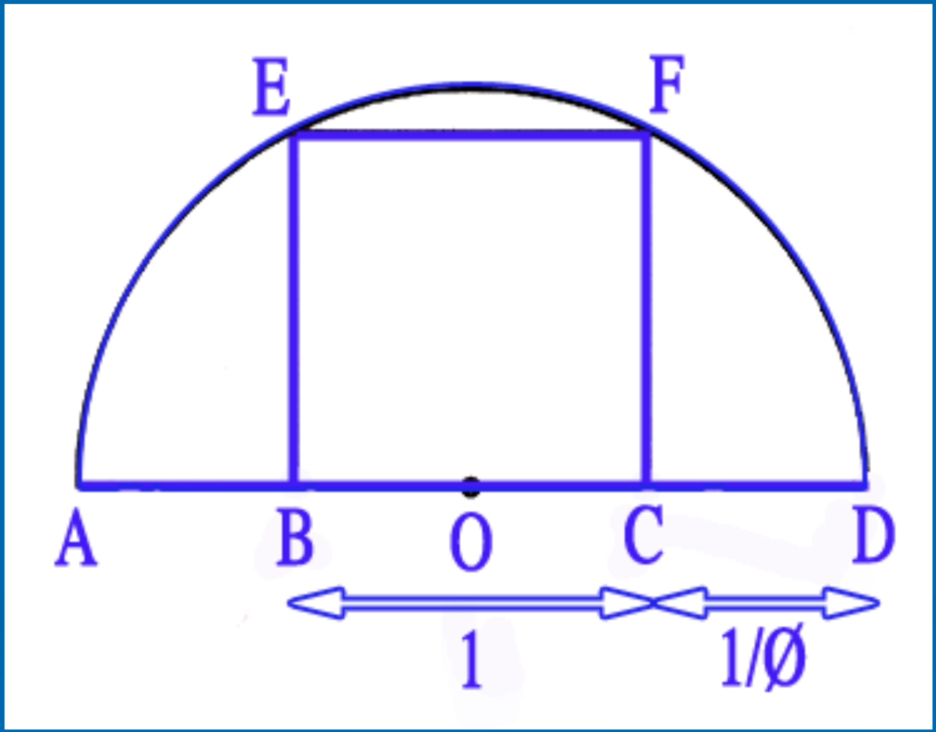


Φ

SG104.3.1.3 PHI - Square (3)

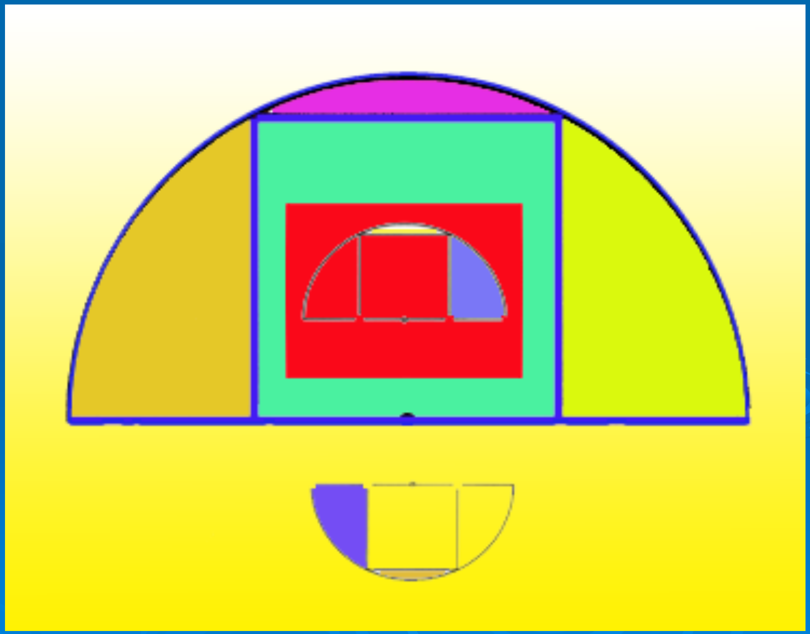
The same PHI construction based on the square and presented in an even simpler way…



Step #1: Construct a square BEFC with sides 1.

Step #2: Mark the midpoint O on BC.

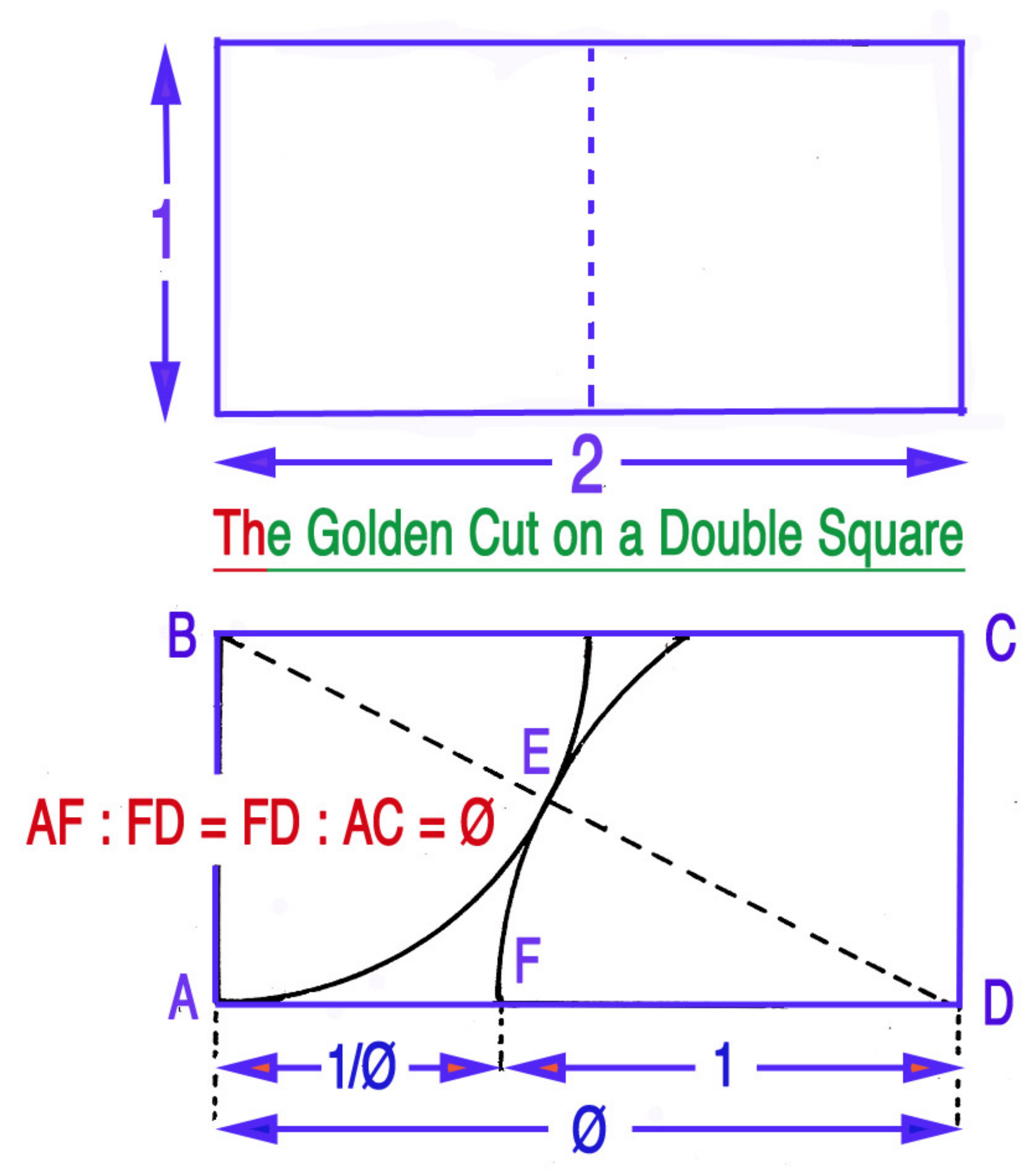
Step #3: With compass, trace a semicircle with radius OA and connecting with the square in E and F.



PhiArt A4

BC = 1
CD = AB = 1/Φ = .618
BD = AC = Φ = 1.618

SG104.3.2.1 PHI - Double Square (1)



The double square is a traditional temple sacred geometry plan.

[◆SG207]

Step #1: Construct a Double Square i.e. a rectangle with sides 1 and 2.

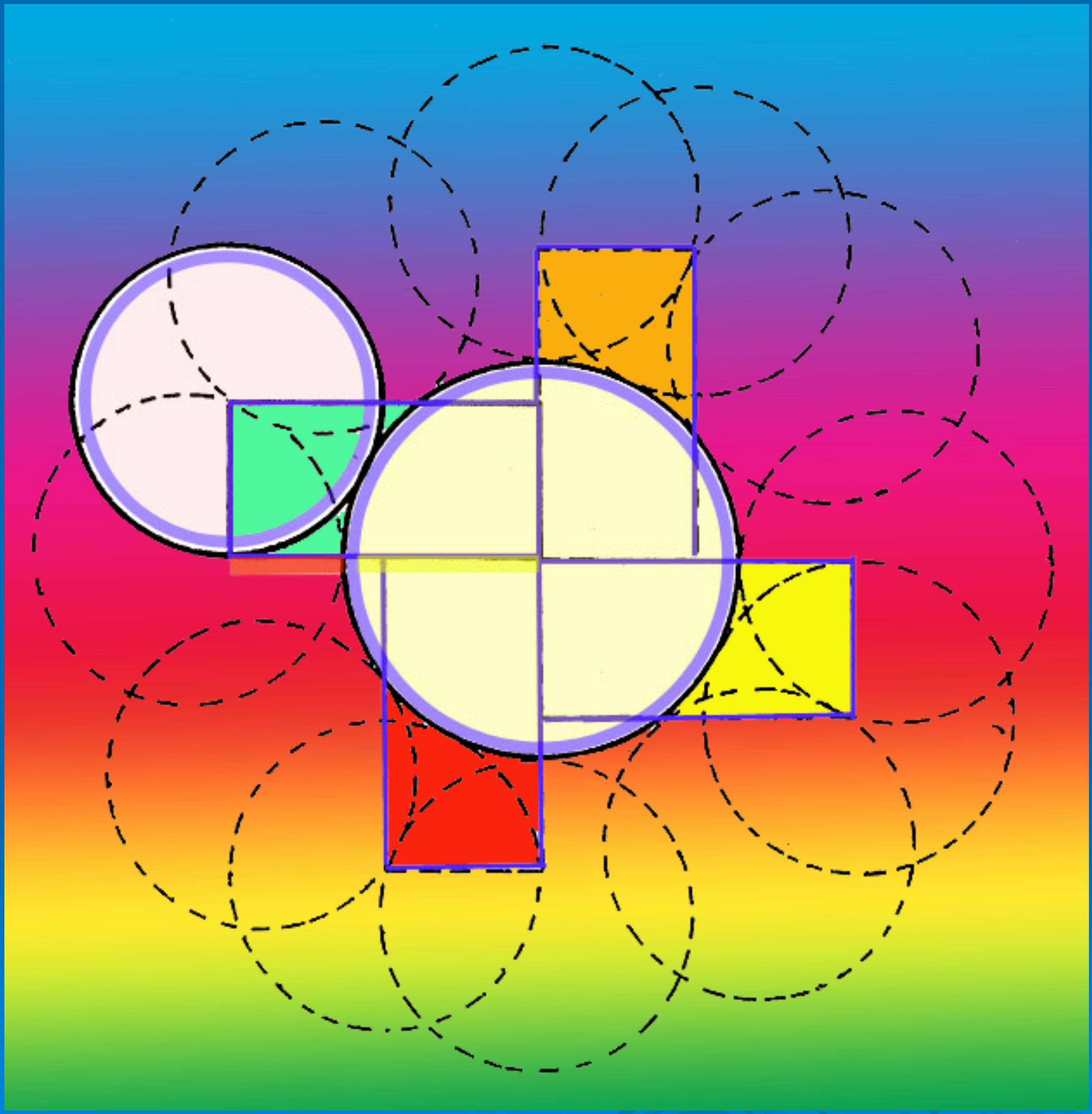
Step #2: Trace the diagonal BD (which equals V5).

Step #3: With center B and radius BA, trace an arc cutting BD in E.

Step #4: With center D and radius DE, trace another arc cutting AD in F.

DF = 1 FA = 1/Φ = .618

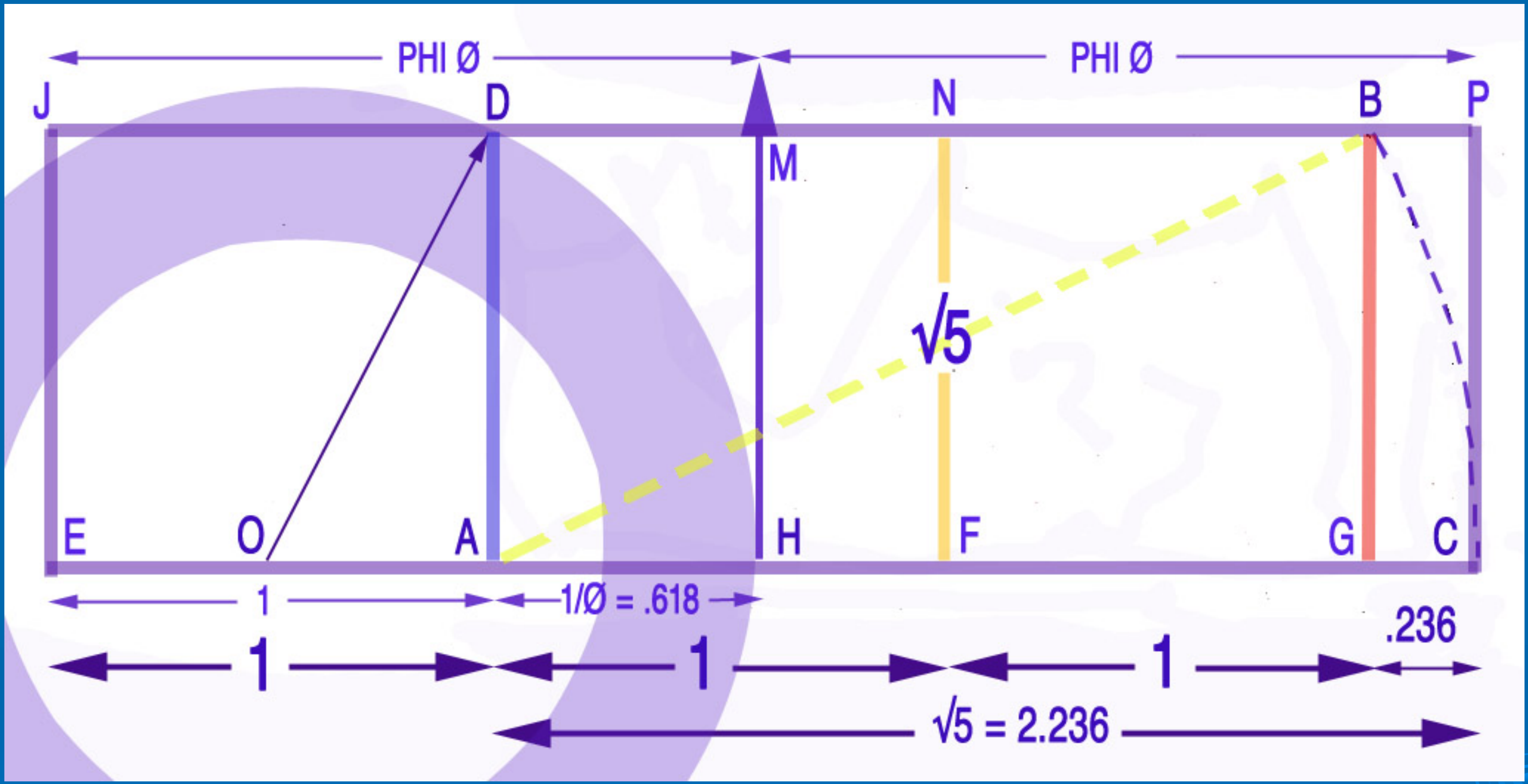
AD = Φ = 1.618



SG104.3.2.2 PHI - Double Square (2)

← Repeating the circles from the preceding figure and "*mandalizing*" the PHI construction, we get *mandalic symmetries* for the double square.

SG104.3.2.3 PHI - Double Square (3)



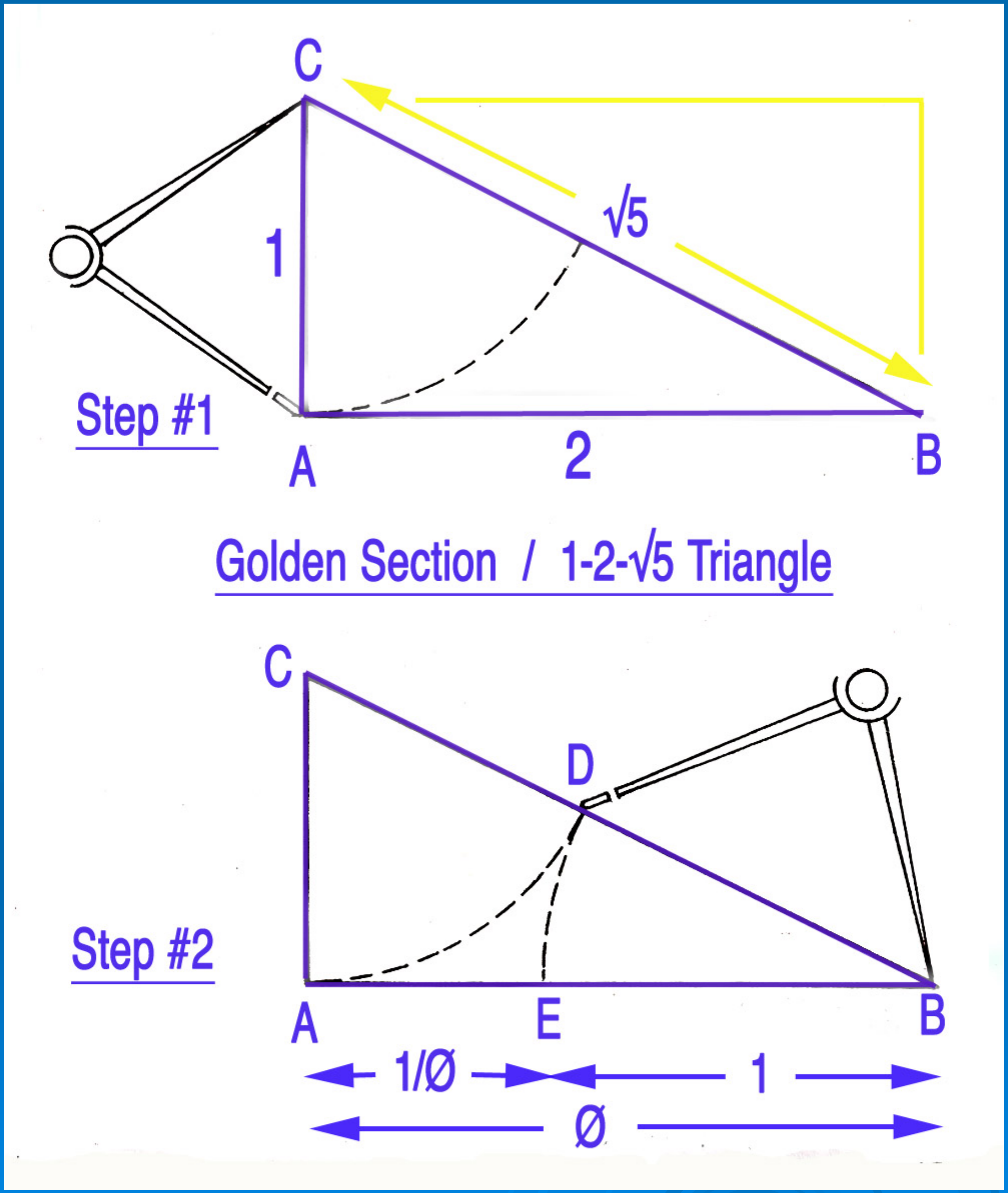
The diagonal AB (in yellow) of the double square ADBG is √ 5 = 2.236. With compass at center A, draw an arc to base line at C. AC is √ 5 as well. EA is 1. So the whole base line EC is √ 5 +1 = 2.236 + 1 = 3.236.

$(\sqrt{5} + 1) / 2 = 3.236 / 2$ is the numerical solution for PHI $= \Phi = 1.618$

$EH = HC = JM = MP = \Phi \qquad .236 = 1 / \Phi^3$

EJMH & HMPC are Golden Rectangles with sides 1 & Φ

(This is verified by the classic Golden Cut construction from radius OD brought to point H)



SG104.3.3.1 PHI - Triangle (1)

This is the same construction as the double square but tracing only half of it i.e. the right triangle with sides 1 and 2 and hypotenuse $\sqrt{5}$. This construction is attributed to Heron of Alexandria (10-70 CE).

Step #1: Construct the triangle ABC, with sides 1 and 2. Draw an arc from C, with radius CA, intersecting the hypotenuse in D.

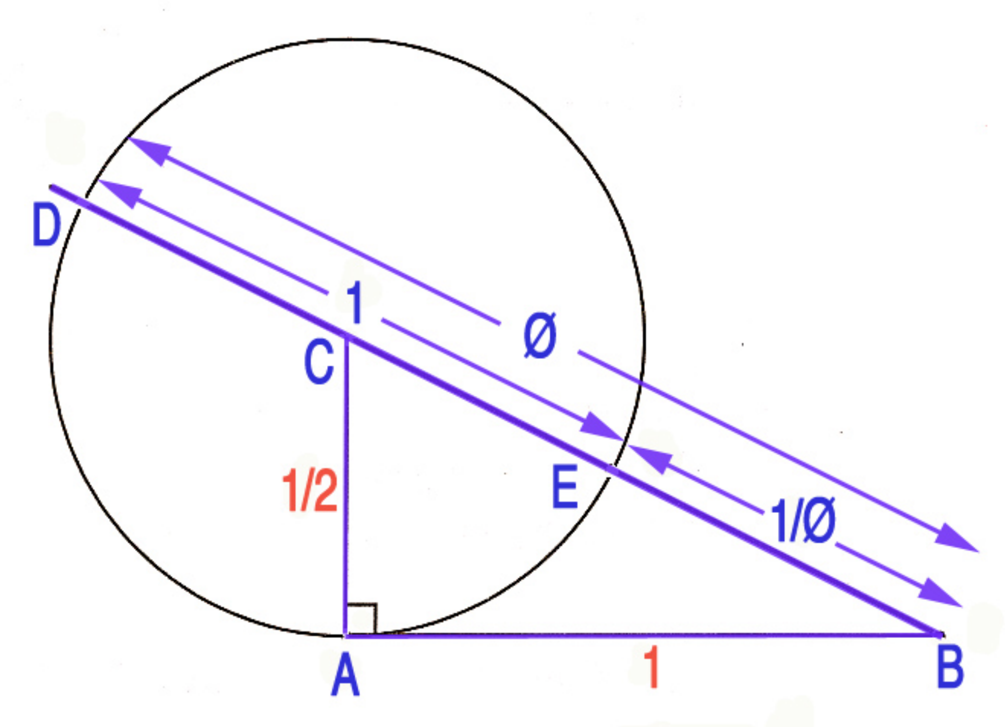
Step #2: Draw another arc from B, with radius BD, intersecting AB in E.

E marks the Golden Section point on AB such as:

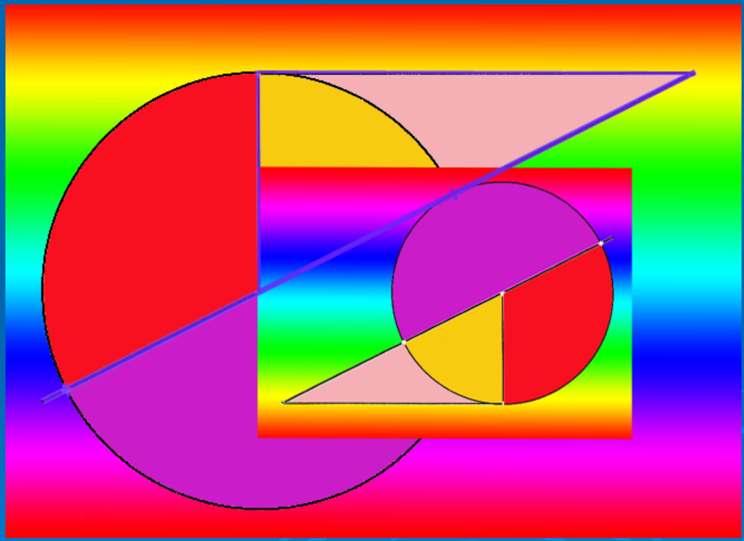
$$EB = 1, AE = 1 / \Phi$$

$$AE / EB = EB / AB = \Phi$$

SG104.3.3.2 PHI - Triangle (2)



← On the same right triangle (with a base twice the length of the side) but here scaled down, we can construct another Golden Section point, this time on the hypotenuse.



PhiArt A6

Step #1: Construct the right triangle ABC with base = 1 and side = 1/2.

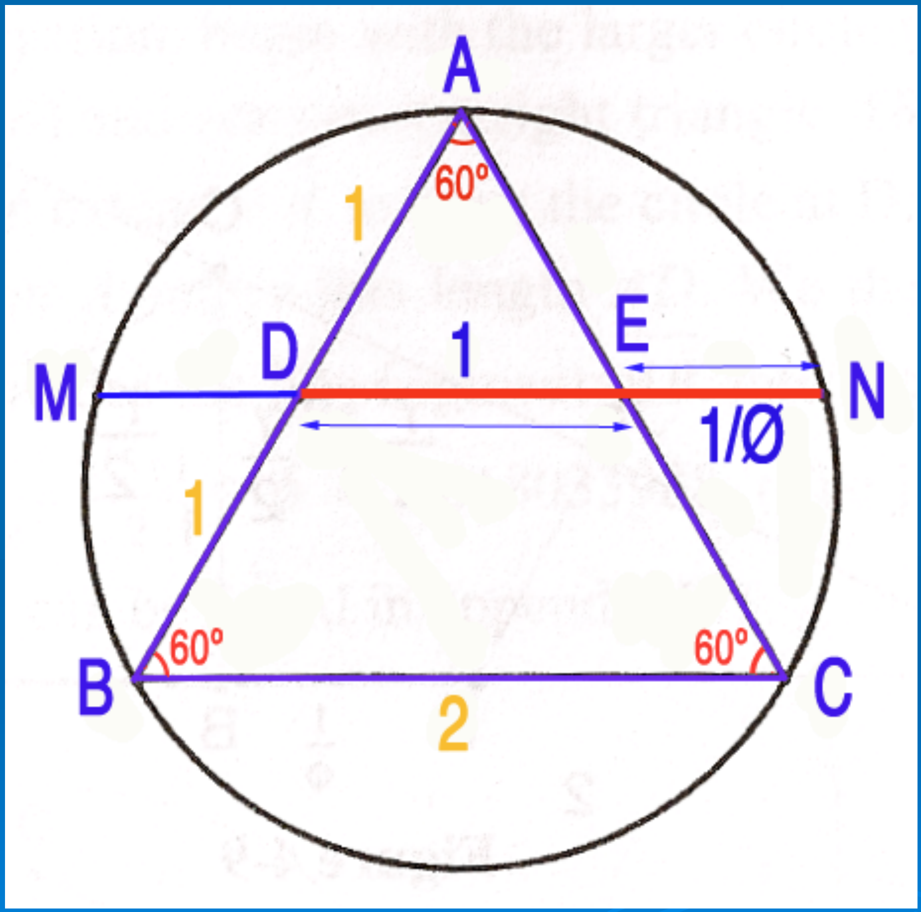
Step #2: From C and with radius CA, draw a circle cutting BC in E.

Step #3: Extend BC until it cuts the circle in D.

E is the Golden Section of DB, so that:
DE = 1, EB = 1/F and DB = F
EB / ED = ED / DB

SG104.3.3.3 PHI - Triangle (3)

And here is another PHI construction based on the equilateral triangle inscribed in a circle. It is very simple and elegant.



DE = 1 EN = 1/Φ DN = Φ

Step #1: Trace an equilateral triangle and inscribe it in a circle.

Step #2: Through the mid-point of each of the two sides, trace a line that intersect the circle.

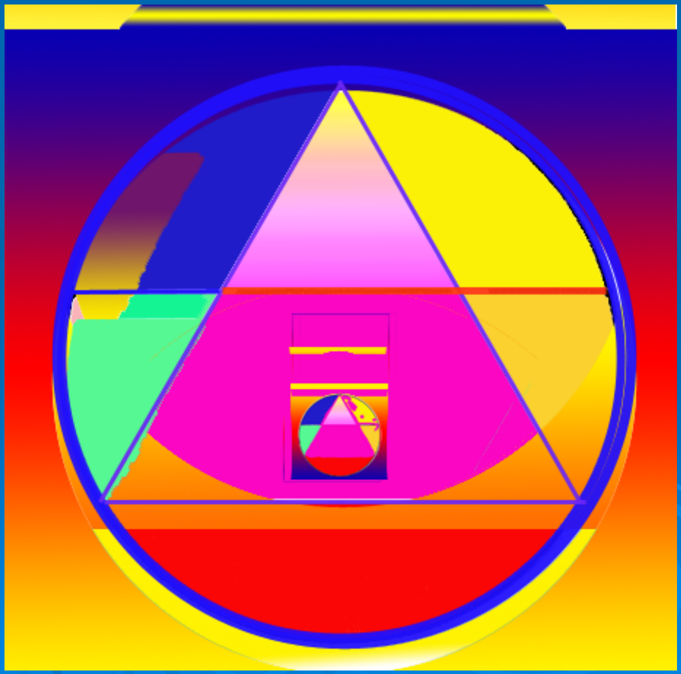
Step #3: Using the relationship stating that the products of the segments of two intersecting chords of a circle are equal, we have:

$$AE \cdot EC = ME \cdot EN$$

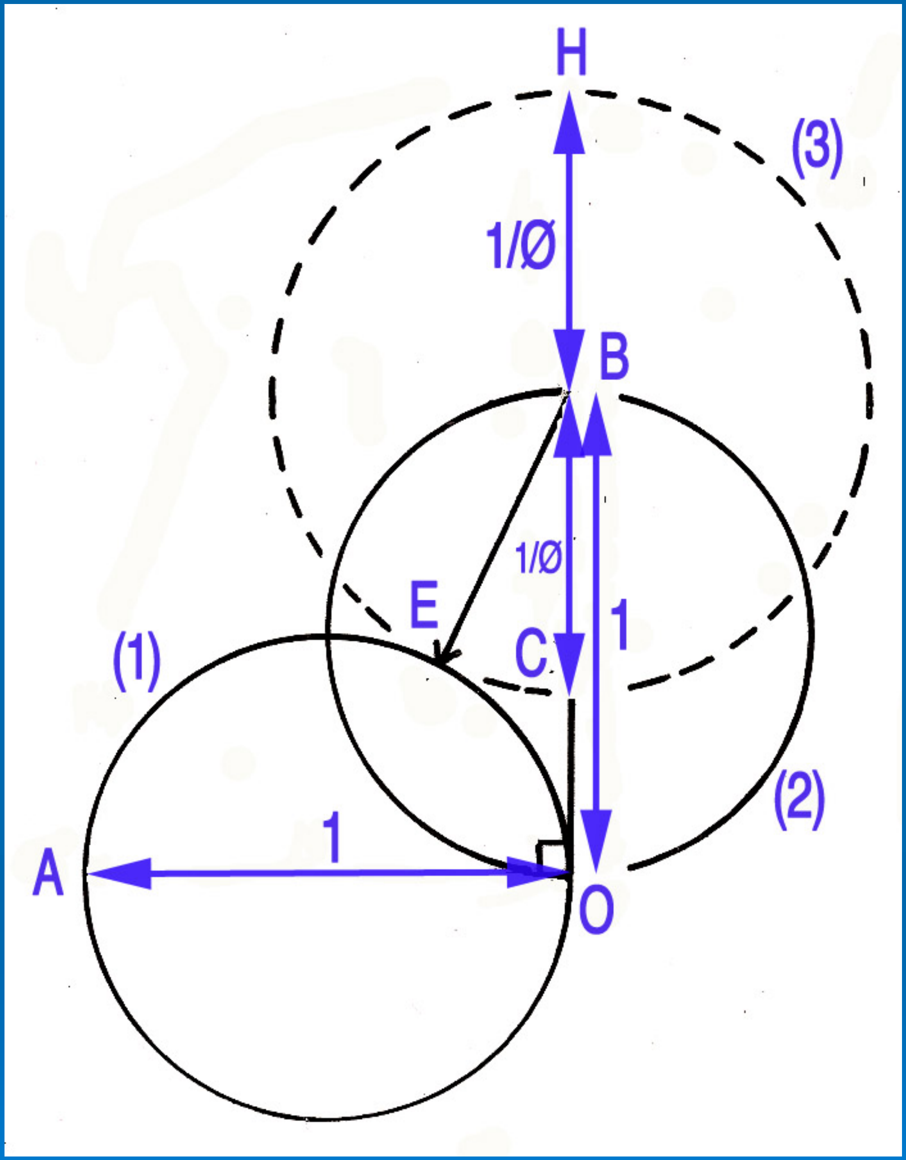
$$1 \times 1 = (x + 1)x$$

$$x^2 + x - 1 = 0$$

$$x = (\sqrt{5} - 1)/2 = 1/\Phi = .618$$



PhiArt A7



AO = OB =1
BC = BH = BE = 1/Φ = .618
CO = 1/Φ² = .382

SG104.3.4.1 PHI - Circle (1)

R. A. Schwaller de Lubicz is the father of "symbolist" Egyptology. He resided 15 years in Thebes to study in situ the many facets of the Temple of Luxor.
In his monumental account *"Le Temple de l'Homme"* (Paris, 1957), de Lubicz speaks about the mystical function of PHI and gives a simple construction method based on the circle. He adds: *"Resulting from the cycle, this function gives countless forms".*

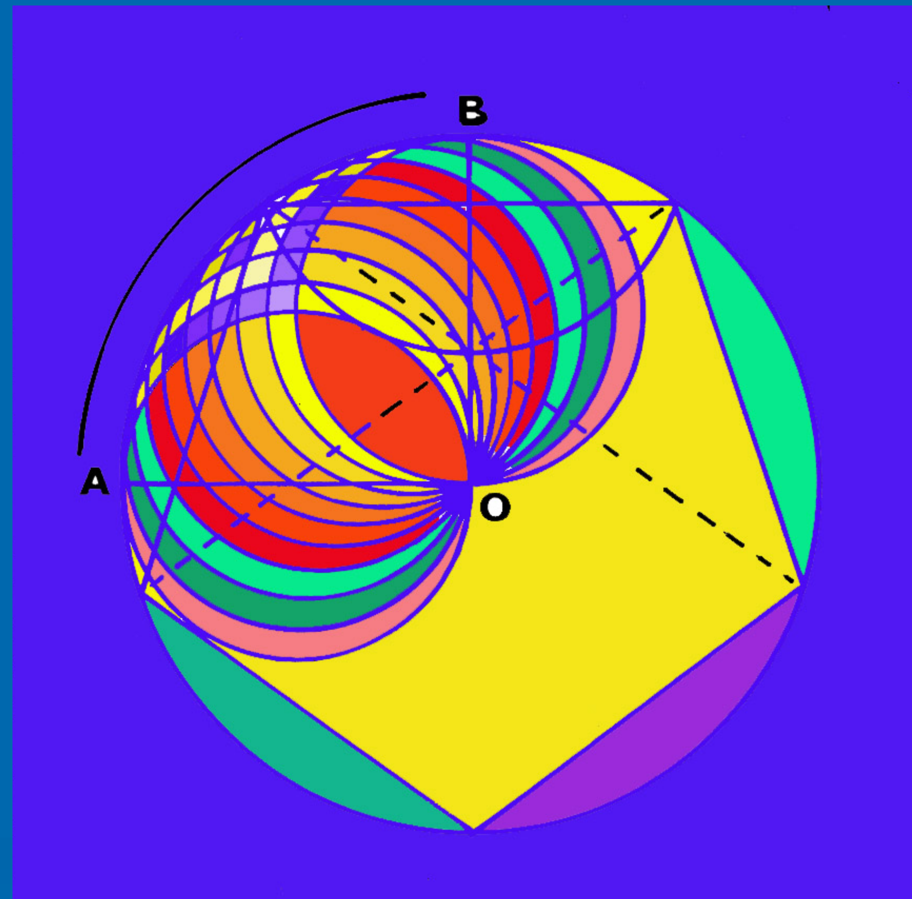
Step #1: On the base line AO, trace circle (1) with diameter AO = 1 unit.

Step #2: trace the tangent in point O, with right angle AOB = 90º to obtain a vertical axis.

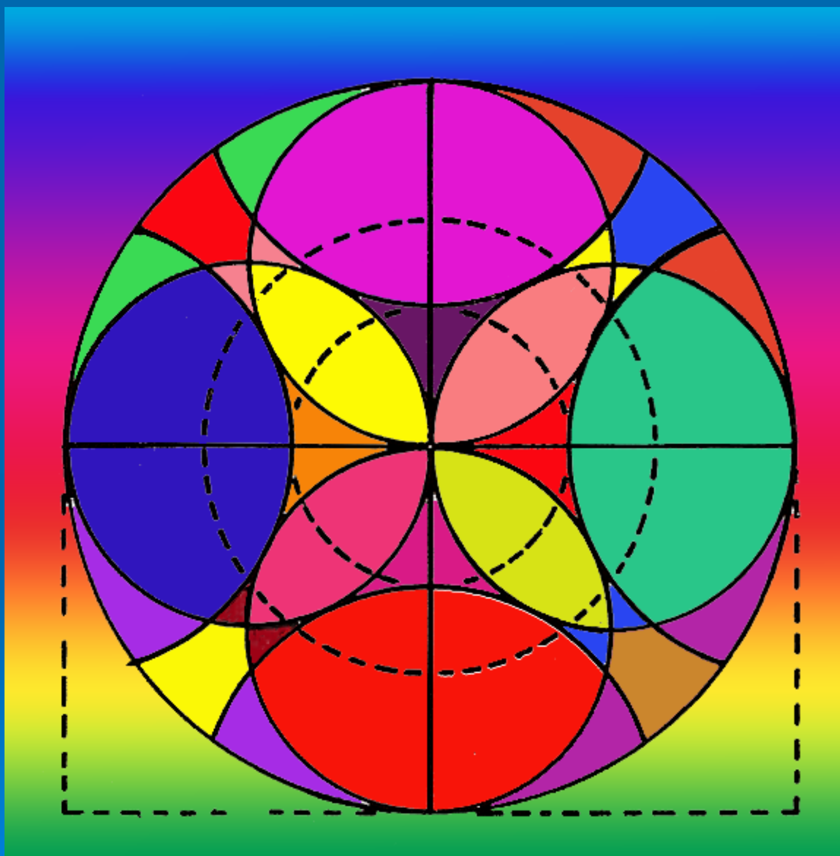
Step #3: Trace a second circle (2) with diameter OB = 1 unit.

Step #4: From B, trace a larger circle (3) tangent to circle (1) in E.
C and all points on the circumference of circle (3) have a 1/Φ = .618 relationship to B.

SG104.3.4.2 PHI - Circle (2)

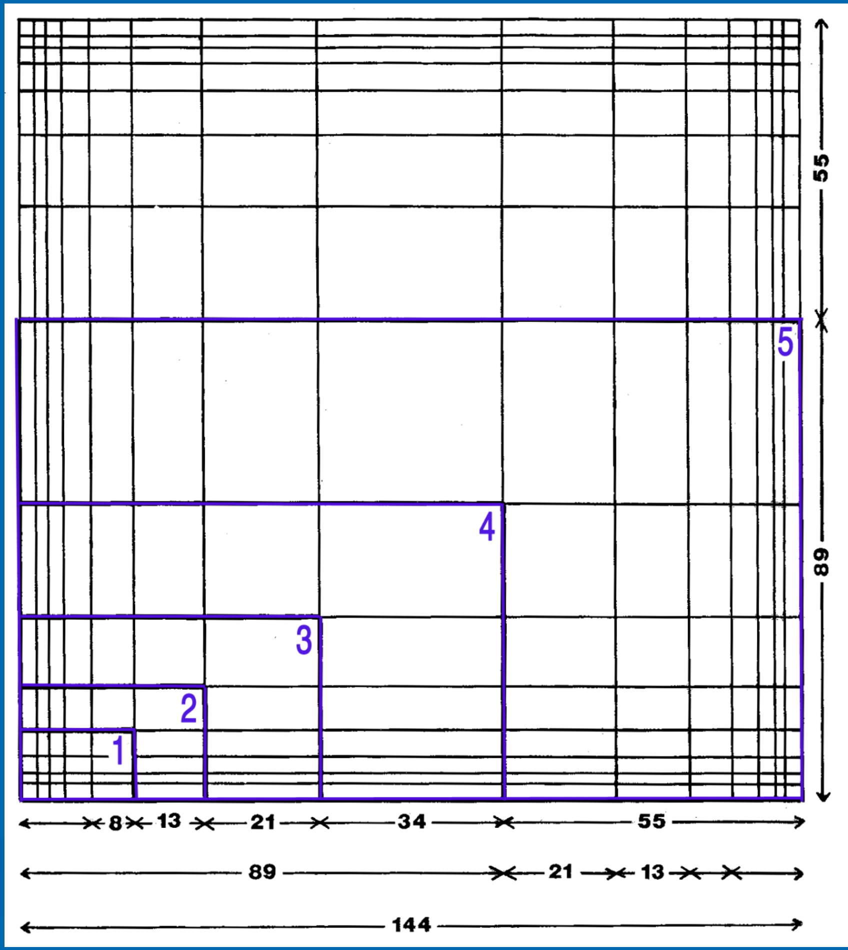


PhiArt A9



PhiArt A10

🡐 🡑 Two of the geometries resultjng from the circle construction of PHI.

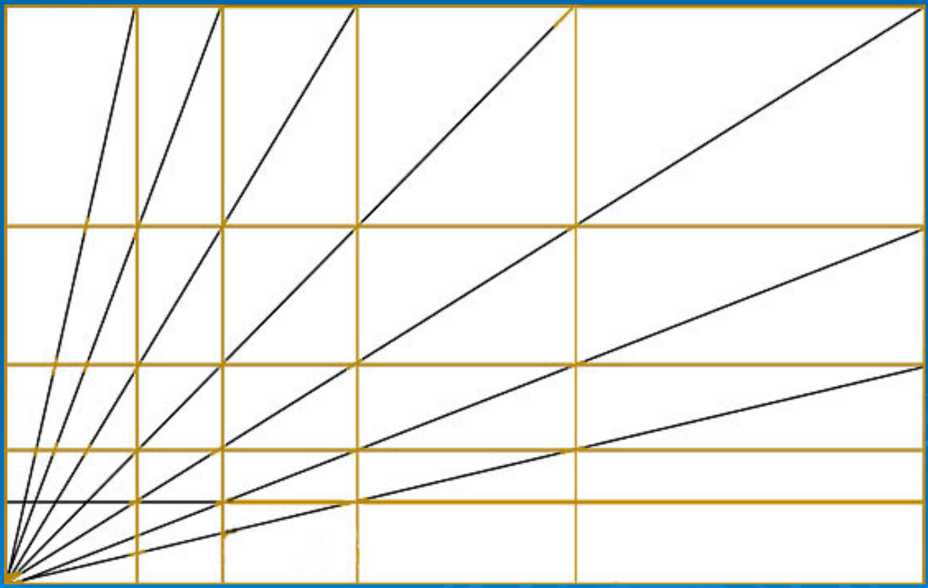


↑ Note the Golden Rectangles
(in blue).

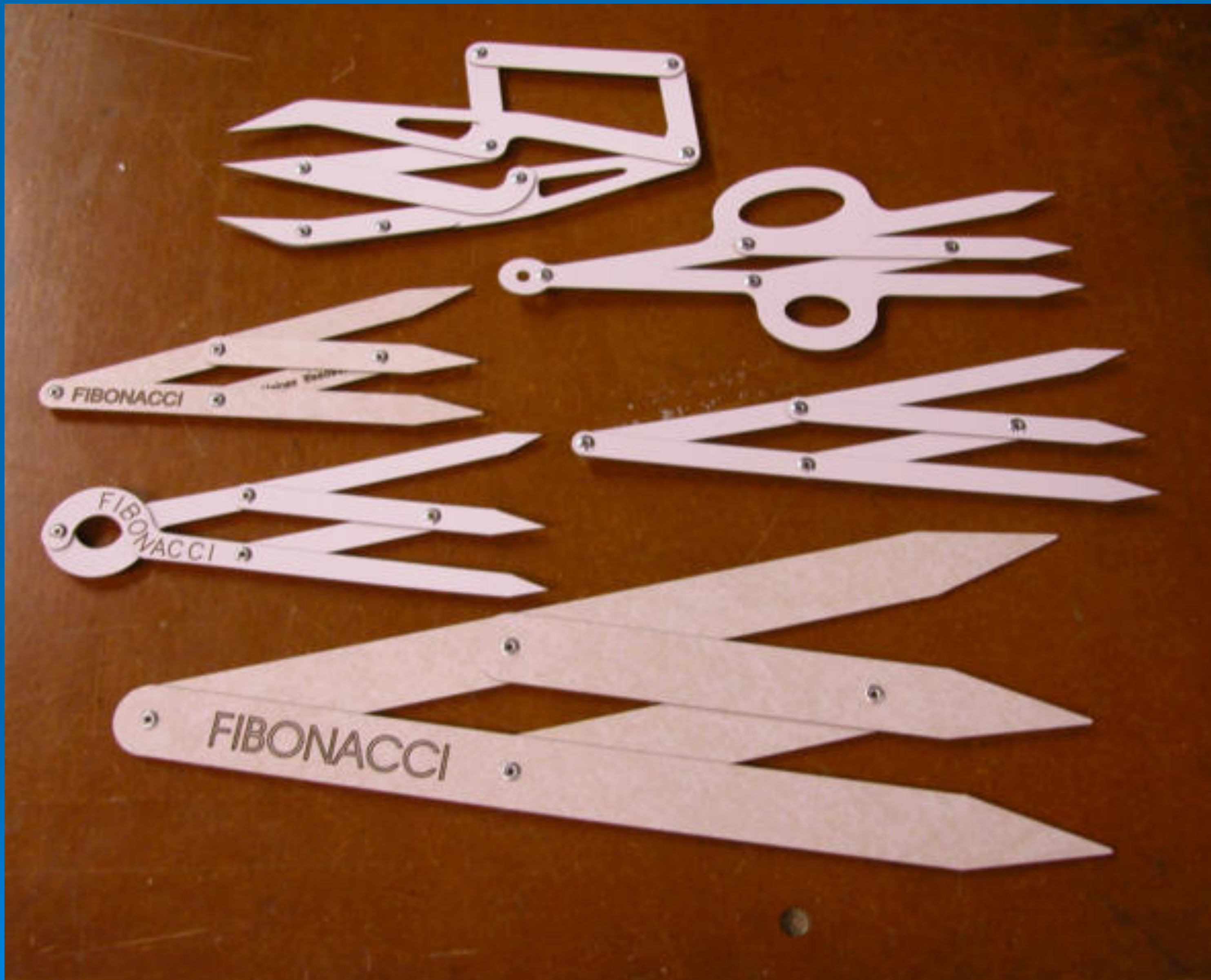
SG104.3.5 PHI Grids

Tools for Harmonic Design
for artists, architects, designers,
Feng Shui practitioners…

When printed on transparent acrylic sheets, these grids allow for a quick "Phi check" on reproductions of artwork, images, architectural plans etc…

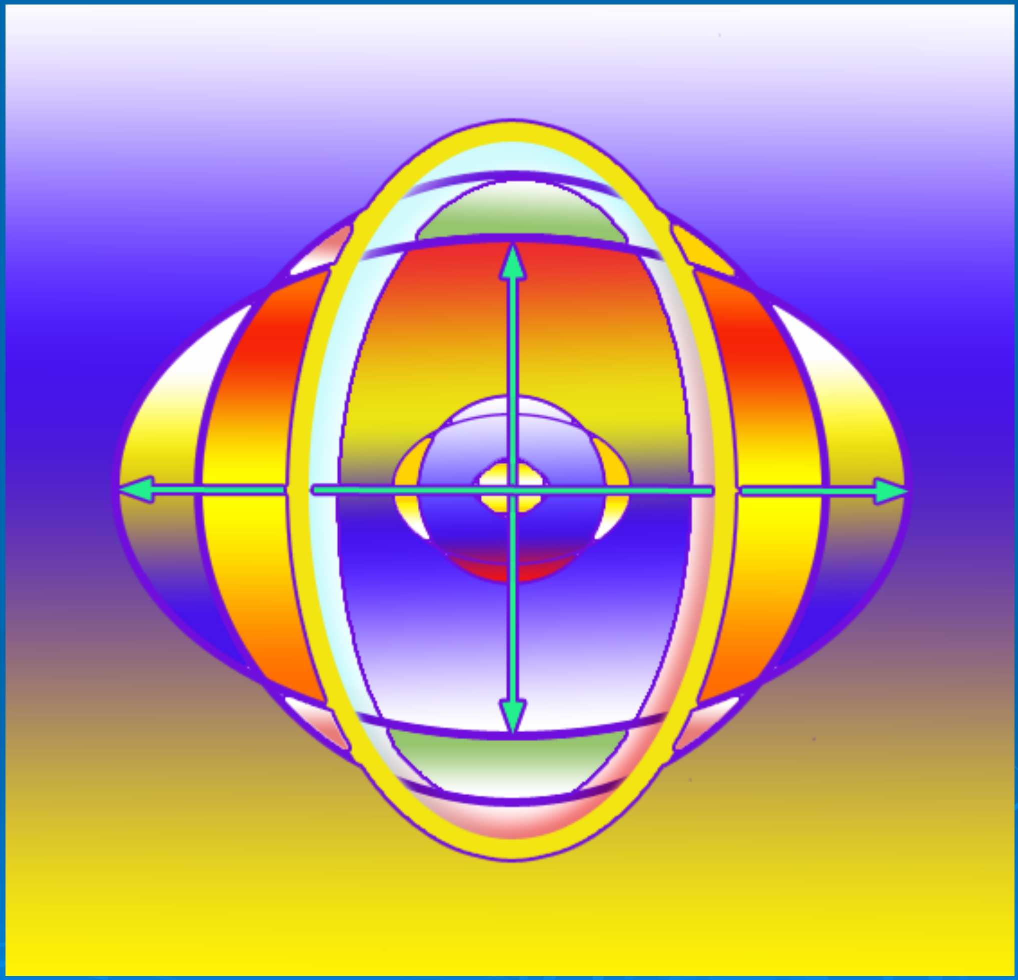


SG104.3.6 PHI & Fibonacci Gauges



SG104.4 Chapter 4. More PHI Constructions

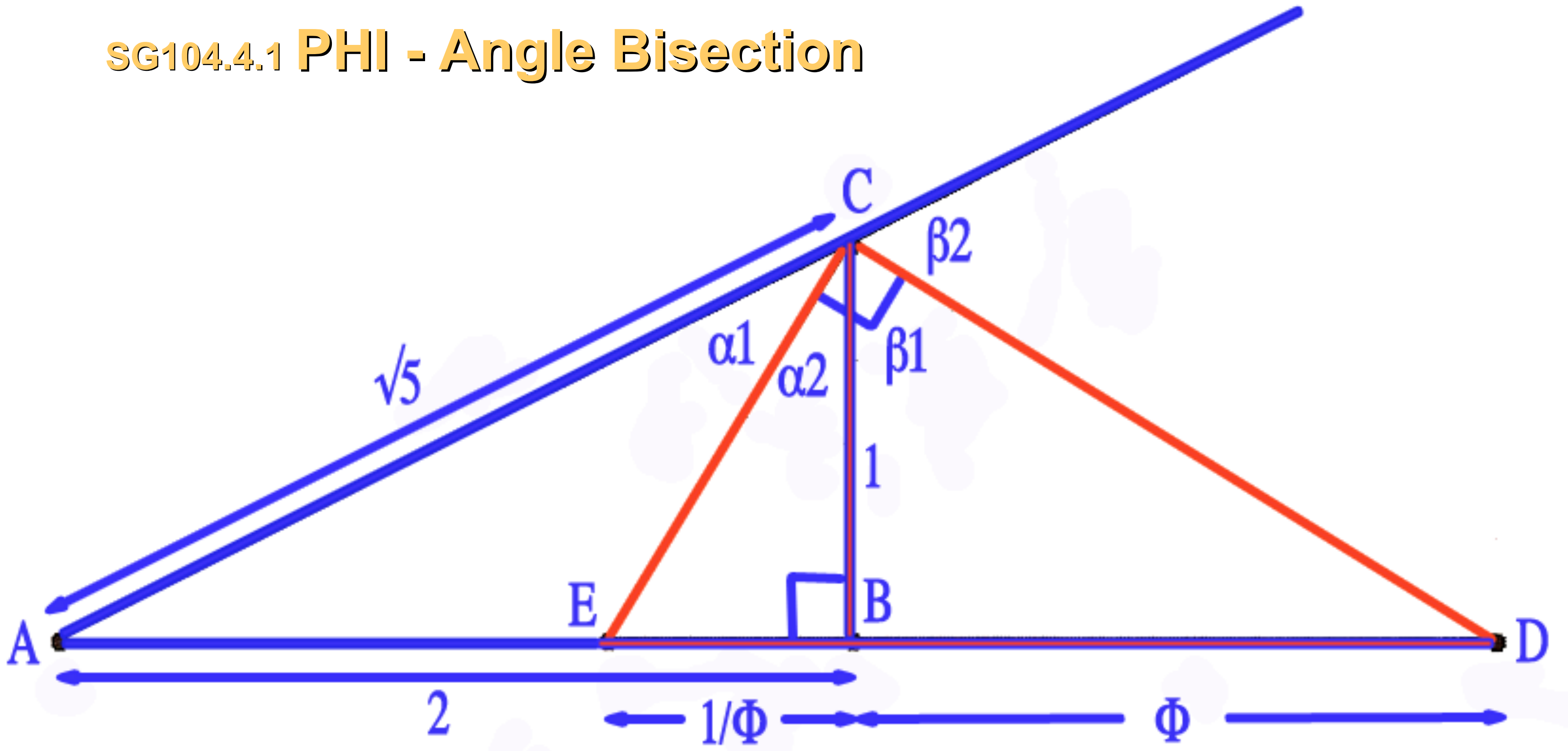
In this chapter, we will get acquainted with a few more interesting ways PHI can be constructed or just appears unexpectedly… You will get glimpses of Sacred Geometry figures that we will explore much more in depth in later SG modules: the Golden *Rectangle & Spiral*, the Vesica Piscis, the *Pentagon/Pentagram*, the *Arbelos*, and the 3D sacred geometric volumes: the *Platonic & Archimedean* Solids.



All these shapes, figures and constructions are intimately related through the harmonizing presence of the PHI Ratio.

We keep encouraging you to do-it-yourself. The wisdom inherent in Sacred Geometry comes from retraining our vision, our perception, our minds and brains so we become fully conscious again of the harmonic patterns encoding visible & invisible creation. Practicing hands-on the Sacred Geometry constructions and/or visualizing them is key to achieve this.

SG104.4.1 PHI - Angle Bisection

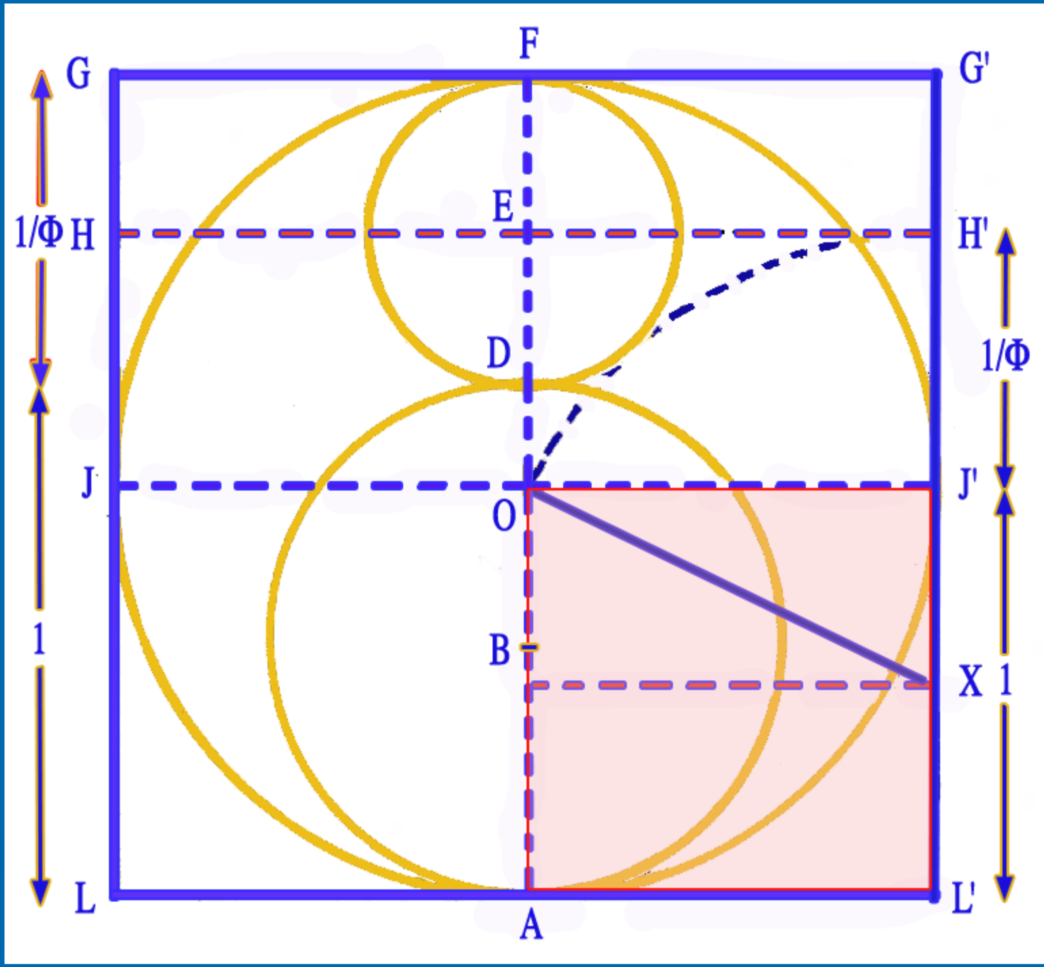


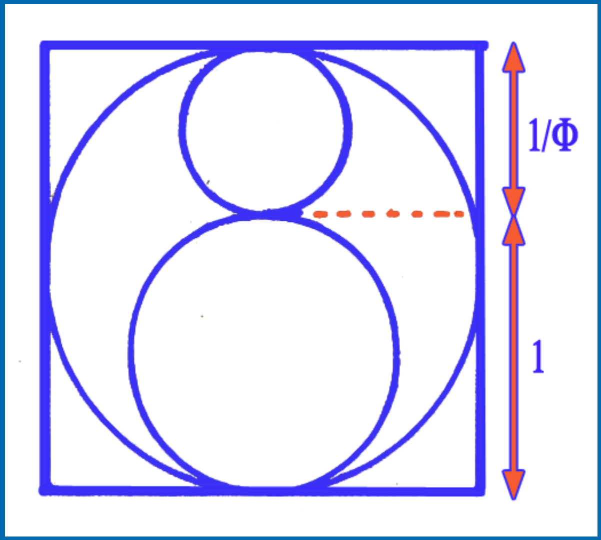
<u>Step #1:</u> Construct the now familiar right triangle ABC with sides 1, 2 and hypotenuse $\sqrt{5}$.

<u>Step #2:</u> Trace the bisectors (in red) of angle ACB and its external angle at C.

$$EB = 1/\Phi \qquad BD = \Phi$$

SG104.4.2.1 PHI - Arbelos (1)



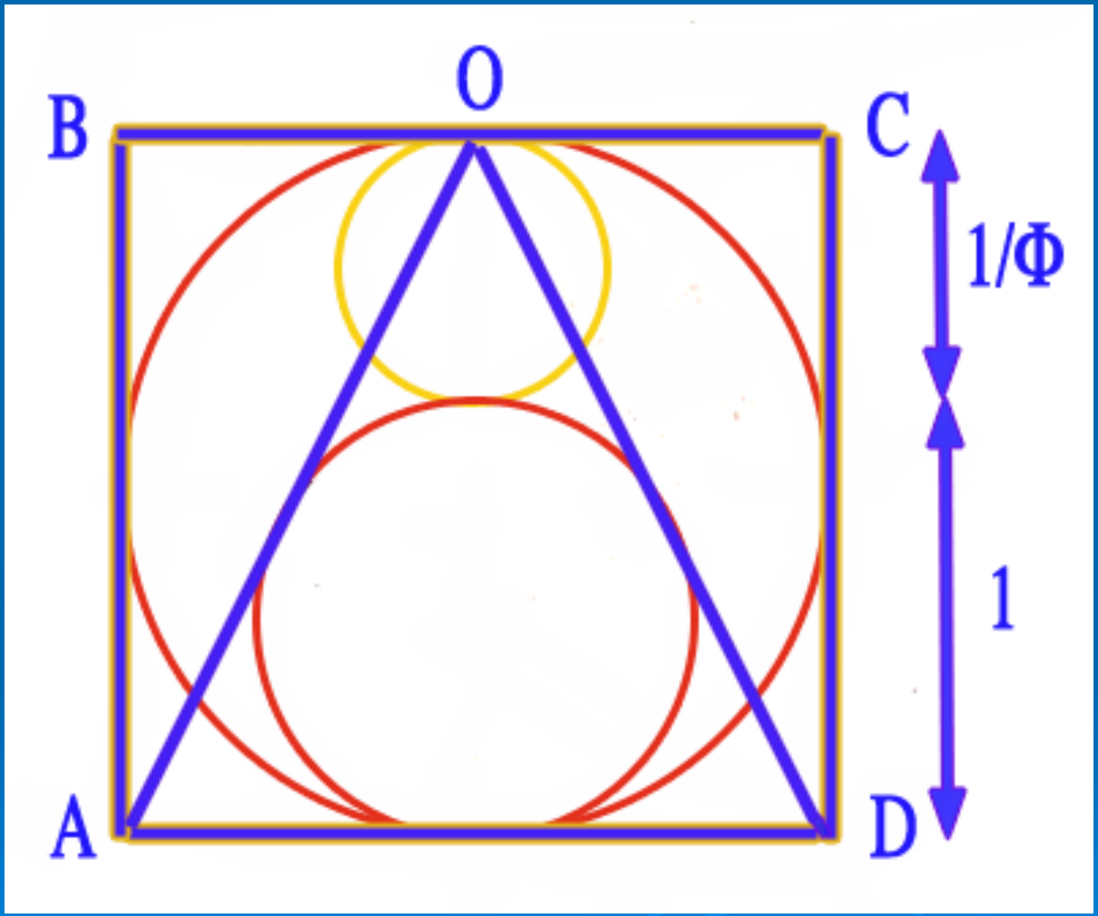


The "*Arbelos*" (Greek = "*cobbler's knife*") of Archimedes is a treasure trove of PHI ratios. We will study it more extensively in SG106.

Step #1: Divide a square LL'GG' into 4 quadrants. From the mid-side (at point X) of the lower right quadrant AOJ'L' (rose color square), trace an arc with radius XO to mark the Golden Section point H' on L'G'.
Step #2: Trace the horizontal line HH' intersecting the vertical axis AF in E. This is the center of the top circle.
Step #3: Trace the lower (bigger) circle by dividing the line AD in two and taking B as center.
Step #4: Circumscribe the Arbelos with a circle inscribed in the square.

SG104.4.2.2 PHI - Arbelos (2)

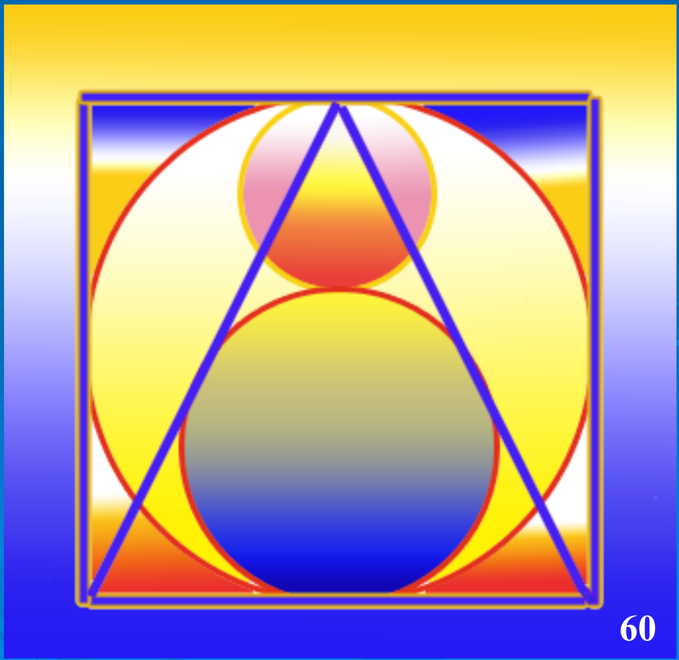
This is a simplified *Arbelos* coming out of nesting (embedding) the 3 basic geometry shapes: circle, triangle and square.

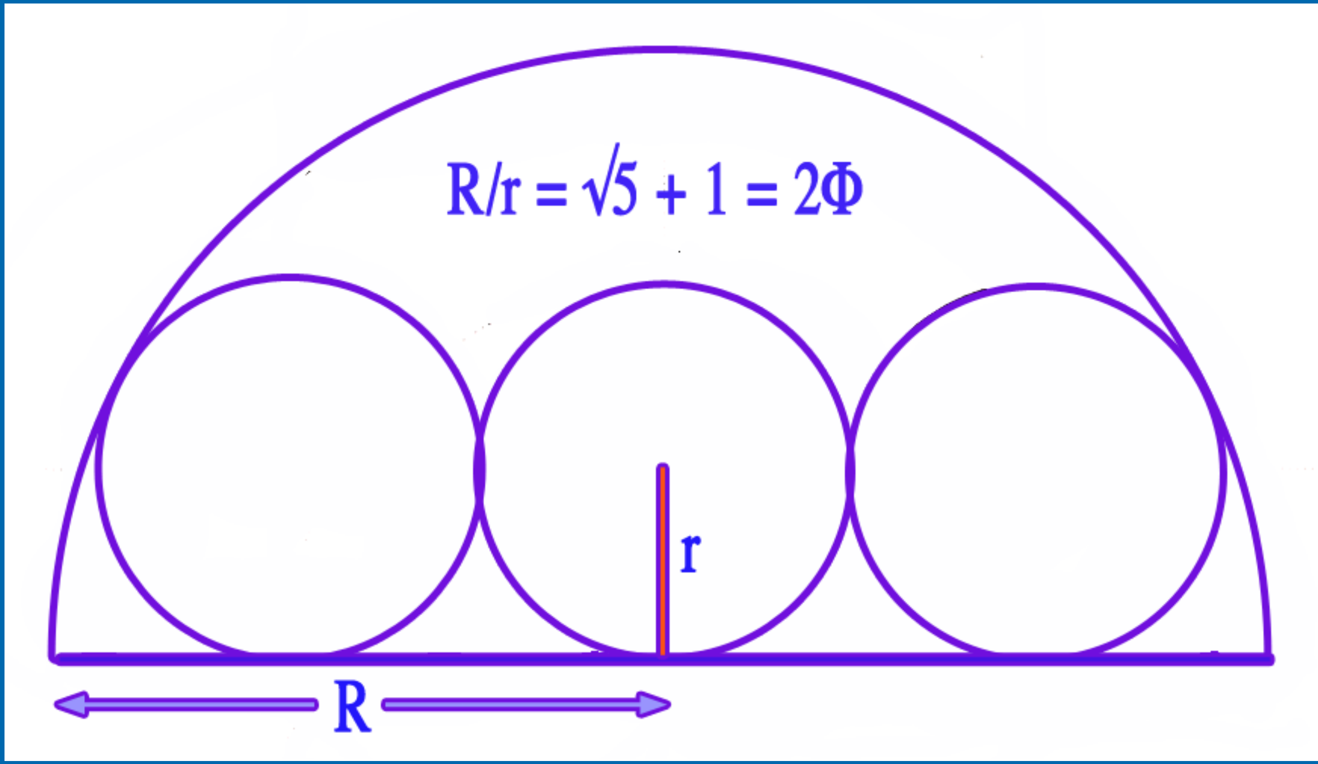


Diameters: Large red circle = Φ   Small red circle = 1
Orange circle = 1/Φ = .618

Step #1: Construct a square ABCD (blue stroked red).
Step #2: Inscribe a circle (larger red) within the square.
Step #3: Trace an isosceles triangle (blue) within the square, with base AD = side of square and two equal sides.
Step #4: Inscribe a circle (smaller red) within the blue triangle.
Step #5: Trace the orange circle touching the square in O and the (smaller) red circle.

Here we have the Arbelos again with its Golden Ratio.





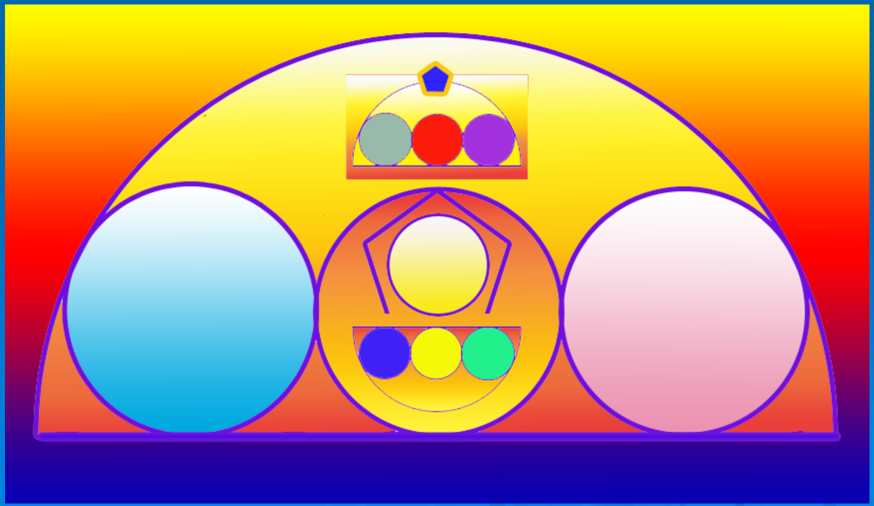
SG104.4.3 PHI
3 Circles

Considering 3 congruent circles inscribed in a semicircle.

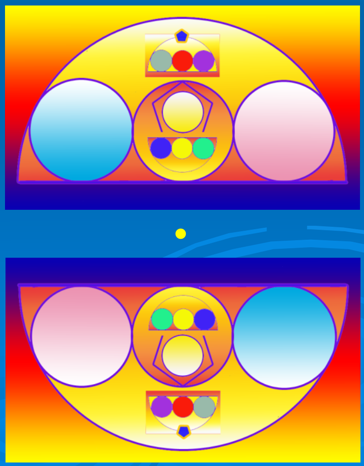
What is the ratio of the of the radius r of one of the small circles to the radius R of the semicircle?

The answer is:

$$R/r = \sqrt{5} + 1 = 2\Phi$$



PhiArt A12



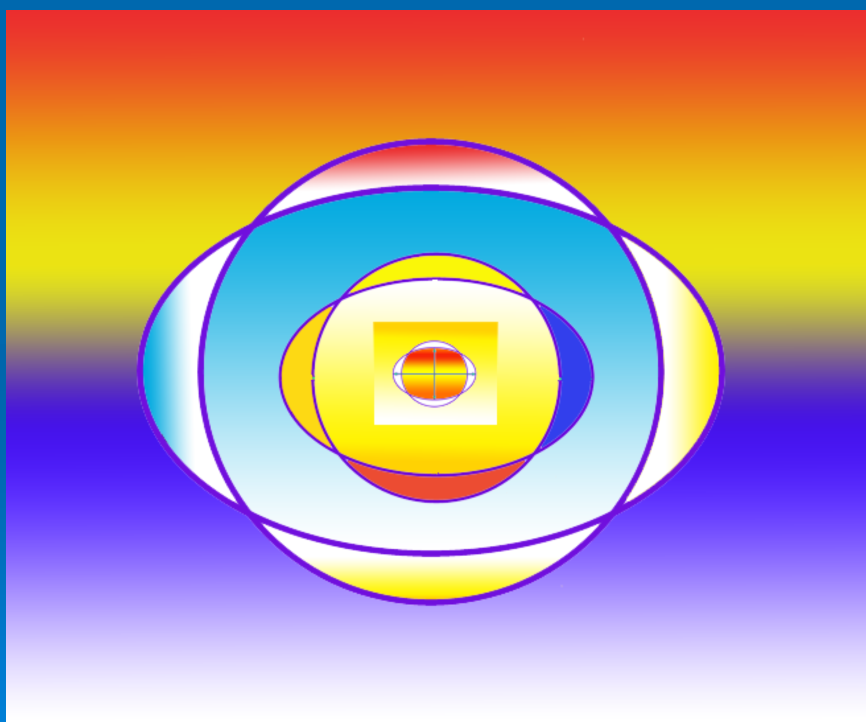
PHIArt A13

SG104.4.4.1 PHI - Ellipse & Circle (1)

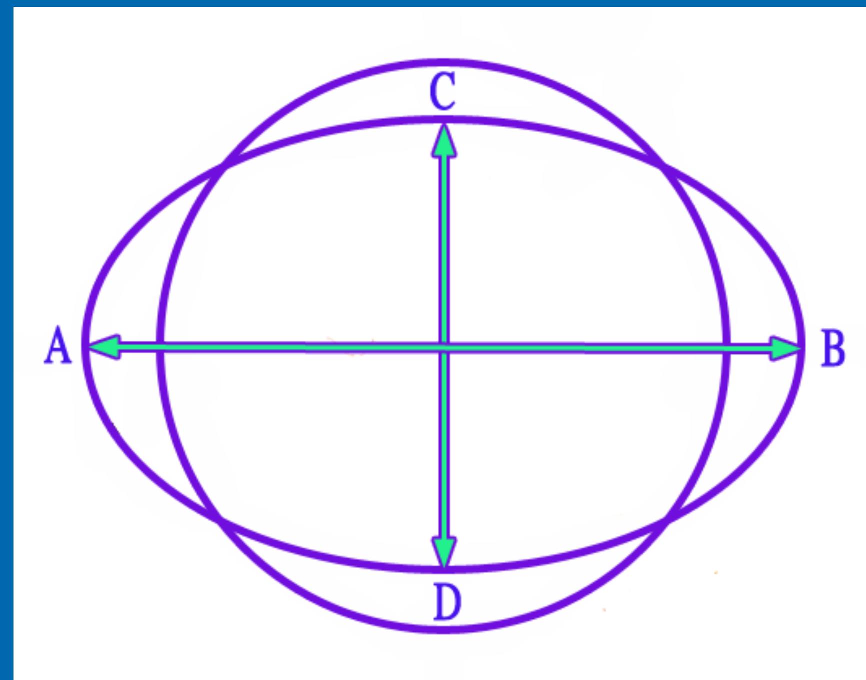
Comparing the area of an ellipse with the area of a circle of same center, we will find a point where the two areas are the same.

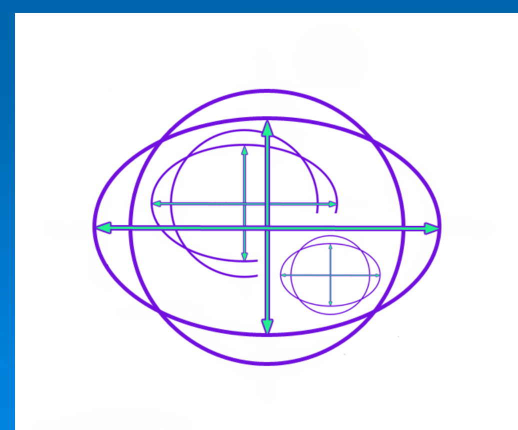
Interestingly enough, this point of balance is when the two axes of the ellipse AB & CD are in the Golden Ratio.

$$AB / CD = \Phi$$

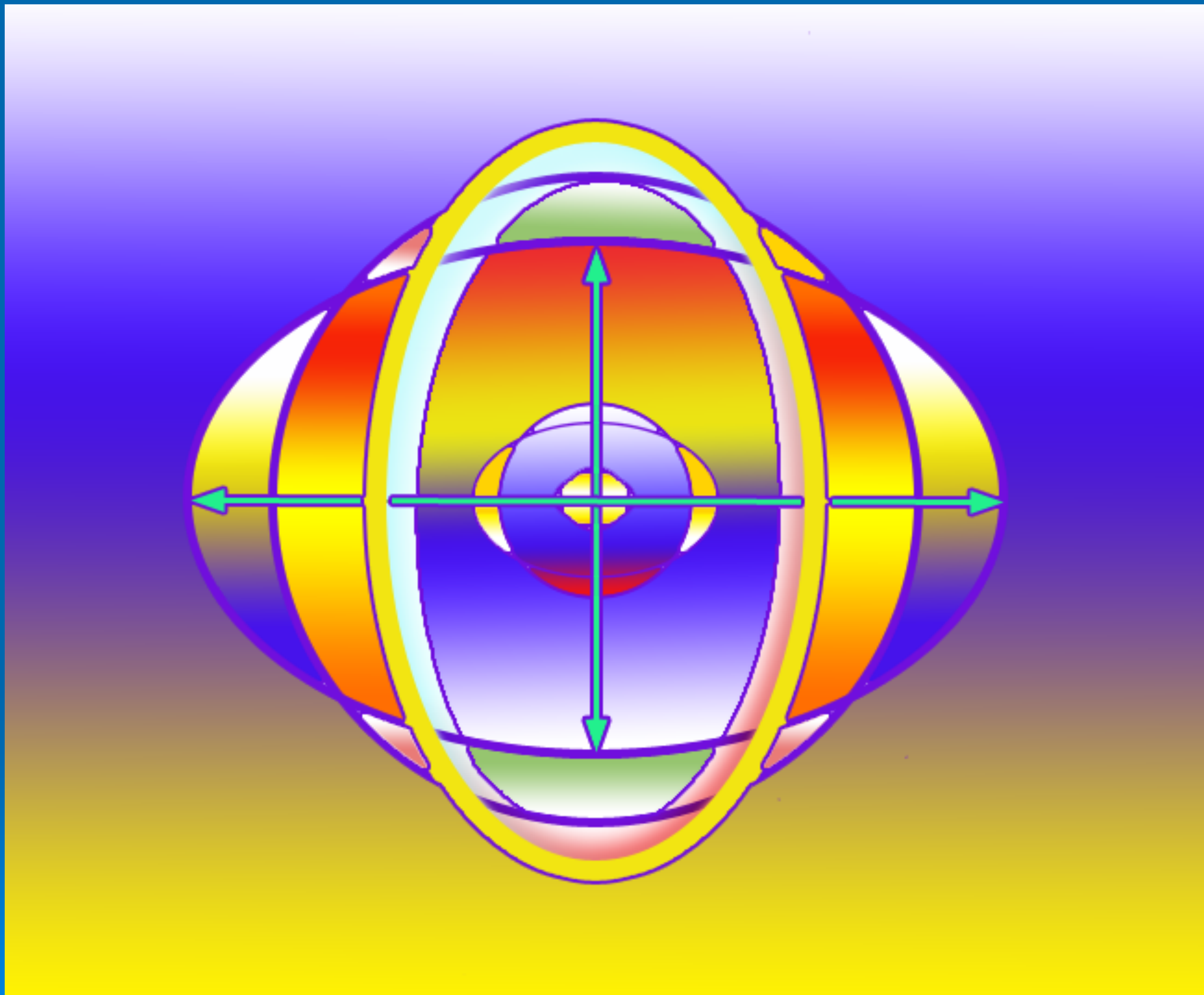


PhiArt A14





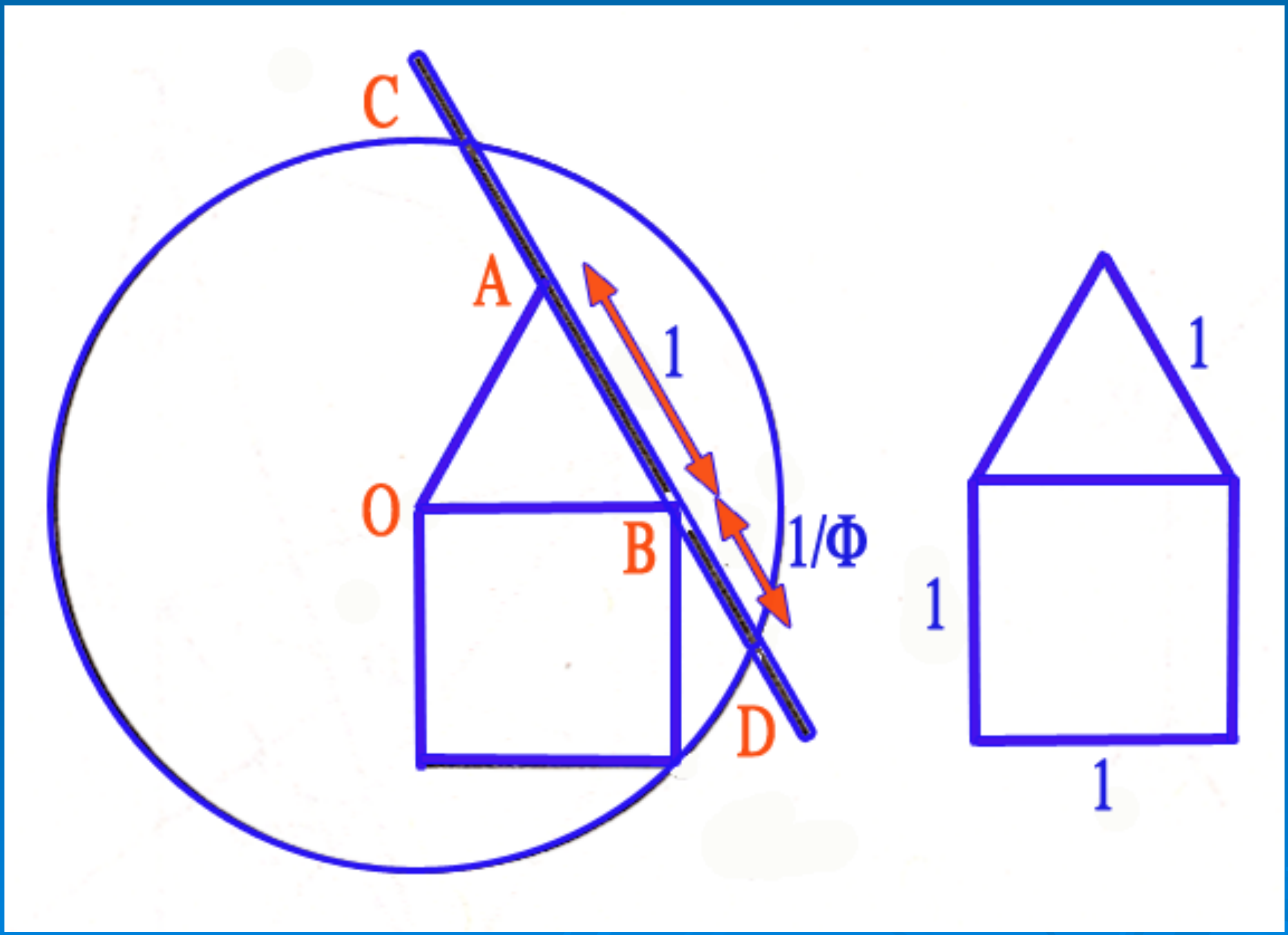
SG104.4.4.2 PHI - Ellipse & Circle (2)



PhiArt A16

SG104.4.5 PHI - Little PHI House

Here we have a combination of basic geometric shapes again: circle, square & triangle. But this time, the square and the triangle (the "*little house*") are attached to a circle.



Step #1: Construct an equilateral triangle of sides = 1 on top of a square of sides = 1.

Step #2: With compass centered on O (top left corner of the square), draw a circle of radius OD.

Step #3: Extend the side AB of the triangle to cut the circle in C and D.

$$AB = 1$$
$$BD = 1/\Phi = .618$$

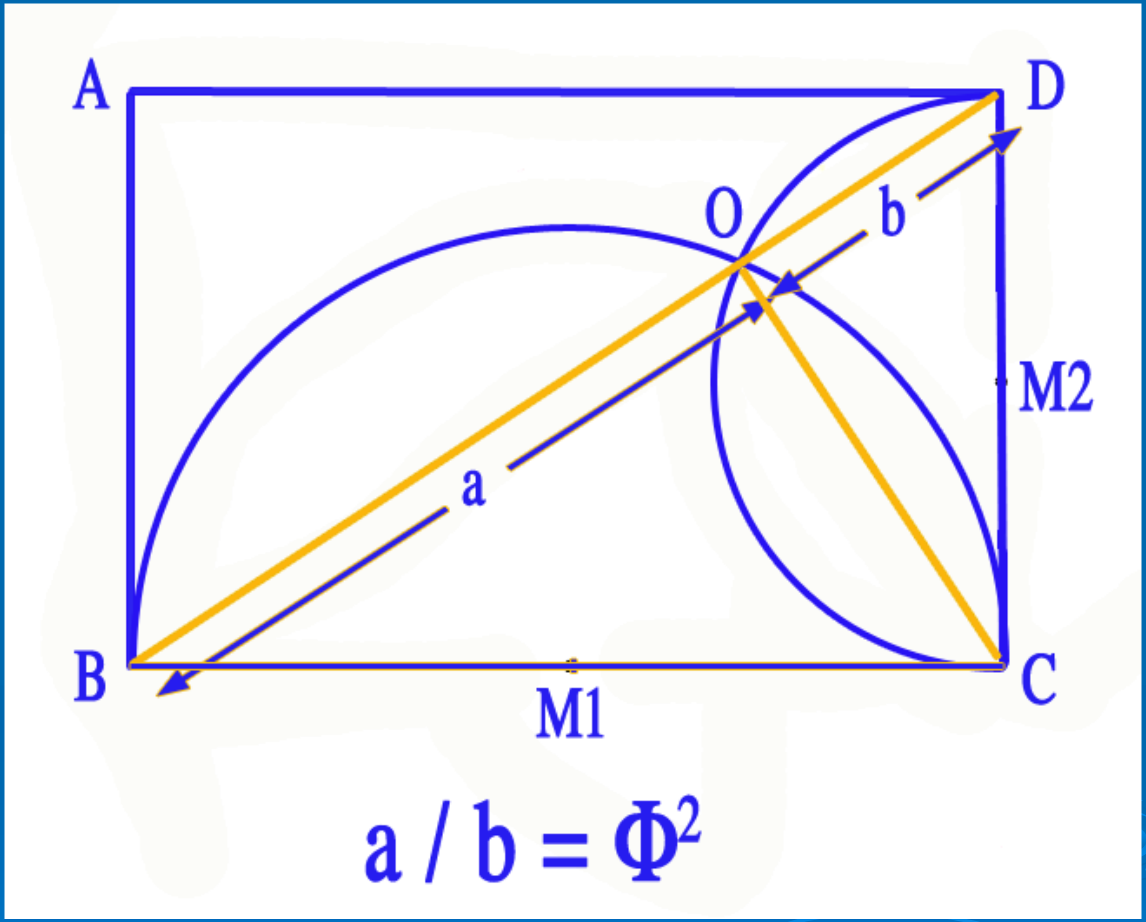
SG104.4.6.1 PHI - Diameter Golden Rectangle (1)

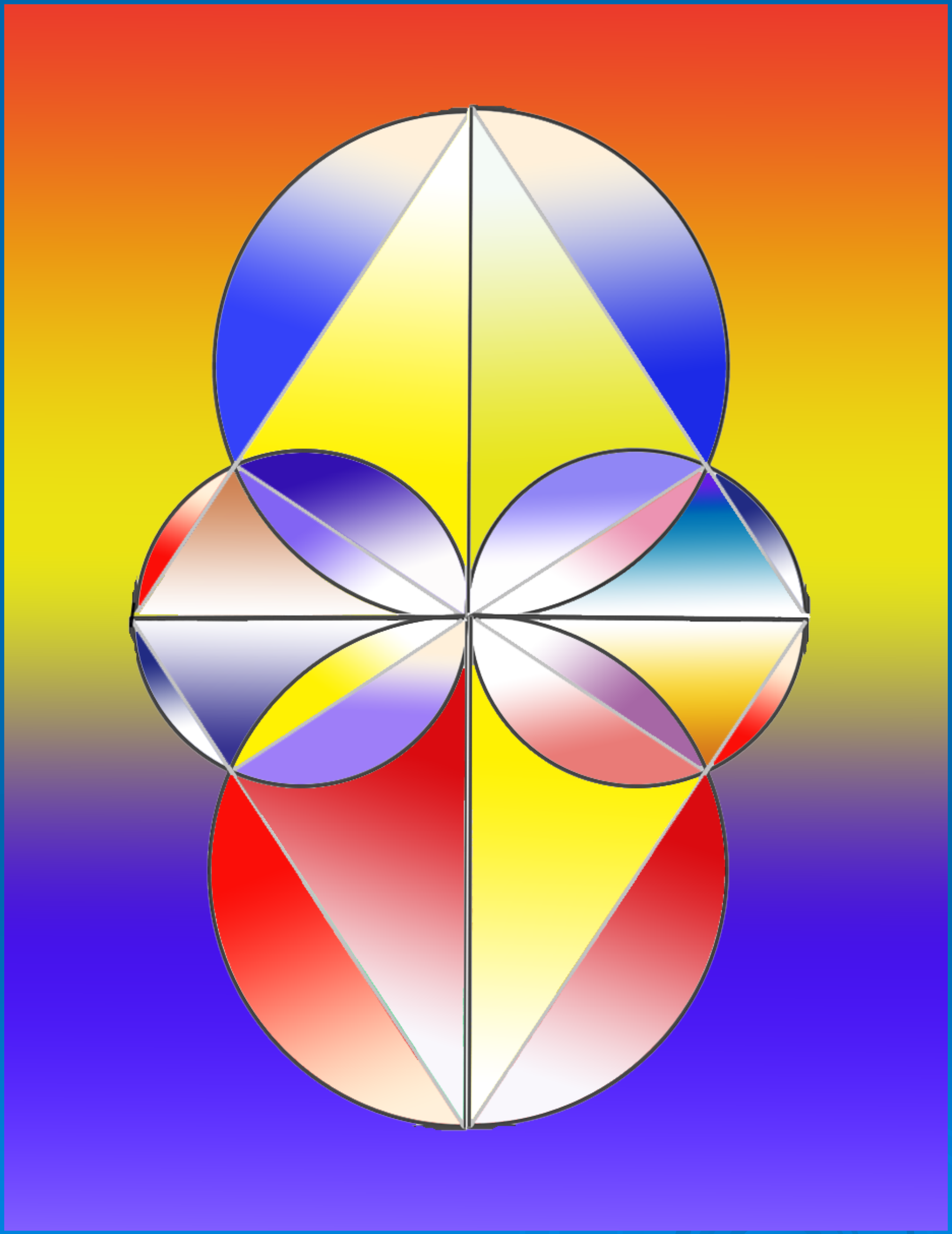
Step #1: Construct a Golden Rectangle with larger side BC in a Phi ratio to the smaller side DC.
BC / DC = Φ

Step #2: Draw two circles, edach centered on the midpoints (M1 and M2) of the sides, and intersecting at O.

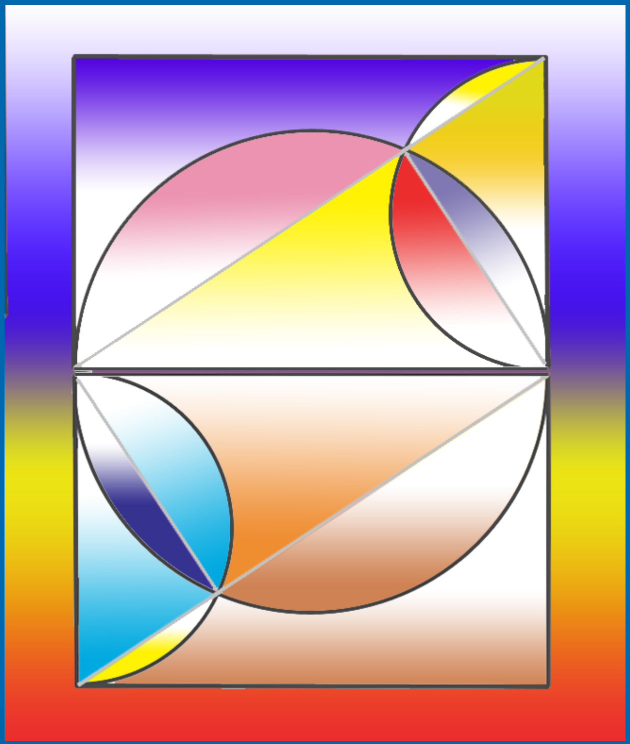
Step #3: Draw the diagonal BD of the Golden Rectangle.
The point O divides the diagonal in the Golden Section in such a way that

$$a / b = BO / OD = \Phi^2$$





PhiArt A18



PhiArt A17

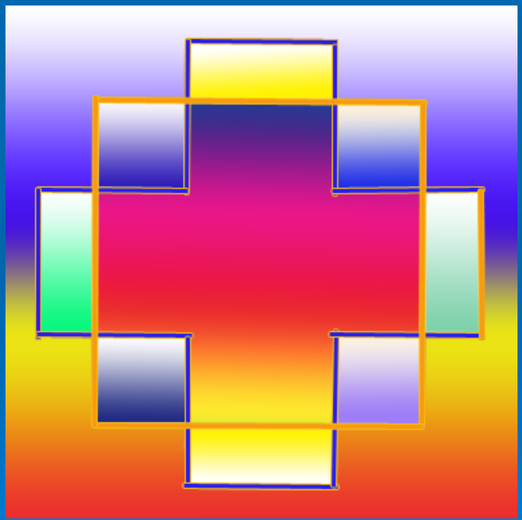
SG104.4.6.2 PHI - Diameter Golden Rectangle (2)

SG104.4.7 PHI - Cross and Square

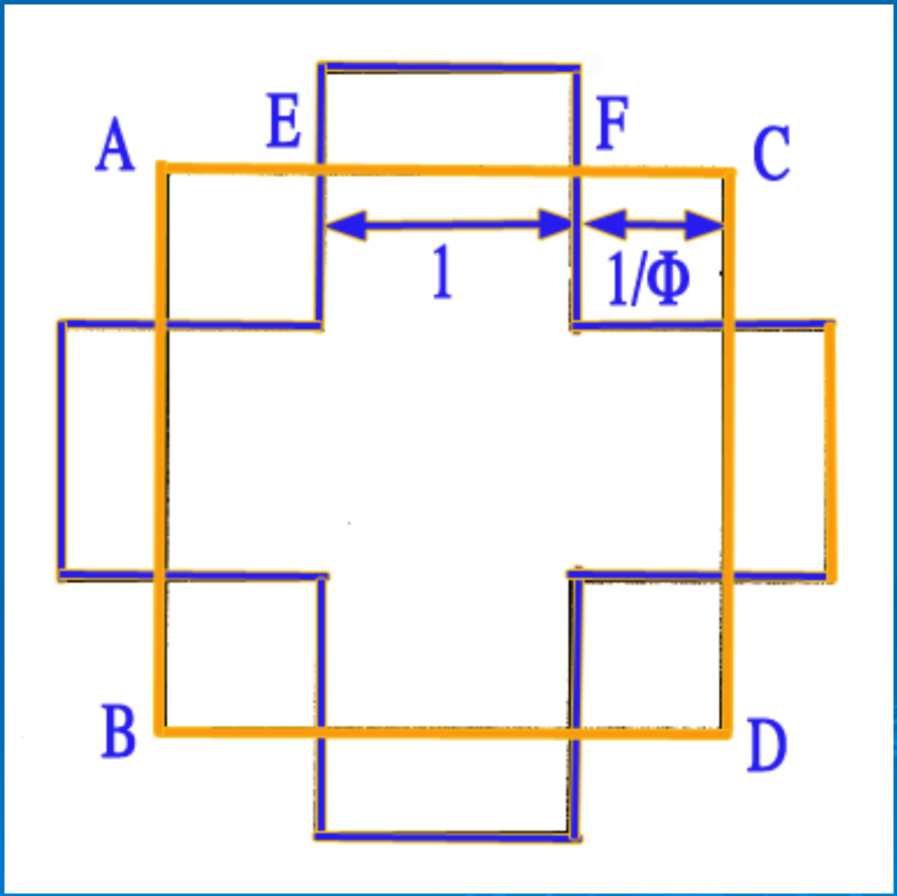
Step #1: Construct an equal arms cross from 5 squares of side = 1.

Step #2: Cut the cross with a square enclosing an area equal that the area of the cross.

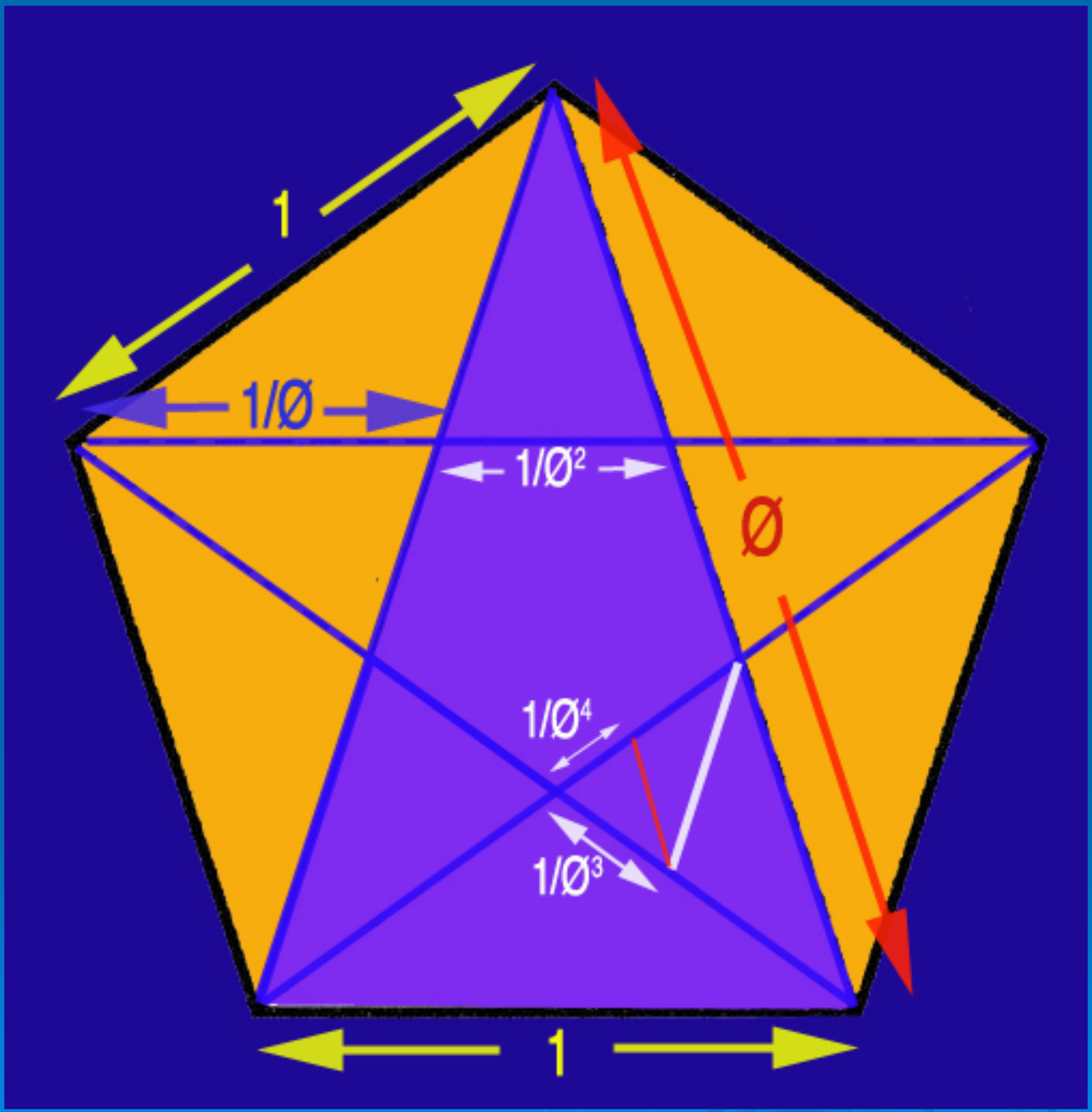
$$EF = 1 \quad FC = 1/\Phi$$
$$EF / FC = \Phi$$



PhiArt A19

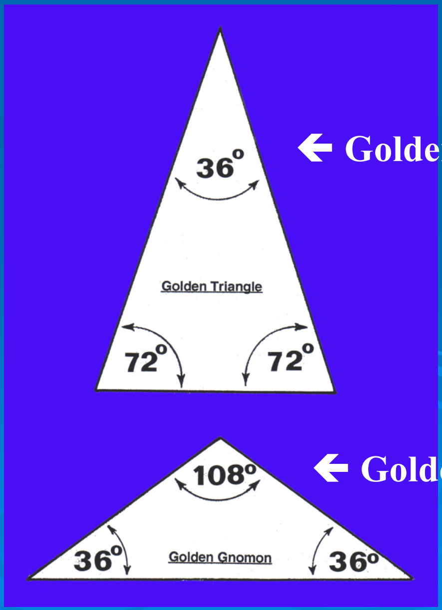


SG104.4.8 PHI - Penta



In SG106 we will pay an extensive visit to the Pentagon and Pentagram.

As an introduction, let us say that the Pentagon and Pentagram (5-pointed star) are replete with Golden Ratios. These Sacred Geometry shapes are nested into each other ad infinitum, up and down the scales of magnitude. They are made of the multiples and submultiples of PHI forming two basic complementary triangles *("Penta-Modules")*: the Golden Triangle and the Golden Gnomon.

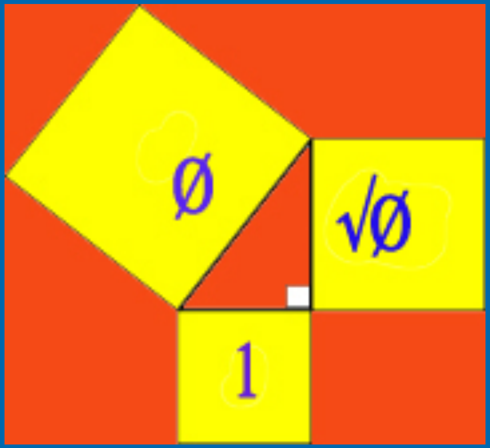


← Golden Triangle

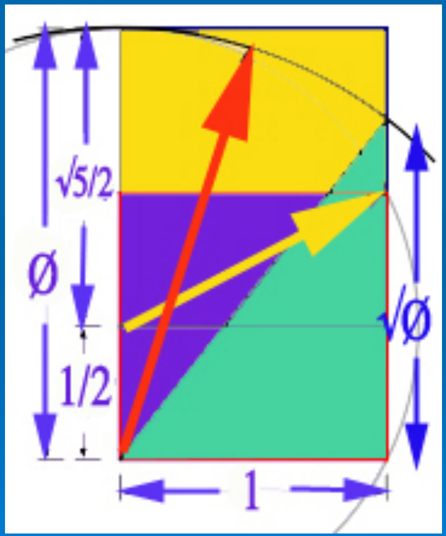
← Golden Gnomon

SG104.4.9 PHI in Kepler's Triangle

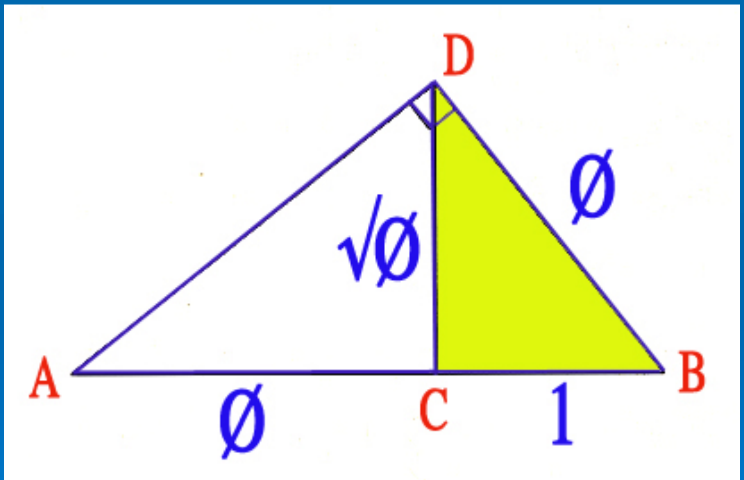
Kepler's fascination with the Golden Ratio guided him to describe what is now known as Kepler's Triangle.
In a 1597 letter to Michael Maestlin, Kepler wrote:
"If, on a line which is divided in extreme and mean ratio [PHI], one constructs a right angled triangle, such that the right angle is on the perpendicular put at the section point, then the smaller leg will equal the larger segment of the divided line".
A Kepler Triangle is a right triangle with edge lengths in geometric progression: 1, √ Φ, Φ.
Kepler triangles combine two key mathematical concepts: the Pythagorean Theorem and the Golden Ratio.



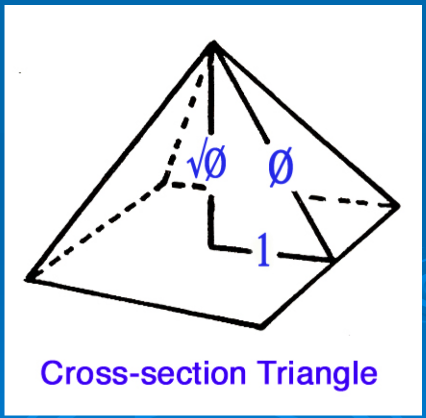
A Kepler Triangle is a right triangle formed by three squares with areas in geometric progression according to the Golden Ratio.



↑ A Kepler triangle can be constructed with only straightedge and compass by first creating a Golden Rectangle.



↑ Kepler's construction: Line AB is divided at the Golden Section point C. Right angled triangle ABD is traced with AB as hypotenuse. CDB (in yellow) is Kepler's Triangle



↑ "*Kepler's Triangle*" is also the cross-section triangle of the Great Pyramid.

SG104.5 Chapter 5. The PHI-Fibonacci Flowering

SG104.5.1.1 The Fibonacci Quarterly (1)

The Fibonacci Quarterly

Official Publication of The Fibonacci Association

THE FIBONACCI ASSOCIATION

"The Fibonacci Quarterly is meant to serve as a focal point for interest in Fibonacci numbers and related questions, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas."

In 1963, with mathematician Verner E. Hoggatt, Alfred Brousseau, founded the Fibonacci Association with the intention of promoting research into the Fibonacci Numbers and related fields. Brousseau was an American monk, avid photographer (he amassed over 20,000 slides of the California flora) and mathematician.

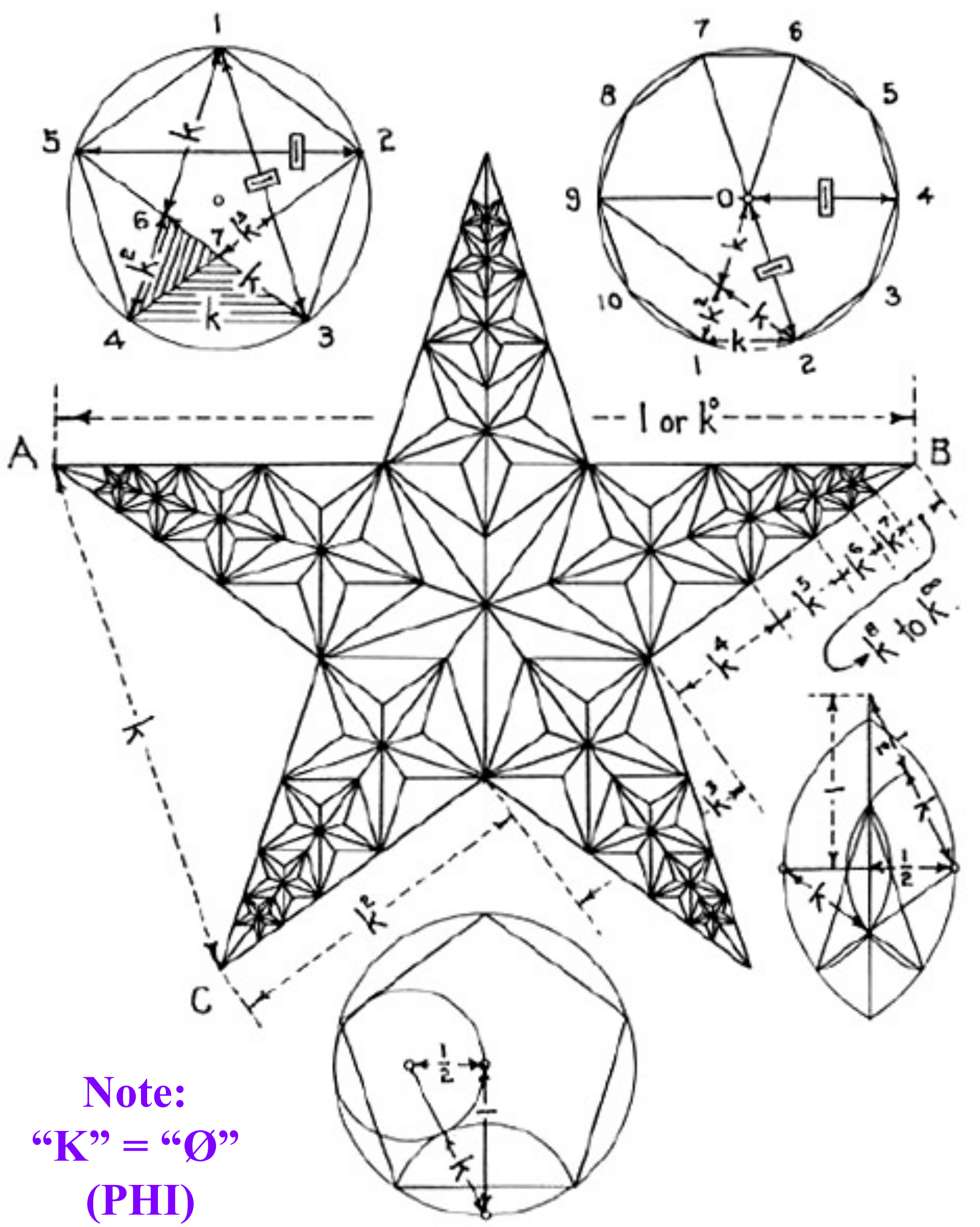
In 1969 Brousseau commented on the Fibonacci Association (and its associated journal, the Fibonacci Quarterly) in the April edition of Time Magazine, *"We got a group of people together in 1963, and just like a bunch of nuts, we started a mathematics magazine... [People] tend to find an esthetic satisfaction in it. They think that there's some kind of mystical connection between these numbers and the universe."*

For a complete list of the entire collection of the Fibonacci Quarterly (and PDF access) - starting with issue #1 / 1963:
http://www.fq.math.ca/list-of-issues.html

fib

🡐 Logo of the Ninth International Conference on Fibonacci Numbers and Their Applications, held in Flagstaff, AZ, in 2002.
This logo combines the "Logarithmic Spiral" and the "Fibonacci Tiling" - in other terms: the Golden Spiral and the Golden Rectangle [◆SG105]

SG104.5.1.2 The Fibonacci Quarterly (2)



Note: "K" = "Ø" (PHI)

The "<u>Star of 21 Stars</u>" *(21 is a Fibo number)* has been the logo of the Fibonacci Association and the Fibonacci Quarterly since 1963.

← In Vol. 4. Number 1, February1966 issue of the Fib. Quart., Colonel R. S. Beard (author of *"Patterns in Space"*) explained the 'star geometries' this logo is pointing to:

- **The PentaStar is proportioned to the 10 successive powers of PHI.**
- **1 diagonal of each of the smaller stars is a side of the bounding pentagon of the next larger star. All the corresponding dimensions of of these successive stars are in the Golden Ratio PHI.**
- **This figure demonstrate that any power of PHI is the sum of all the higher powers from Φ^{n+2} to $\Phi^{infinity}$.**

[◆SG106]

SG104.5.2 Special properties of Fibonacci Numbers

There are countless characteristics of the Fibonacci numbers.
Amazing, unexpected and curious properties…
Mathematicians are still filling volumes & websites with these re-discoveries and their applications.
We are presenting a few of them below:

1. **Ever third F number is EVEN.**
2. **The sum of any ten F numbers is divisible by 11:**
   21 + 34 + 55 + 89 + 144 + 233 + 377 + 610 + 987 + 1597 = 4147 / 11 = 377
3. **The sum of the squares of F numbers = the product of the last and next number in the sequence:**
   $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 + 34^2 = 34 \times 55 = 1{,}870$
4. **Subtracting the squares of two alternating F numbers creates another F number:**
   $F_7^2 - F_5^2 = 13^2 - 5^2 = 144 = F_{12}$
5. **The sum of the subscripts of two F numbers whose squares are added gives the subscript of the F number representing their sum:** $F_{10}^2 + F_{11}^2 = 10{,}946 = F_{21}$
6. **For any four consecutive F numbers, the difference of the squares of the middle two numbers = the product of the outer two numbers:**
   Starting sequence: 13, 21, 34, 55. $34^2 - 21^2 = 1156 - 441 = 55 \times 13 = 715$
7. **The product of two alternating F numbers = square of the F number between them, plus or minus 1:**
8. $144 \times 377 = 54{,}288 = 233^2 = 54{,}289$

… and much more…
So, clearly, we are dealing with a complex matrix,
intricately weaving harmonic patterns & music-like resonances
in, trough and in between many dimensions.
The Fibonacci Series is not just a line of numbers: it is a poly-D Hyper-MANDALA.

8

SG104.5.3.1 The Magic of 1/89 (1)

9

89 is the 11th Fibonacci number and the 5th Fibonacci Prime. It has a curious property:

The period of its reciprocal (1/89) is generated by the 5 first Fibonacci numbers. But then it seems to go off.

1/89 = 0.011235955056179775280898876404494382022471 91

However, if one computes 1/89 in terms of fractions: 1/89 = $0/10^1 + 1/10^2 + 1/10^3 + 2/10^4 + 3/10^5 + 5/10^6 + \ldots$, and writes out the decimals, one obtains the table on the right ➔, clearly showing all the Fibonacci numbers cascading nicely.

Curiously, the above sequence of 44 decimals repeats itself. It is said to have a period of 44. And, right in the middle of the reoccurring period, guess who we find?
Our friend 89 (in yellow color).

Of course, the full Fibonacci Series can also be obtained if one adds to 89 the Fibonacci numbers as decimals:

1/89.00112358 = .01123581321…

0
\+ .01
\+ .001
\+ .0002
\+ .00003
\+ .000005
\+ .0000008
\+ .00000013
\+ .000000021
\+ .0000000034
\+ .00000000055
\+ .000000000089
\+ .0000000000144
\+ .00000000000233
\+ .000000000000377
\+ .0000000000000610
\+ .00000000000000987

.011235955056179 77 . . .

SG104.5.3.2 The Magic of 1/89 (2)

Indeed 89 plays a pivotal role in the Fibonacci Number Field because of its link with the base-10 system:

$$89 = 10^2 - 10 - 1$$

Also consider the equation:

8 9

$$1 = \frac{89}{10^{n+1}} \cdot (F_1 \cdot 10^{n-1} + F_2 \cdot 10^{n-2} + \ldots + F_{n-1} \cdot 10 + F_n) + \frac{10F_{n+1} + F_n}{10^{n+1}}.$$

Another curiosity of 89 is the following: if one takes any number and find the sum of the squares of its digits, and repeat the process continuously, one always ends up with 1 or 89.
Example: Take the number 64. $6^2 + 4^2 = 52$ $5^2 + 2^2 = 29$ $2^2 + 9^2 = 85$ $8^2 + 5^2 = 89$

Also note the symmetry:

$$89 = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

And 89 is the largest prime equal to the sum of its digits plus the product of its digits:

$$89 = (8 \text{ x } 9) + (8 + 9)$$

[More interesting regularities & harmonic configurations in the Number Field are presented in ◆SG202]

In ◆SG205, we will see that 89 is the largest Fibonacci number occurring in phyllotaxy (natural plant arrangement) as the most common number of petals of the *Michaelmas daisy* (*asteraceae* family).

SG104.5.4.1 Deep Structure of the Fibonacci Numbers (1)

Several Sacred Geometry researchers have (re)discovered the "*Deep Structure*" or "*Pulse*" pattern of the Fibonacci Sequence: Jain of www.mathemagics.com, W. B. Conner and Ralph B. Nunnelley. Fibonacci mathematicians call it the *Pisano Period modulo 9* ("*Pisano*" refers to Leonardo of Pisa).
While the discovery is one, the interpretations and developments are different although complementary.

Basic Discovery - The Deep Structure

Step 1: Take the first 24 numbers of the Fibonacci Series:

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765 10946 17711 28657 46368

Step 2: Reduce each Fibonacci Number to its Digital Root. Examples:
55 = 5 + 5 = 10 = 1 + 0 = 1. The Digital Root of 55 is "1".
1597 = 1 + 5 + 9 + 7 = 22 = 2 + 2 = 4. The Digital Root of 1597 is "4".

1 1 2 3 5 8 4 3 7 1 8 9 8 8 7 6 4 1 5 6 2 8 1 9

Step 3: Continue with the next 24 numbers in the Fibonacci Sequence and compute their Digital Roots. Then the next 24 numbers etc… You will prove to your own satisfaction that the first set of 24 Digital Roots reoccurs again and again.
Clear Conclusion: there is a definite and predictable pattern (the "*Deep Structure*" or "*Pulse*" or "*Period*") to the Fibonacci Sequence. Since the Fibonacci Sequence is the matrix for the Golden Number, we can say: a transcendental/ irrational number (PHI), thought to elude any order, has been found to display a very precise pattern, a harmonic pulsation… But there is more…

1 1 2 3 5 8 4 3 7 1 8 9 8 8 7 6 4 1 5 6 2 8 1 9

1-1-2-3-5-8-13-21-34-55-89-144-233-377…

SG104.5.4.2 Deep Structure of the Fibonacci Numbers (2)

#	F(n)	Code
1	1	1
2	1	1
3	2	2
4	3	3
5	5	5
6	8	8
7	13	4
8	21	3
9	34	7
10	55	1
11	89	8
12	144	9
13	233	8
14	377	8
15	610	7
16	987	6
17	1,597	4
18	2,584	1
19	4,181	5
20	6,765	6
21	10,946	2
22	17,711	8
23	28,657	1
24	46,368	9

#	F(n)	Code
25	75,025	1
26	121,393	1
27	196,418	2
28	317,811	3
29	514,229	5
30	832,040	8
31	1,346,269	4
32	2,178,309	3
33	3,523,578	7
34	5,702,887	1
35	9,227,465	8
36	14,930,352	9
37	24,157,817	8
38	39,088,169	8
39	6,324,5986	7
40	102,334,155	6
41	165,580,141	4
42	267,914,296	1
43	433,494,437	5
44	701,408,733	6
45	1,134,903,170	2
46	1,836,311,903	8
47	2,971,215,073	1
48	4,807,526,976	9

← Table of the first 48 Fibonacci Numbers: the 24 digits Deep Structure Pulse is repeated twice.

SG104.5.4.3 Deep Structure of the Fibonacci Numbers (3)

We are just beginning to scratch the surface of a whole new understanding of the Beauty of Cosmic Harmonics. The Deep Structure Pulse of the Fibonacci Sequence (*"Fibo Pulse"*) is a window into the multi-dimensional resonance of the Number Field. Below are pointers from various researchers:

- **The most obvious observation is that each number in the Fibo Pulse is the sum of the two preceding ones (doing the digital root if needed), just like the parent Fibonacci Sequence. Not surprising but neat.**

- **Starting with '0', the sum total of all 24 digits = 108, an essential Sacred Geometry number and the inner angle of the pentagon (replete with PHI ratios). Two sets = 216 = 6 x 6 x 6 = $6^3 = 3^3 + 4^3 + 5^3$.**

- **Starting with '0' and extracting every other number in the Fibo Pulse gives a mini cycle of 12 units having 6 pairs of '9', symmetrically arranged around the central 9: 0 1 3 8 3 1 9 8 6 1 6 8 9. Extracting the remaining numbers gives a mini cycle of 12 units with mirror symmetry : 1 2 5 4 7 8 / 8 7 4 5 2 1**

One can arrange one set of the Fibo Pulse in 2 rows of 12 each (starting with 0):

0 1 1 2 3 5 8 4 3 7 1 8

9 8 8 7 6 4 1 5 6 2 8 1

or two sets in 2 rows (starting with 1):

1 1 2 3 5 8 4 3 7 1 8 9 8 8 7 6 4 1 5 6 2 8 1 9

8 8 7 6 4 1 5 6 2 8 1 9 1 1 2 3 5 8 4 3 7 1 8 9

This creates complementary pairs summing up to '9'.

Also the difference between a pair = the number of spaces to the next similar pair.

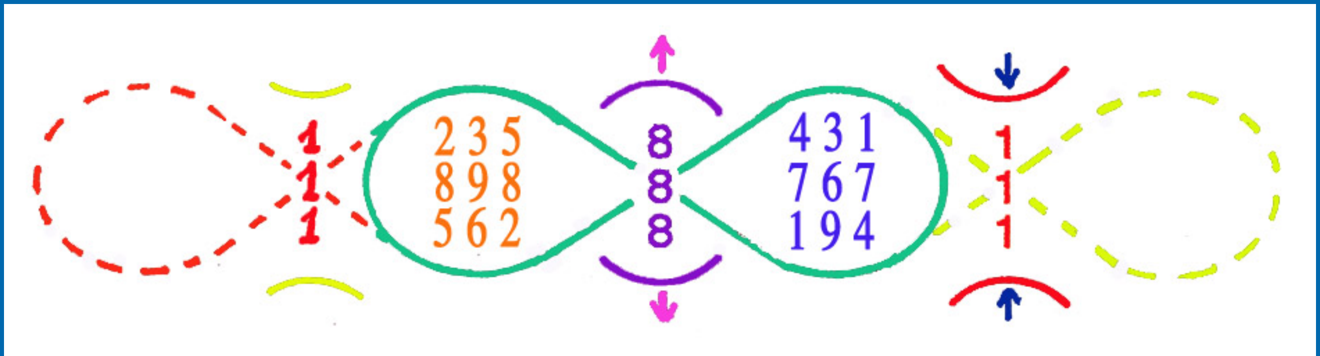
Example: the 7 and 2 pair is spaced by 5 pairs from the other 7-2 pair and 7 - 2 = 5.

SG104.5.4.4 Deep Structure of the Fibonacci Numbers (4)

• Starting with 1, the Fibo Pulse is divisible into 3 'octaves' of 8 units each:

1 1 2 3 5 8 4 3
7 1 8 9 8 8 7 6
4 1 5 6 2 8 1 9

This brings up two "beats' of 3 rows of 3 digits separated by a column of '1' and a column of '8':

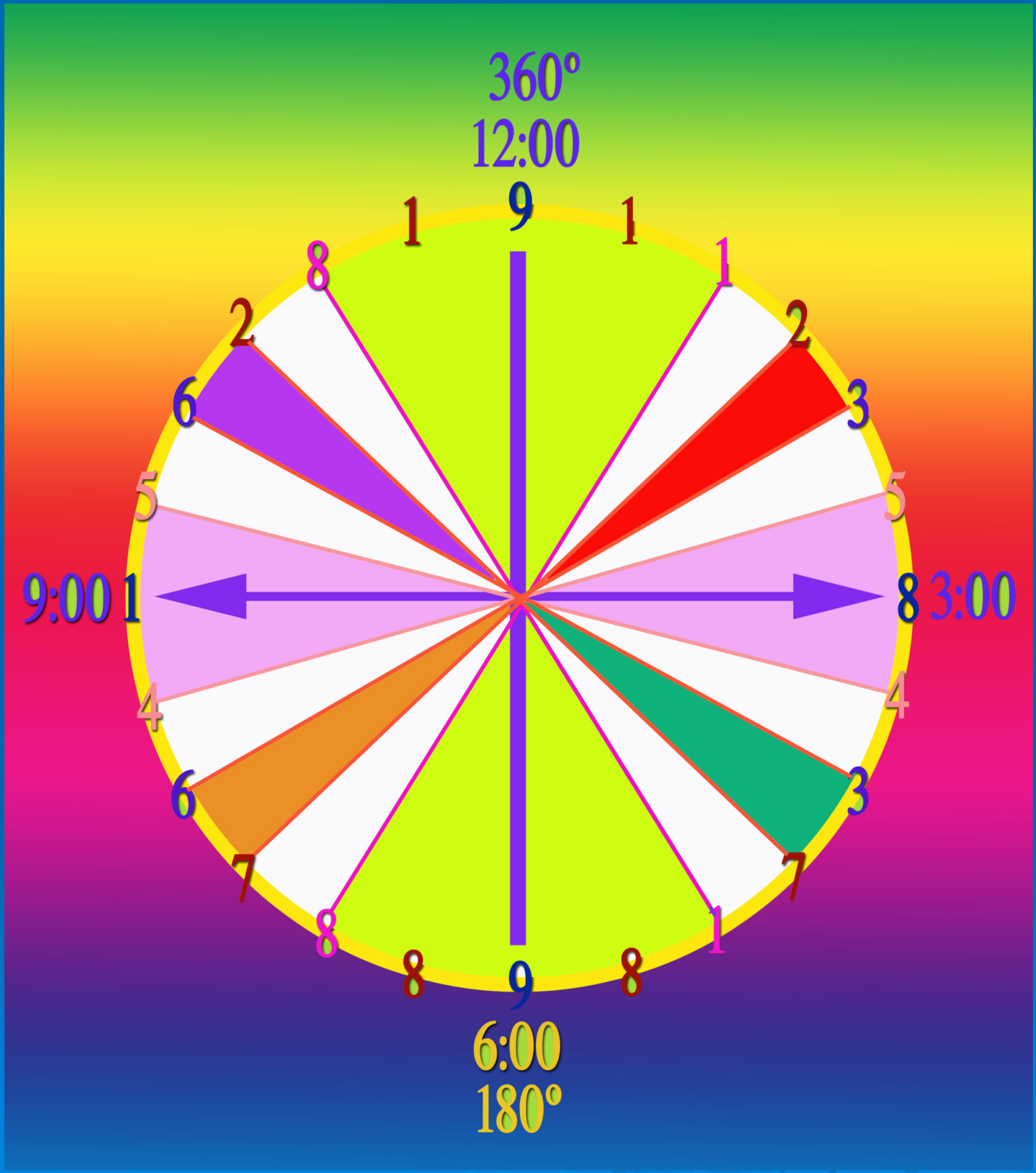


• Superimposing these 2 Fibo Beats on top of each other (with Digital Root) gives 3 rows of '6'. As "9-dots Patterns", these beats may well be connected with the traditions of Magic Squares 3 x 3.

• Let's practice 3D visualization. Imagine each of the two Fibo Pulse Beat is a 4-sided pyramid (First beat apex = 9; second beat apex = 6). Put them base-to-base and apex-to-apex to form a chain of octahedrons.

• Starting with 1, the sum total of all 24 digits = 117 = 39 x 3. You may recall that 39 is the Pulse of the Triangular Numbers. [◆SG102].

• The Fibo Pulse is a deeper level of understanding the PHI SEED contained in the surface pulsation waveform of the Fibonacci Sequence. It may well pertain to the new research about the Fibonacci & Lucas resonance patterns within DNA sequences.

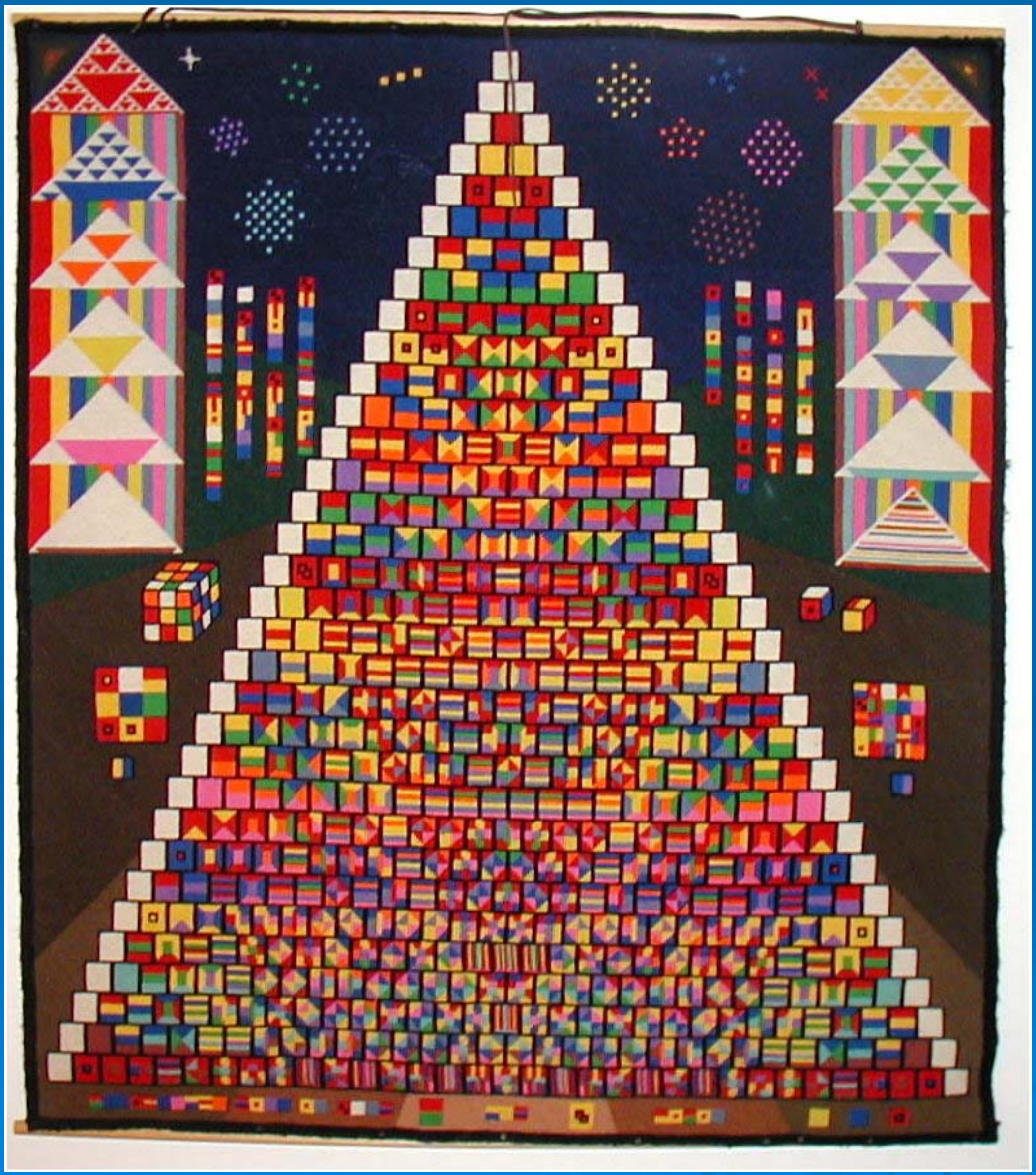


SG104.5.4.5 Deep Structure of the Fibonacci Numbers (5)

← The Fibo Pulse Mandala Clock.

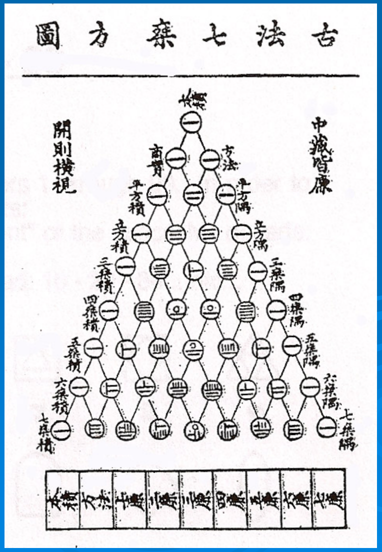
Pairing to complementary '9' in the Fibonacci Deep Structure Pulse.
See it as a vortex.

SG104.5.5.1 Pascal's Triangle & Fibonacci Numbers (1)



Known in the West as "*Pascal's Triangle*", this triangular pyramid of numbers was actually known long before Blaise Pascal (1623-1662) wrote about some of its properties in his *"Traité du Triangle Arithmétique"*.

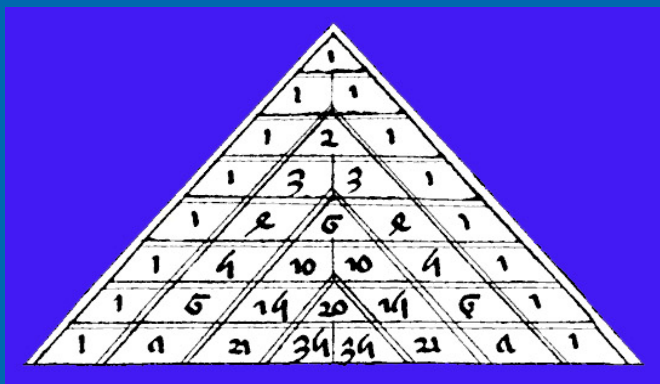
Indian mathematician Pingala (3rd century BC) had already recognized it. In a 10th century commentary of Pingala's Chandah-Shastra written by Jaina mathematician Halayudha, "Pascal's Triangle" is called *Meru-Prastaara* (the *Staircase to Mount Meru*). The Persian astronomer & poet Omar Khayyam (1048-1123) described it as well.



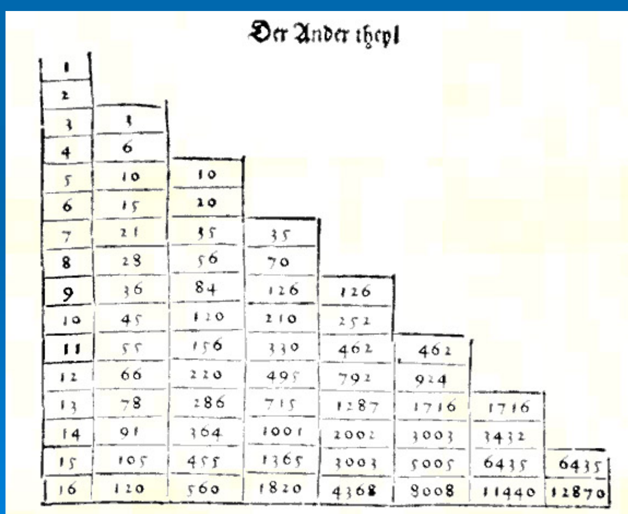
← Further East, it was also known and studied by the Chinese under the name of "*Triangle of Yang Hui*" (1261) and "*Triangle of Zhu Shi-Jiei*" (1303)

SG104.5.5.2 Pascal's Triangle & Fibonacci Numbers (2)

Three historical examples of the "*numeri trianguli*" published before Pascal's time.



↑ Jordanus' edition of De Arithmetica (1407)



Der Ander theyl

1								
2								
3	3							
4	6							
5	10	10						
6	15	20						
7	21	35	35					
8	28	56	70					
9	36	84	126	126				
10	45	120	210	252				
11	55	165	330	462	462			
12	66	220	495	792	924			
13	78	286	715	1287	1716	1716		
14	91	364	1001	2002	3003	3432		
15	105	455	1365	3003	5005	6435	6435	
16	120	560	1820	4368	8008	11440	12870	

↑ Michael Stifel's "*Figurate Triangle*" (1544)

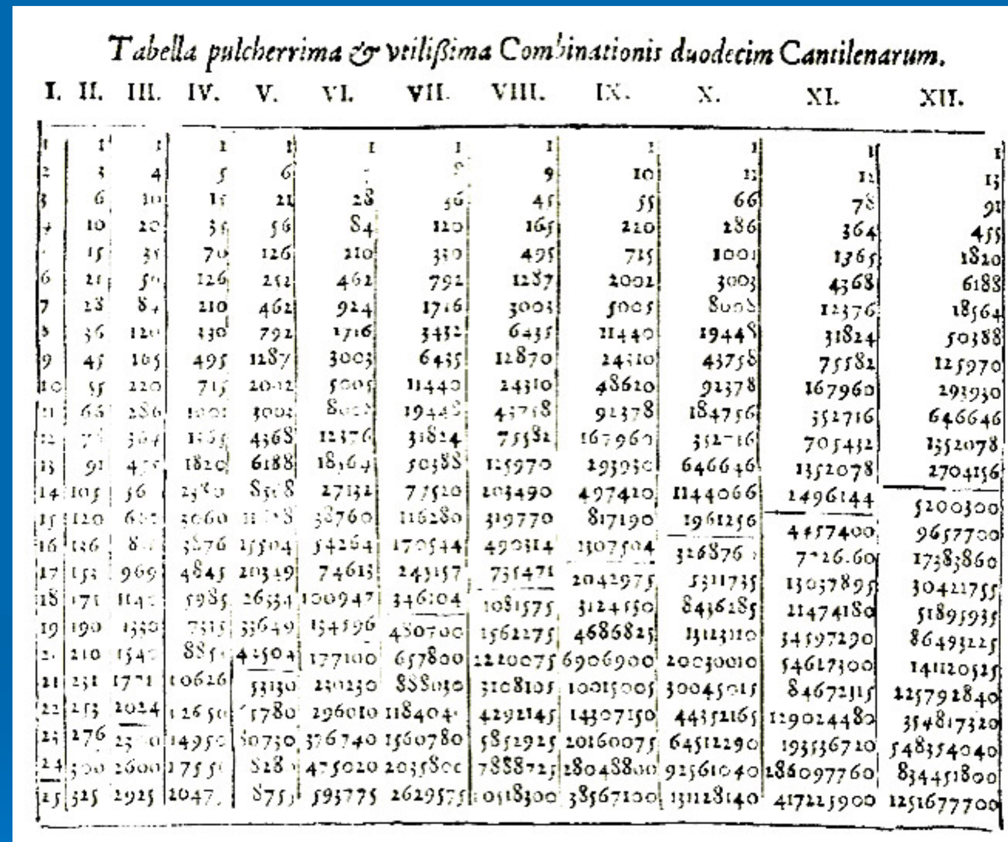
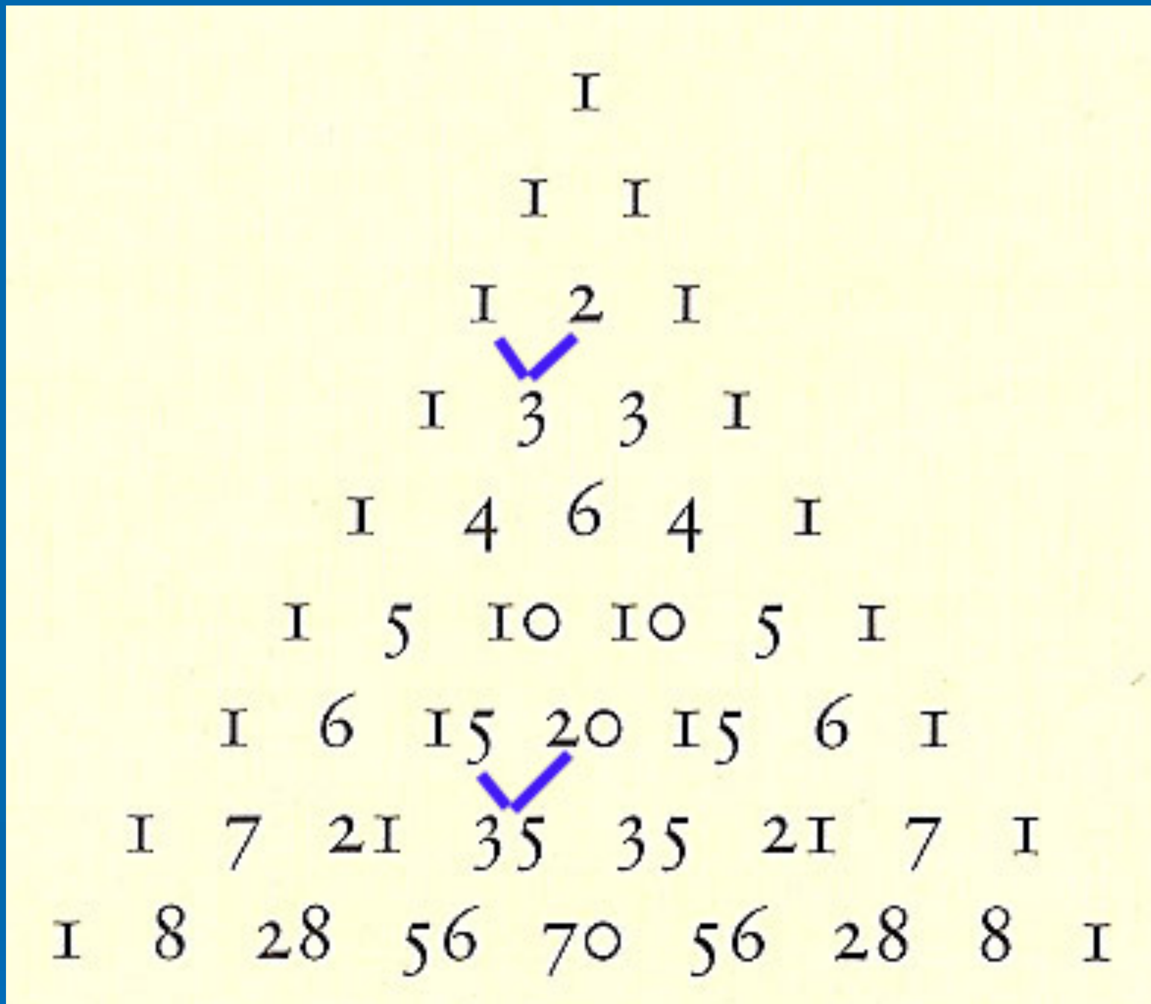


Tabella pulcherrima & vtilißima Combinationis duodecim Cantilenarum.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1	1	1	1	1	1	1	1	1	1	1	1
2	3	4	5	6	[illegible]	[illegible]	9	10	11	12	13
3	6	10	15	21	28	36	45	55	66	78	91
4	10	20	35	56	84	120	165	220	286	364	455
[illegible]	15	35	70	126	210	330	495	715	1001	1365	1820
6	21	56	126	252	462	792	1287	2002	3003	4368	6188
7	28	84	210	462	924	1716	3003	5005	8008	12376	18564
8	36	120	330	792	1716	3432	6435	11440	19448	31824	50388
9	45	165	495	1287	3003	6435	12870	24310	43758	75582	125970
10	55	220	715	2002	5005	11440	24310	48620	92378	167960	293930
11	66	286	1001	3003	8008	19448	43758	92378	184756	352716	646646
12	78	364	1365	4368	12376	31824	75582	167960	352716	705432	1352078
13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078	2704156
14	105	56	[illegible]	8568	27132	77520	203490	497410	1144066	1496144	5200300
15	120	[illegible]	3060	[illegible]	38760	116280	319770	817190	1961256	4457400	9657700
16	136	[illegible]	3876	15504	54264	170544	490314	1307504	326876	[illegible]	17383860
17	153	969	4845	20349	74613	243157	735471	2042975	5311735	13037895	30421755
18	171	[illegible]	5985	26334	100947	346104	1081575	3124550	8436285	21474180	51895935
19	190	[illegible]	7315	33649	154596	450700	1562275	4686825	13123110	34597290	86493225
20	210	1540	8855	42504	177100	657800	2220075	6906900	20030010	54627300	141120525
21	231	1771	10626	53130	230230	888030	3108105	10015005	30045015	84672315	225792840
22	253	2024	12650	65780	296010	1184040	4292145	14307150	44352165	129024480	354817320
23	276	[illegible]	14950	80730	376740	1560780	5852925	20160075	64512290	193536720	548354040
24	300	2600	17550	[illegible]	475020	2035800	7888725	28048800	92561040	186097760	834451800
25	325	2925	2047[illegible]	875[illegible]	593775	2629575	10518300	38567100	131128140	417225900	1251677700

↑ The "Fibonacci Table" in Father Marin Mersenne's *Harmonicorum Libri XII* (1636). Both Blaise Pascal and his father Etienne Pascal knew Mersenne.

SG104.5.5.3 Pascal's Triangle & Fibonacci Numbers (3)



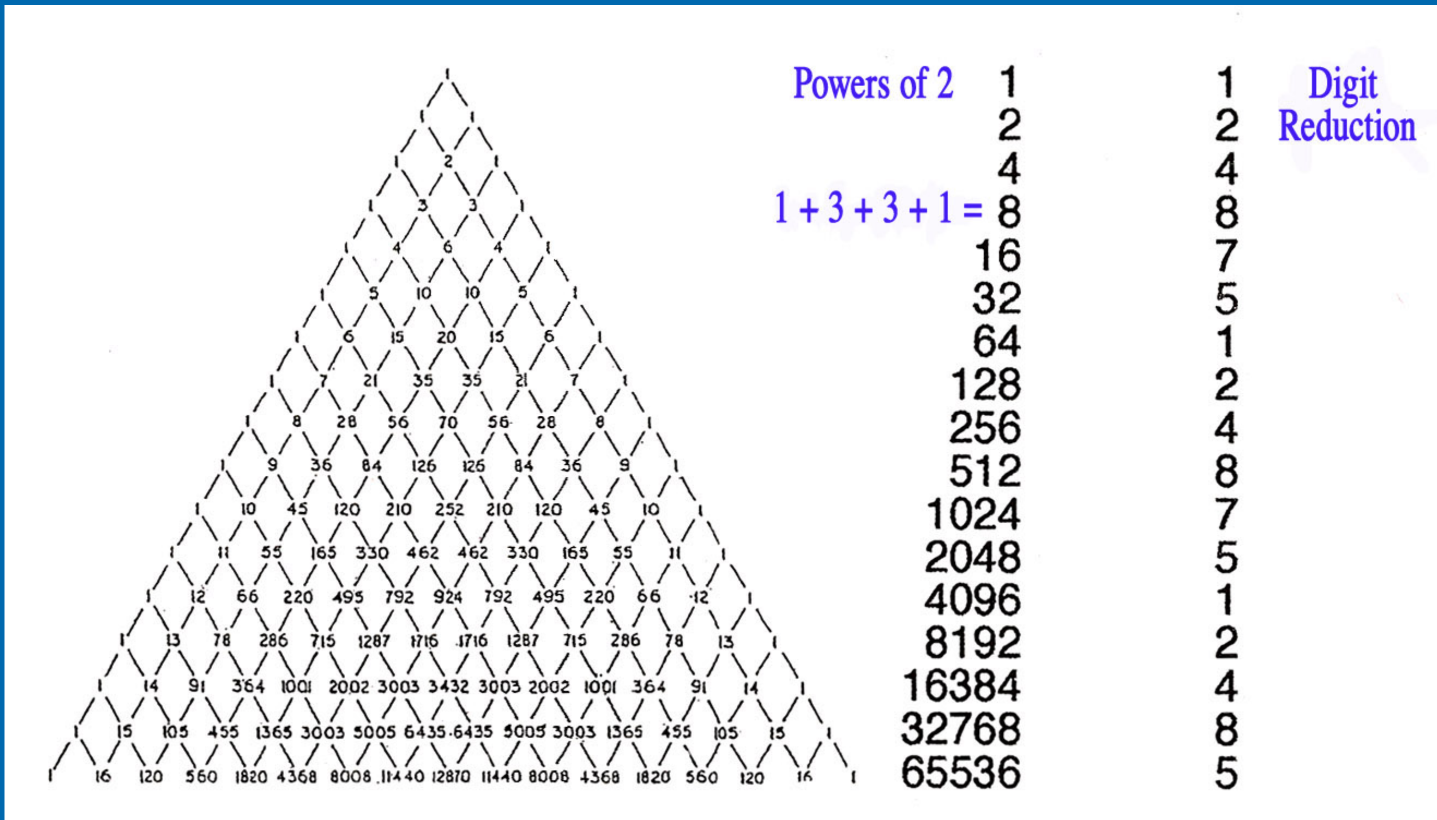
<u>Notice the two sides of "1".</u>
Then it's just a matter of adding the two "parent" numbers.
Examples: 3 (on row #4) results from 1 + 2
35 (on row #8) results from 15 + 20

This grid-matrix is a beautiful example of visual geometry that has survived all the way to modern mathematics as a table of the numerical coefficients for the binomial expansion (a "*binomial*" is an algebraic expression consisting of two terms).

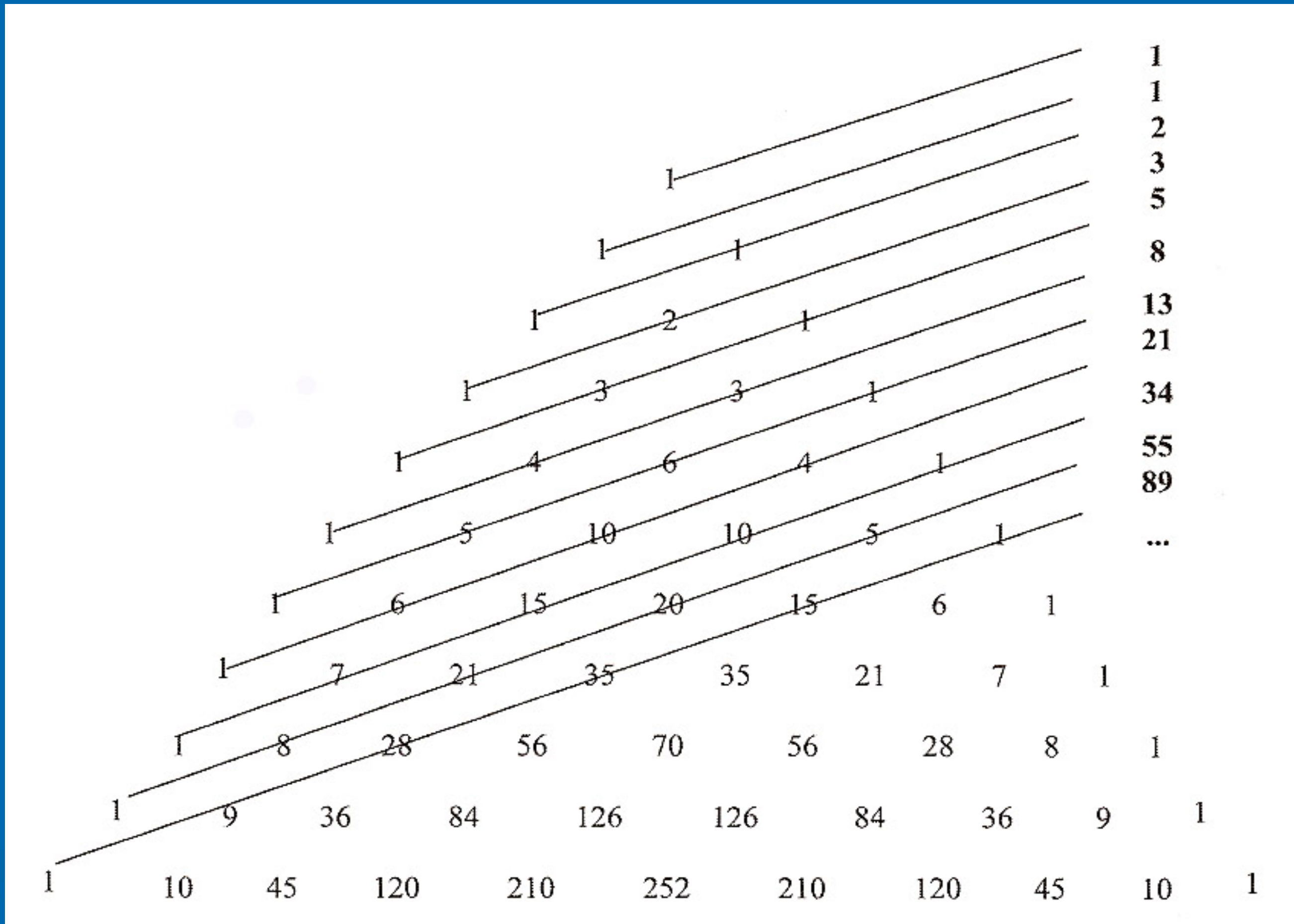
Pascal's Triangle is a great exercise in pattern recognition, digit reduction and periodicity hunting in the field of numbers…

… and it features the Fibonacci Sequence!

SG104.5.5.4 Pascal's Triangle & Fibonacci Numbers (4)



- **Every other number is the sum of the two parent numbers, on its right and its left in the row above it.**
- **Each row read across is a power of 11. Examples: 121 = 11^2 or 14641 = 11^4.**
- **The sum of each row is a power of 2. Examples: 1 + 3 + 3 + 1 = 8 = 2^3 or 1 + 5 + 10 + 5 + 1 = 32 = 2^5**
- **Each row shows the coefficients for $(a + b)^n$. Example: The coefficients of $(a + b)^5$ are 1, 5, 10, 10, 5 and 1. Then we have: $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$**
- **Adding any two successive numbers in the diagonal 1-3-6-10-15-21-28... results in a perfect square (1, 4, 9, 16, etc.)**
- **Notice that the Digit reduction of the sum of each row gives a repeated pattern: 1--2-4-8-7-5 (Pulse-6)**



SG104.5.5.5 Pascal's Triangle & Fibonacci Numbers (5)

This is how Fibonacci and Pascal are connected throughout the ages…

The shallow diagonals of Pascal's Triangle display the Fibonacci Numbers!

The story goes that Blaise Pascal and Pierre de Fermat were sitting at a Paris café playing a game of chance by flipping a coin for money. Suddenly, Fermat was called away. This led the two men to consider how the stakes should be divided. Eventually they laid down the foundation for the study of probabilities (the "odds"). Interestingly enough, the Fibonacci numbers are quite involved in the "laws" of probability and Pascal's Triangle is used to find combinations in probability problems.

Example: if you pick any two of five items, the number of possible combinations is 10, the second number in the fifth row. Note: Ignore the 1's.

SG104.5.6 The Golden String

The Golden String (also called the "*Rabbit Sequence*") is a remarkable sequence of 0s and 1s which is intimately related to the Fibonacci numbers and to Phi and has amazing harmonic properties: 1 0 1 1 0 1 0 1 1 0 1 1 0 1 0 1 1 0 1 0 1 1 0 1 …

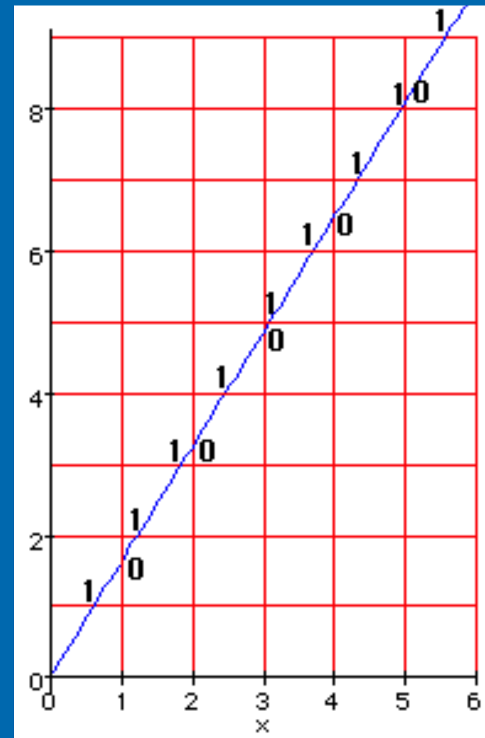
The Rabbit Sequence uses 0s and 1s instead of M(ature rabbit) and N(ew rabbit).

s(0)=0	=0	1	0
s(1)=1	=1	0	1
s(2)=1+0	=1	1	1
s(3)=1+0+1	=2	1	2
s(4)=1+0+1+1+0	=3	2	3
s(5)=1+0+1+1+0+1+0+1	=5	3	5
s(6)=1+0+1+1+0+1+0+1+1+0+1+1+0	=8	5	8
...			

↑ Number of 0s and 1s in S(n)

s(0)=0	0
s(1)=1	0
s(2)=1+0	1
s(3)=1+0+1	2
s(4)=1+0+1+1+0	4
s(5)=1+0+1+1+0+1+0+1	7
s(6)=1+0+1+1+0+1+0+1+1+0+1+1+0	12
...	

↑ Number of addition operations ("A series")



← Graph of y = PHI (x)

1 = PHI line crossing horizontal grid line

0 = PHI line crossing vertical grid line.

Lo and behold: here we have the Rabbit Sequence!

1 0 1 1 0 1 0 1 1 0 1 1 0 1 0 1 1 0 1 …

n:	0	1	2	3	4	5	6	7	8	9	10
A(n):	0	0	1	2	4	7	12	20	33	54	88
f(n+1):	1	1	2	3	5	8	13	21	34	55	89

↑ The "A series" numbers are just 1 less than a Fibonacci number.

www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibrab.html

SG104.5.7 Other Fibonacci Family Members

Generalization of Fibonacci Numbers

NegaFibonacci Numbers are the negatively indexed elements of the Fibonacci sequence.
They are defined inductively by the recurrence relation: F-1 = 1, F-2 = -1, Fn = F(n+2)−F(n+1).
The first 10 NegaFibonacci numbers are 1, -1, 2, -3, 5, -8, 13, -21, 34, -55
They can represent any integer and they provide a system of coding integers using a binary representation.

N-Nacci Numbers are higher order Fibonacci Series:

Tribonacci Numbers: each element of the Tribonacci Sequence is the sum of the previous 3 elements.
0-0-1-1-2-4-7-13-24-44-81-148-274-504-927-1705-3136-5768-10609-19513-35890…
The Tribonacci constant is 1.83929… satisfying the equation $x + x^3 = 2$

Tetranacci Numbers: starts with 4 predetermined terms and each term is the sum of the previous 4.
0-0-0-1-1-2-4-8-15-29-56-108-208-401-773-1490-2872-5536…
The Tetranacci constant is 1.92757

Pentanacci Numbers: 0-0-0-0-1-1-2-4-8-16-31-671-120-236-461-912-1793-3522…
The Pentanacci constant is 1.96430

Hexanacci Numbers: 0-0-0-0-0-1-1-2-4-8-16-32-64-125-248-492-976-1936-3841…
The Hexanacci constant is 1.98398

Heptanacci Numbers: 0-0-0-0-0-0-1-1-2-4-8-16-32-64-127-253-504-1004-2000-3984…
The Heptanacci constant is 1.992

Octanacci Numbers: 0-0-0-0-0-0-0-1-1-2-4-8-16-32-64-128-255-509-1016-2028-4048…
The Octonacci constant is 1.99605

Little did dear Leonardo Fibonacci realized he would have so many grand-children!

SG104.5.8.1 The Lucas Series

2 - 1 - 3 - 4 - 7 - 11 - 18 - 29 - 47 - 76 - 123 - 199 - 322 - 521…

Francois Edouard Anatole Lucas (1842-1891) was a 19th century French mathematician from whom the Lucas Series is named - which is quite fair as Lucas gave the name "Fibonacci Numbers" to the series studied by Leonardo of Pisa in the 12th-13th century.

The Lucas series is defined almost identically to the Fibonacci series, except that the Lucas series begins with 2 and 1 instead of 1 and 1.

The Lucas series converges to PHI just like the Fibonacci series, only in a slower way:

Comparative examples of PHI Convergence:
Fibonacci: 233 / 199 = 1.618055
(closer to PHI and faster)
Lucas: 322 / 199 = 1.618090
(not as exact as fast)

This PHI convergence is true for any Fibonacci-like series starting with any pair of numbers: The limit of growth rate will always be PHI.
(See ◆SG104.5 "*Lusus Numerorum*")

n	Fib(n)	2n	Fib(2n)	k=Fib(2n)/Fib(n)
1	1	2	1	1
2	1	4	3	3
3	2	6	8	4
4	3	8	21	7
5	5	10	55	
6	8	12	144	
7	13	14	377	

↑ Fib (2n) = Fibonacci numbers in an even position. If we divide Fib (2n) by the regular Fibonacci numbers, we get the "k" series or Lucas series.

SG104.5.8.2 Lucas & Fibonacci: the PHI Link

(Fibonacci Series)

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584

1 3 4 7 11 18 29 47 76 123 199 322 521 843 1364 2207 3571

(Lucas series)

F			F																
1	1	2	3	4	5	7	8	11	13	18	21	29	34	47	55	76	89	123	144
L			L	L		L		L		L		L		L		L		L	

199	233	322	377	521	610	843	987	1364	1597	2207	2584	3571
L		L		L		L		L		L		L

Combined Fibo-Lucas: 1. Lucas = Fn_{-1} + Fn_{+1} , where Fn_{-1} is Fibo closest down.

Examples: Lucas 322 = 233 + 89 Lucas 1364 = Fibo 987 + 377

2. Any Lucas / preceding Fibo two places down (Fn_{-2}) = $\sqrt{5}$ = 2.236

Examples: Lucas 521 / Fibo 233 = 2.236 Lucas 3571 / Fibo 1597 = 2.240

3. Lucas = Sum of corresponding Fibo + Fn_{-1} + Fn_{-2} + Fn_{-3}

Example: Lucas 521 = 233 + 144 + 89 + 55

SG104.5.9 Lusus Numerorum Experiment

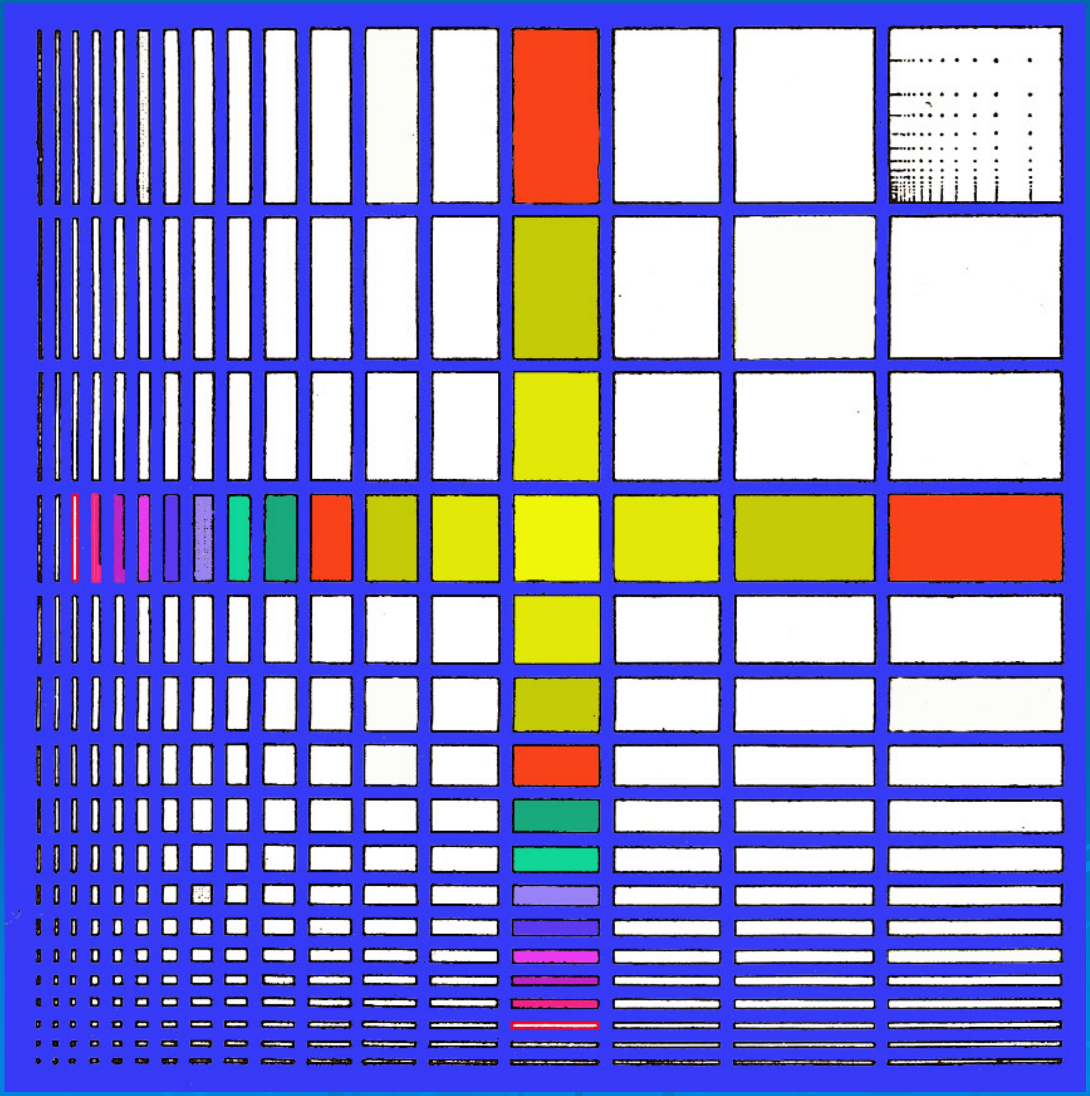
This is a "Game of Numbers" group experiment to provide more insights on the cumulative proportional property of the Golden Ratio PHI

Step #1: Provide calculators & paper/pen.
Step #2: Ask each member of the class/group to pick any 2 digits (from 1 to 9).
Step #3: Ask the players to add their 2 numbers in order to get the third number in a series, and to keep adding the two last numbers until they have a written list of 20 numbers.
Step #4: The last number (#20) will be divided by the penultimate (next to last = #19) to obtain a quotient.
Step #5: Compare the results as a group.

Example 1: Numbers 3 & 7. **96917 ÷ 59898 = 1.6180339...**	10 - 17 - 27 - 44 - 71 - 115 - 186 301 - 487 - 788 - 1275 - 2063 8739 - 14140 - 3338 - 5401 22879 - 37019 - 59898 - 96917
Example 2: Numbers 1 & 3. **(The "Lucas series")** **39603 ÷ 24476 = 1.6180339...**	4 - 7 - 11 - 18 - 29 - 47 - 76 - 123 199 - 322 - 521 - 843 - 1364 2207 - 3571 - 5778 - 9349 - 39603 15127 - 24476 - 39603
Example 3: Numbers 8 & 1. **65066 ÷ 40213 = 1.6180339...**	9 - 10 - 19 - 29 - 48 - 77 - 125 202 - 327 - 529 - 856 - 1385 2241 - 3626 - 5867 - 9493 - 15360 24853 - 40213 - 65066

Surprise! All quotients converge to PHI. How is that possible?

SG104.6. Chapter 6. The Ubiquity of Fibonacci Numbers



SG104.6.2 Phi in Quarks & E-Infinity

M. S. El Naschie, the founder of the journal Chaos, Solitons & Fractals, is known for his E-infinity theory, sometimes called Cantorian space-time, which is the first theory that makes use of the concepts of Cantor set and golden ratio in high energy physics. The theory successfully predicted the accurate values of mass spectrum of elementary particles.

The E-Infinity theory presents the Golden Ratio as the "winding number" in the harmonic manifestation of quark & subatomic particle masses.

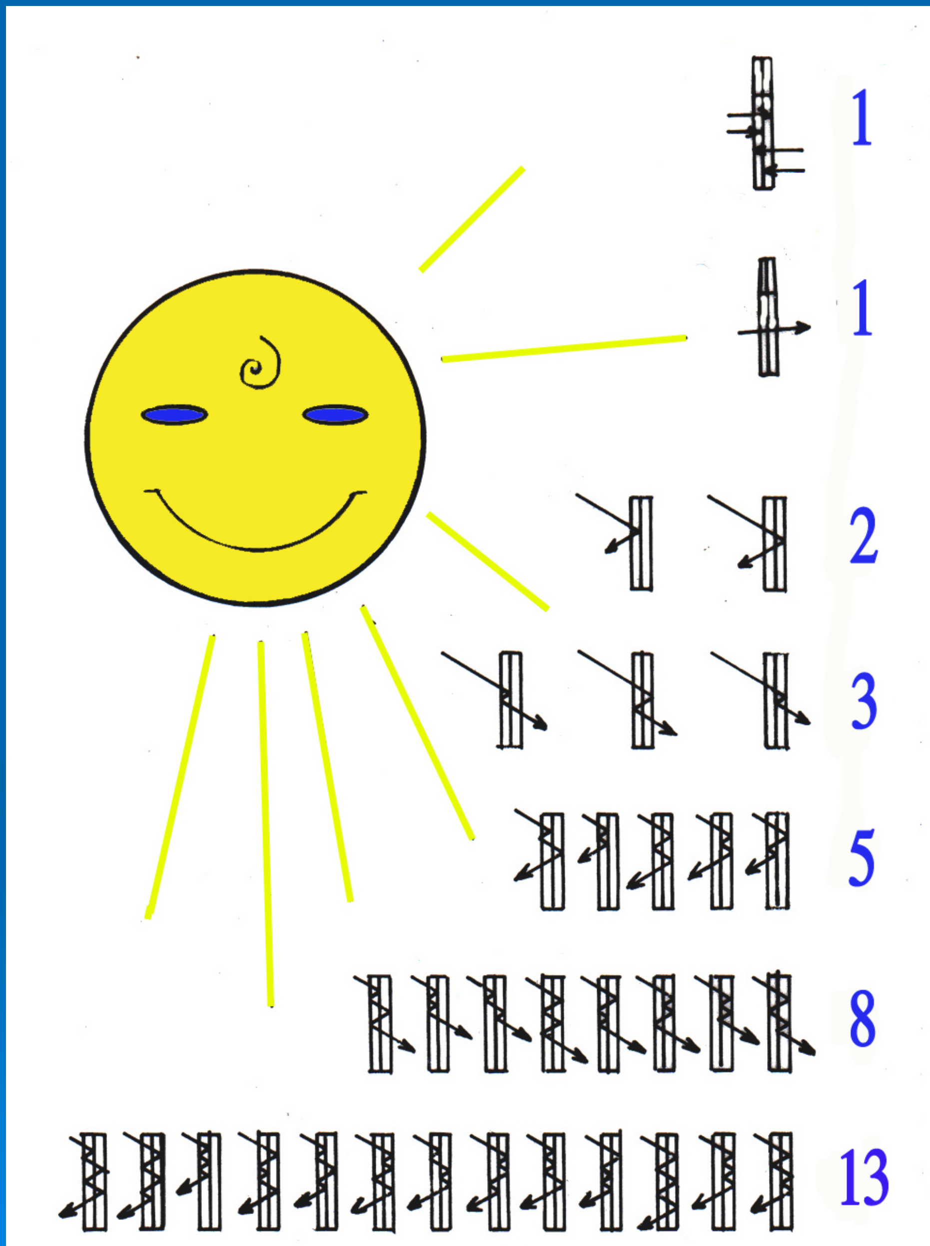
"The appearance of the Golden Mean as the frequency of vibration and mass-energy factor…. indicates that it is the simplest realistic unit from which a Hamiltonian dynamics can start developing a highly complex structure, a so-called nested vibration… The Golden Mean plays a decisive role in non-linear dynamical stability, chaotic systems and hi energy particle physics… (by creating the most stable periodic orbit due to being the most irrational number, according to the chaos border KAM theorem)."

In the words of El Naschie, particle physics appears as a "*cosmic symphony*" and the values of quark masses form a "*harmonic musical ladder*".

Quark Flavor	Current Mass (MeV)	Constituent Mass (MeV)
Up	$2\ \Phi^2 = 5.236$	$80\ \Phi^3 = 338.885\ldots$
Down	$2\ \Phi^3 = 8.472\ldots$	$80\ \Phi^3 = 338.885\ldots$
Strange	$10\ \Phi^6 = 179.442\ldots$	$10\ \Phi^8 = 469.787\ldots$
Charm	$300\ \Phi^3 = 1{,}270.82\ldots$	$20\ \Phi^9 = 1{,}520.263\ldots$
Beauty/Bottom	$10^3\ \Phi^3 = 4{,}236.067\ldots$	$100\ \Phi^8 = 4{,}697.871\ldots$
Truth/Top	$104\ \Phi^3 = 42{,}360.673$	$10^4\ \Phi^6 = 179{,}442.719\ldots$

↑ Current and constituent quark mass as function of Φ and 1/Φ

SG104.6.3 Fibonacci Numbers in Optics

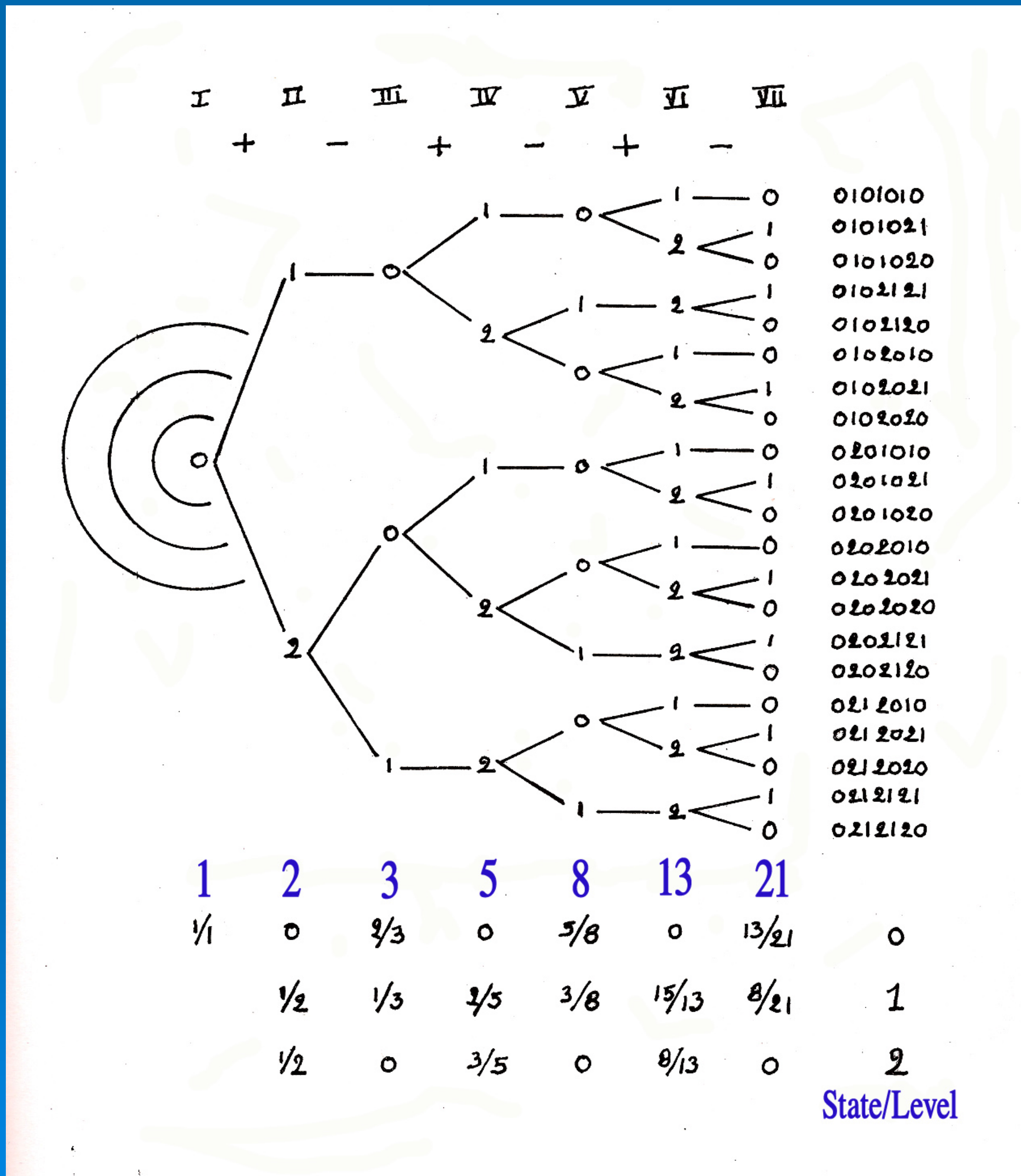


Take two glass plates with different light refraction properties, i.e. made out of different types of glass. Mount them face to face.

Now, if a beam of light is incident upon the two plates, part of the light will be transmitted, part absorbed and the rest reflected. There will multiple reflections allowed and accounted for by the physics of light rays (optics). The number of different paths followed within the glass plates before the rays emerge depends on the number of reflections which the ray undergoes.

It so happens that the number of emergent rays is a Fibonacci Number.

SG104.6.1 Fibonacci Numbers in Atoms



Possible Histories of an electron of Hydrogen Gas

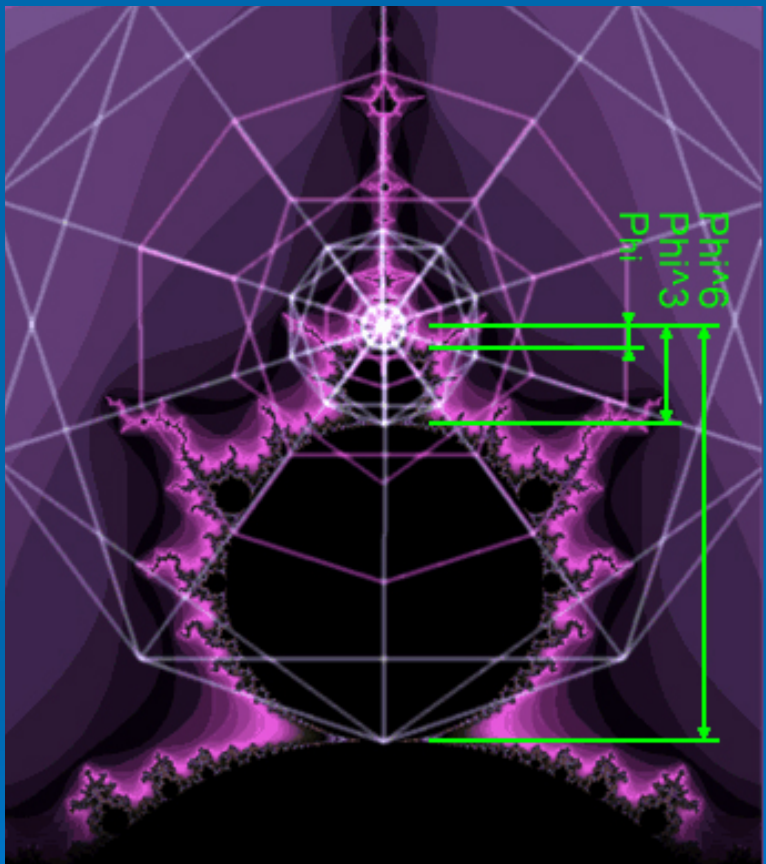
Fibonacci Numbers show up in connection with the possible histories of an electron in an atom of hydrogen gas.

Electrons pass through 3 different states depending on their energy levels (state 0, state 1 and state 2) and certain rules apply as they gain or lose energy (+ or -).

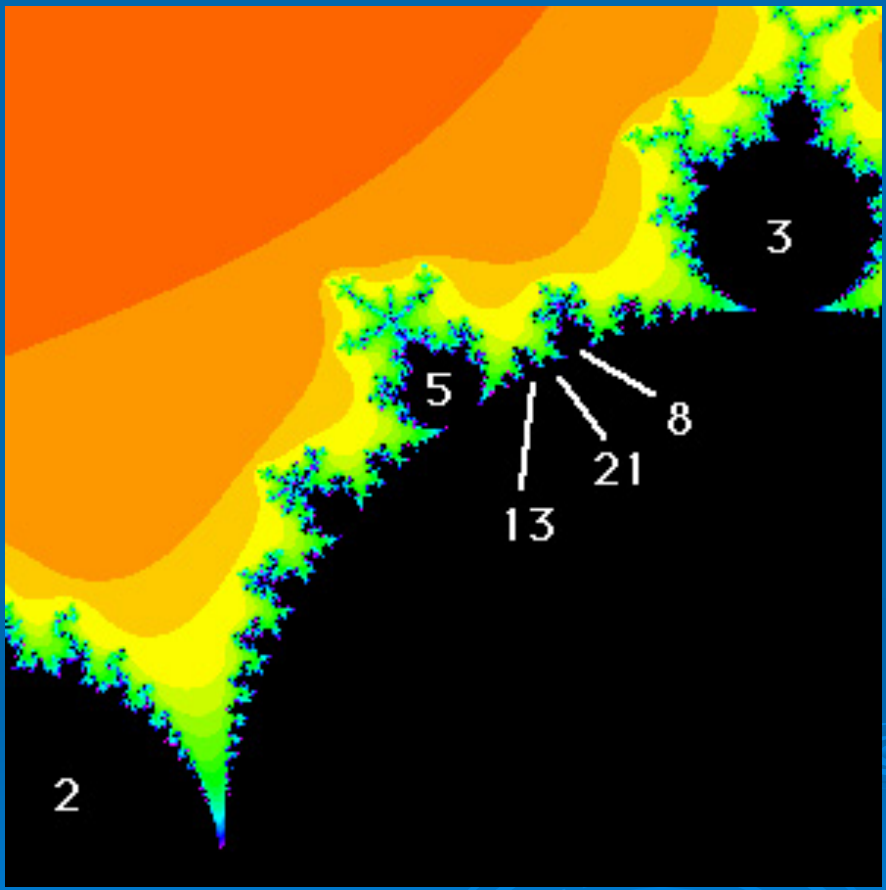
← The fractions on the diagram show the ratios of different states (0, 1, 2) as time passes: they are formed by Fibonacci numbers.

The numbers of possible different histories of an electron are also Fibonacci numbers.

SG104.6.4 Fibonacci Numbers in Fractals



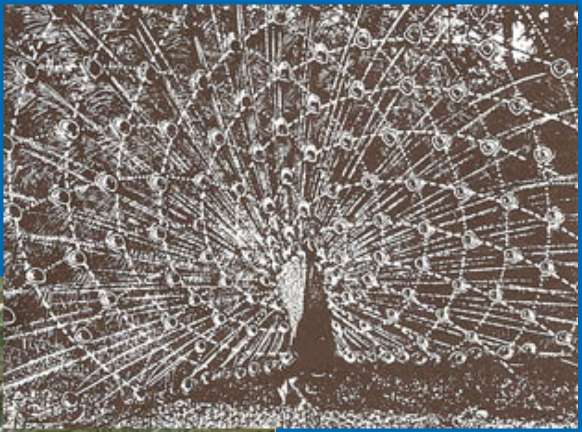
The "Starmother" cosmic fractal (Icosa-dodeca stellation) and the Mandelbrot fractal: both creative symmetries in the Bhaeravii stage, both self-referent by a ratio of Phi to the power of 3...



[◆SG203.1]

SG104.6.5 Fibonacci Numbers in Nature

The are innumerable examples of Fibonacci numbers / Phi ratios in nature, on all scales. We will meet many in some of the next modules: ◆SG204, SG205, SG206… Below, one beautiful instance…



The PHI_based Golden Spiral is a log spiral.

↑ The eyes of the peacock's tail are at the intersections of logarithmic spirals.

SG104.6.6 Fibonacci Numbers in the DNA

In his seminal 1997 book *L'ADN Decrypté* (DNA Decoded), mathematician and computer scientist Jean-Claude Perez methodically explains how he uncovered a revolutionary fact: the DNA is woven with a Supra-Code of multi-layered resonances that constitute a hidden yet precise language.

The sequences of the four DNA bases ACTG are harmonically interconnected through properties based upon the Fibonacci & Lucas Series, and therefore the Golden Ratio. J. C. Perez calls these properties "Resonances" and their global patterns of music-like pulses the "DNA Supra-Code".

This is BIG NEWS! And yet this discovery has not yet penetrated the walls of official genetics, even though it was made in 1990 and regularly published since 1991.

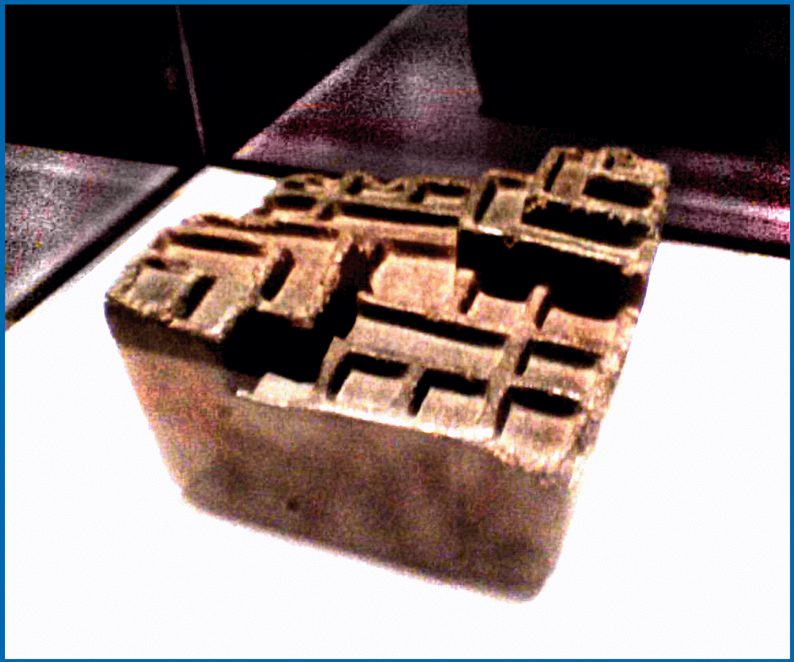
When finally accepted, it will herald the entry of Sacred Geometry in genetics and will open a new, deeper level of perception of their field for geneticists and biologists.

And, like always when the "time comes" for updating obsolete scientific models, it will suddenly seem natural and obvious: YES, of course, the DNA is a harmonic and musical score, a grand orchestration of life with exquisite chords in tune with PHI Major and Phi Minor. The two Leonardos would have been thrilled.

In later modules [◆SG204], we will go into a lot more details about the protocols of research and the implications of this revolutionary discovery.

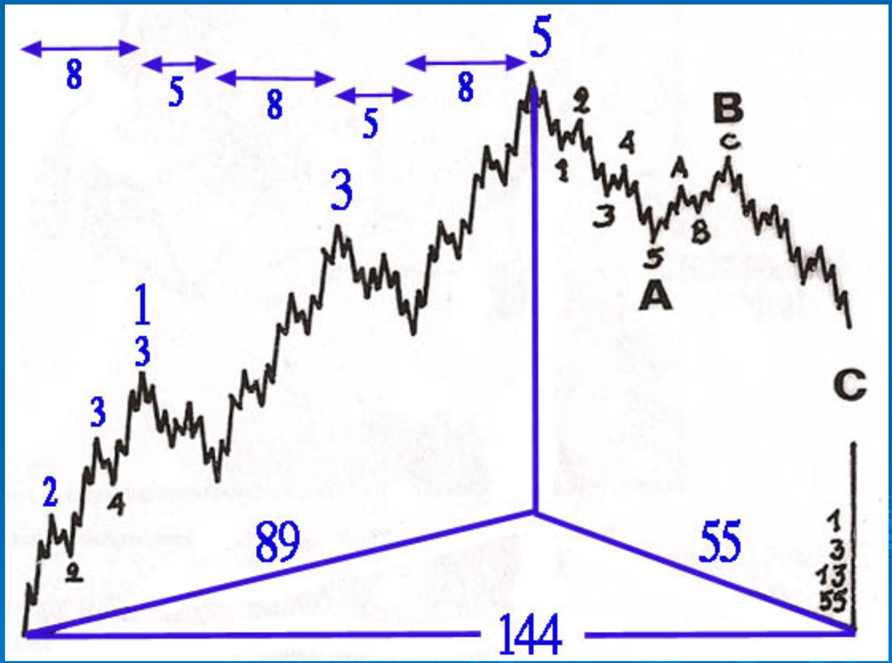
SG104.6.7.1 Fibonacci Numbers in Culture

In later modules, we will be encountering many examples of Fibonacci numbers in cultures, both ancient and contemporary. On this page, we give one example of each.



↑ A *yupana* (Quechua for "*counting tool*") is a calculator which was used by the Incas. Researchers assume that calculations were based on Fibonacci numbers to minimize the number of necessary grains per field.

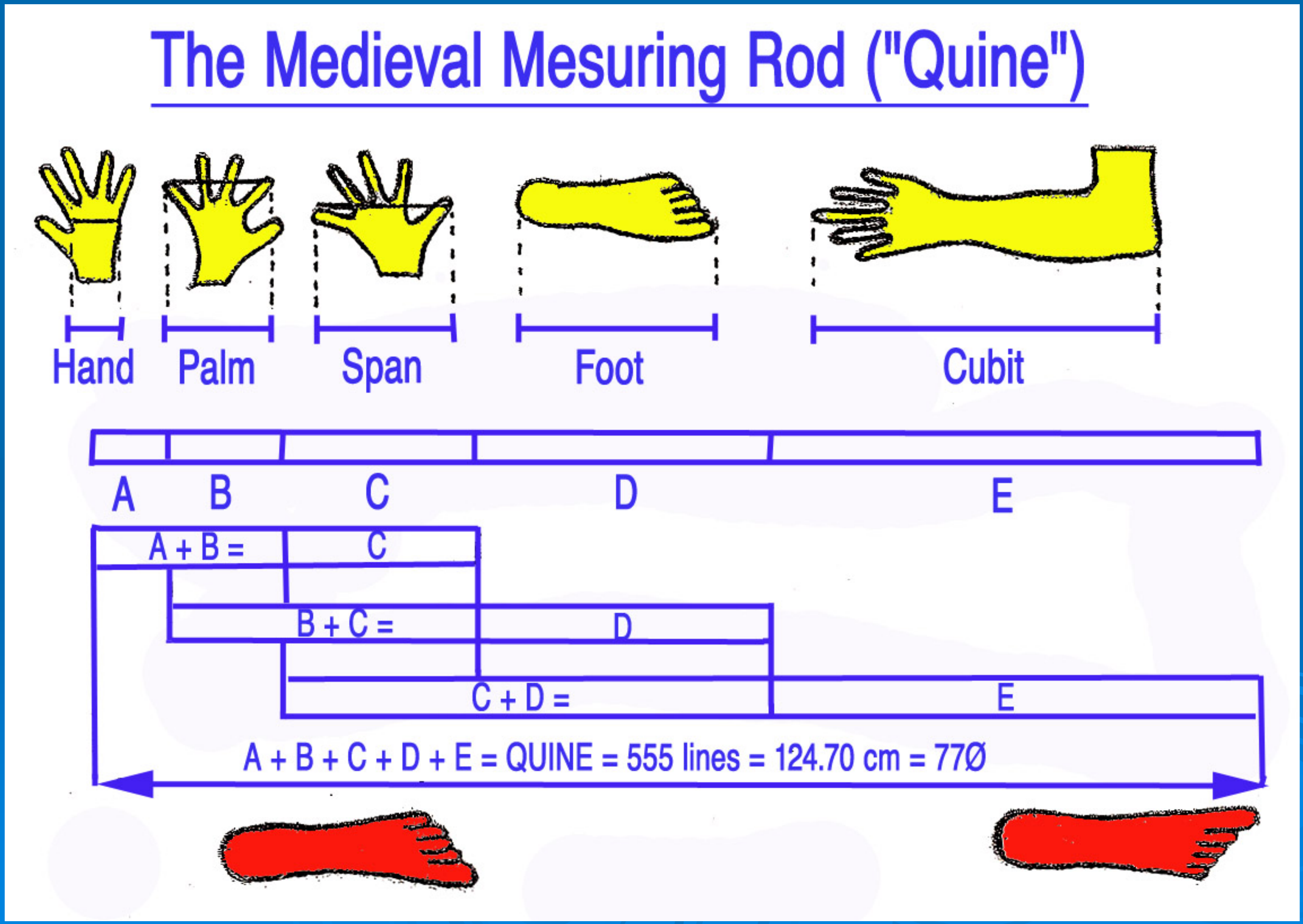
(Wikipedia)



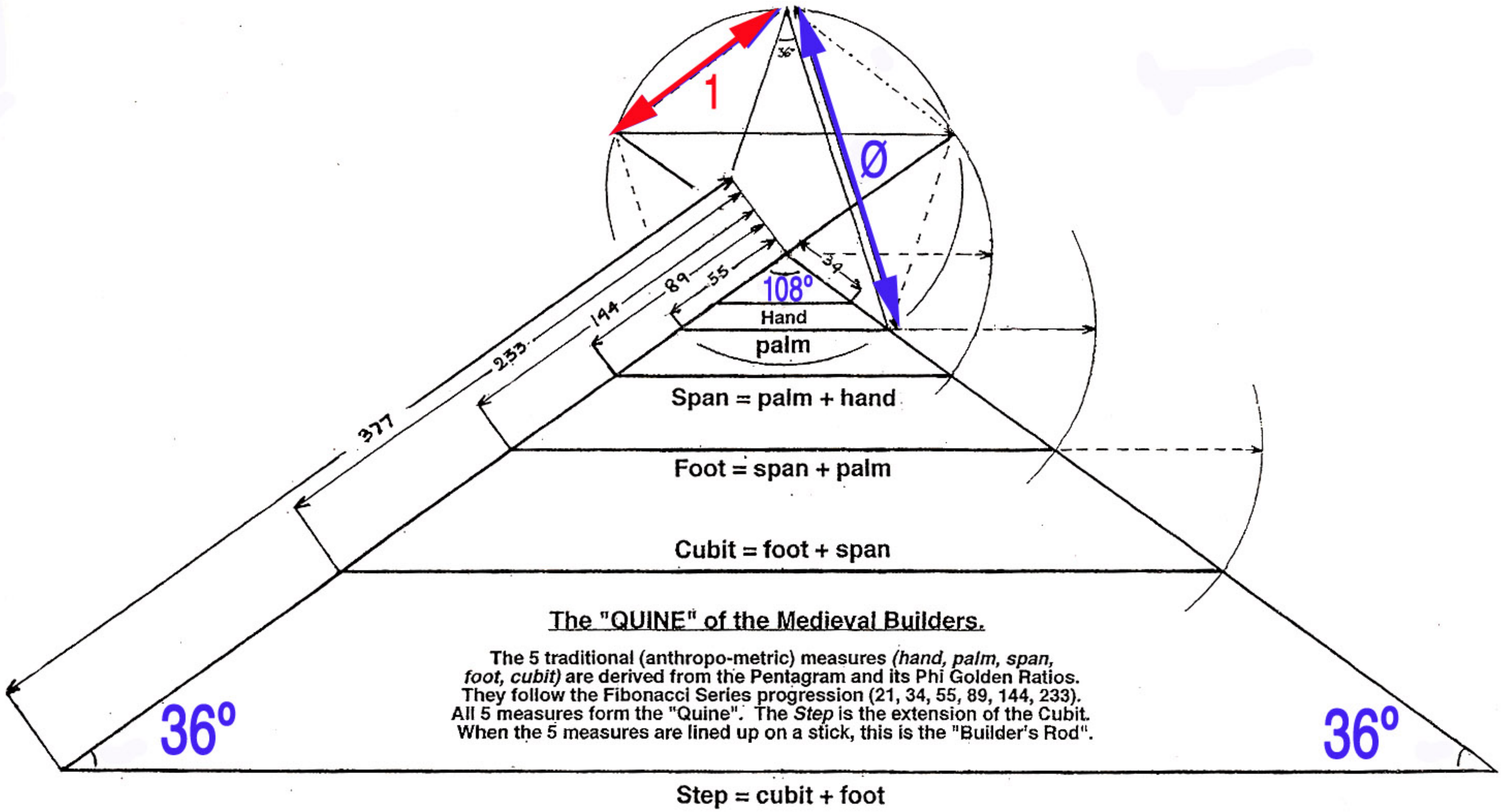
↑ Pattern for a complete stock market cycle, according to the Elliott Wave Theory.

SG104.6.7.2 The Medieval “Fibonacci” Quine (1)

The medieval geometers & architects in Europe were using a builder’s rod called “*la Quine*” (from Latin *quini* = five by five. The Quine is based on the five traditional (anthropo-metric) measures: hand, palm, span, foot and cubit. These in turn are derived from the Fibonacci Series, its converging limit the Golden Ratio and, therefore, the Pentagram. All five measures lined up on a measuring stick form the Quine or Builder’s Rod. The *Step* is the harmonic extension of the cubit.



SG104.6.7.3 The Medieval "Fibonacci" Quine (2)



The 108-36-36 triangle is the Golden Gnomon Penta-Module of the Pentagram.

SG104.6.7.4 The Medieval “Fibonacci” Quine (3)

UNITF	EQUIVALENCE	LENGTH (CM)	PHI SERIES
Line	Barley seed	0.2247	
Thumb	12 lines	2.6964	
Hand	34 lines	7.64	$1/\Phi^2 = 2 - \Phi$
Palm	55 lines	12.36	$1/\Phi = \Phi - 1$
Span	89 lines	20	$1 = 1$
Foot	144 lines	32.36	$\Phi = \Phi$
Cubit	233 lines	52.36	$\Phi^2 = 1 + \Phi$

Notice the number 52.36 cm = .5236 meter. It so happens that .5236 is the length of the Royal Cubit of Memphis used by the Egyptians.

Also consider the following:

$$\pi / 6 = 3.1416 / 6 = .5236$$

$$\sqrt{5} = 2.236 \quad 1 / \Phi^3 = .236$$

$$\sqrt{5} + 3 = 5.236$$

$$\Phi + 1 = 2.618 = .5236 \times 5$$

$$\Phi = (\sqrt{5} + 1) / 2 = 3.236 / 2$$

The Egyptian Royal cubit of .5236 m (Liverpool Museum)

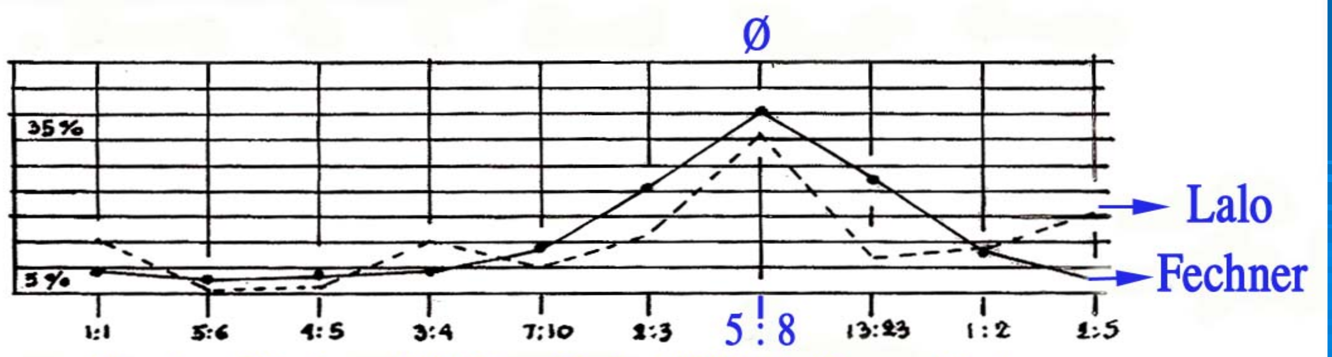
SG104.6.8 PHI Aesthetic Preferences

Is there a human aesthetic / perceptive preference for the proportions of the Golden Ratio? This question has been positively answered, in the western tradition, by Plato and a host of other luminaries through the ages. As St. Thomas Aquinas, a 13th century theologian and founder of Thomist scholasticism, put it:

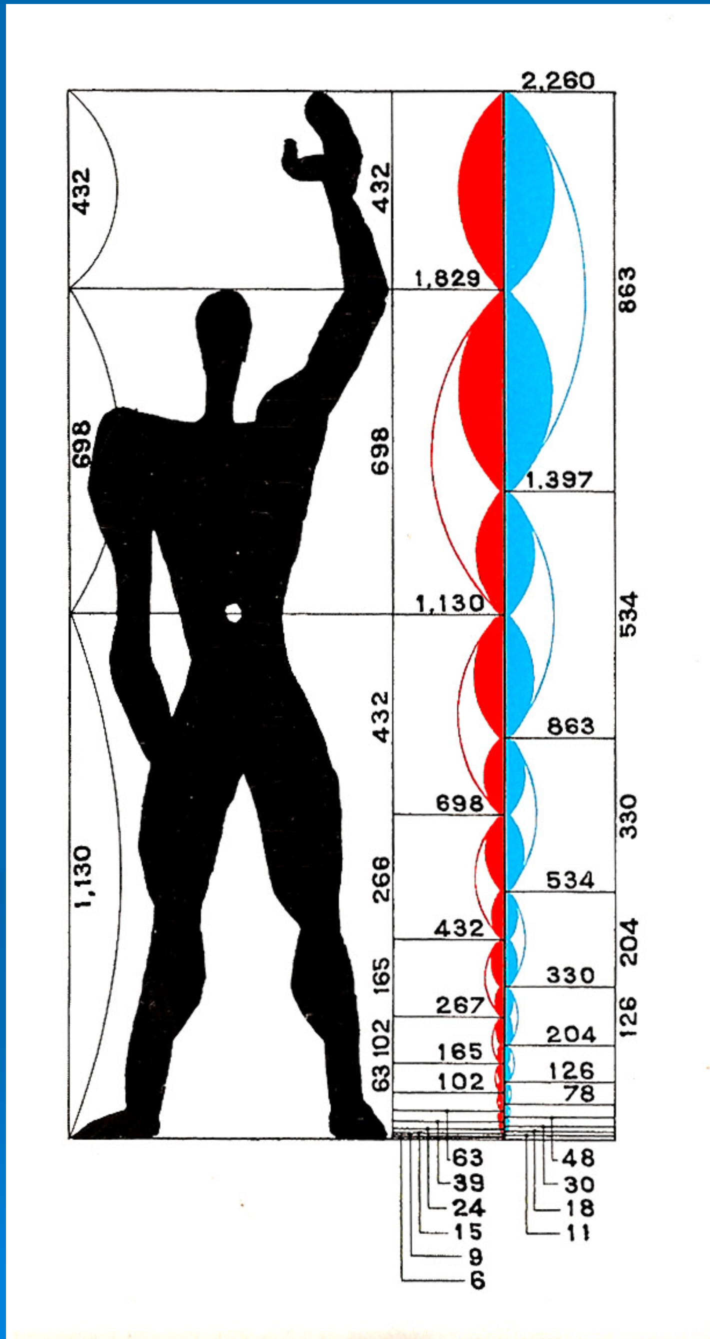
"*The senses delight in things duly proportioned*".

Closer to us, the German natural philosopher Adolf Zeising studied the proportions of the human body and published in 1884 *Der Goldene Schnitt* (The Golden Section) where he extols the virtues of PHI as a universal law. His compatriot psychologist Gustav Theodor Fechner (1801-1887) is considered to be the pioneer of experimental aesthetics.

Fechner investigated the hypothesis of a cross-cultural archetypal preference for Golden Section proportions. He found, by taking thousands of measurements of rectangular objects (such as books, boxes and buildings), that the average rectangle ratio was close to the Golden Section. Moving on to test human aesthetic preferences, he found that most people would choose the Golden Rectangle when asked to pick their preferred rectangle in a series of rectangular shapes. These experiments were later repeated by Lalo and others with similar results.



Graphs of Fechner (1876) and Lalo (1908) Preference Tests

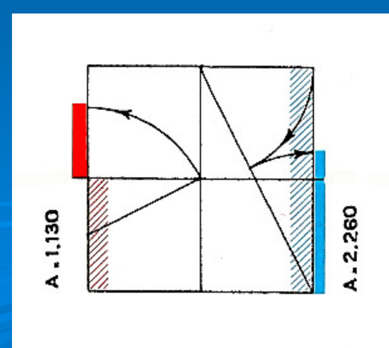


SG104.6.9.1 The Modulor Scale of Le Corbusier (1)

Architect Le Corbusier (1887-1965) created the first modern system of proportions based on the PHI Ratio and the human body. [◆SG207]
As part of this proportioning system, called the "Modulor", Le Corbusier devised a double scale of lengths: the "Red" and the "blue" series.

The Blue Series is simply a PHI sequence built on the Golden Section: $a/\varnothing^2$, $a/\varnothing$, a, $a\varnothing$, $a\varnothing^2$, $a\varnothing^3$… for some practical value of *a*. The Red Series is created so that each length is the *arithmetic mean* of successive lengths of the Blue Series. And each length of the Blue Series is the *harmonic mean* of the two adjacent lengths of the Red Series. So the two series are like *yin-yang* complementaries: they work together with units of one interspersed with units of the other. This prevents the rapid gaps building up if one series is used alone.
The combined Blue & Red Series give the following sequence:

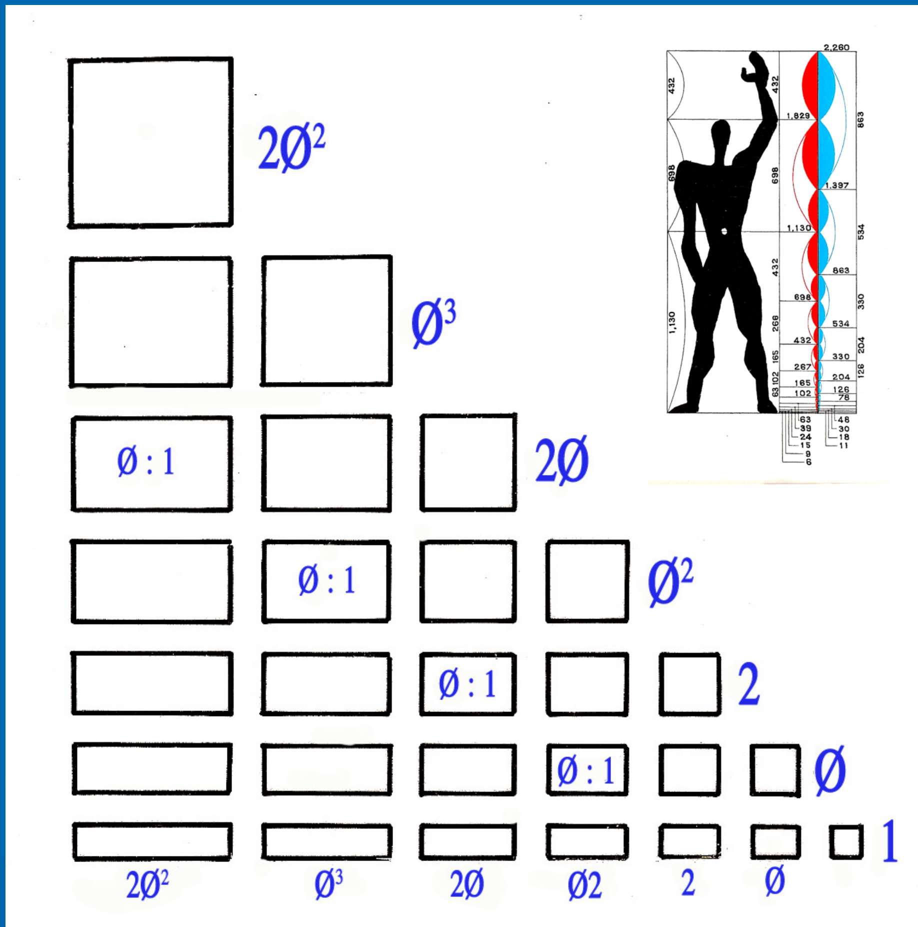
$2/\varnothing^2$ - 1 - $2/\varnothing$ - $\varnothing$ - 2 - $\varnothing^2$ - $2\varnothing$ - $\varnothing^3$ - $2\varnothing^2$ - $\varnothing^4$ - $2\varnothing^3$



[◆SG104]

← The two constructions of PHI used by Le Corbusier: based on the square and based on the triangle.

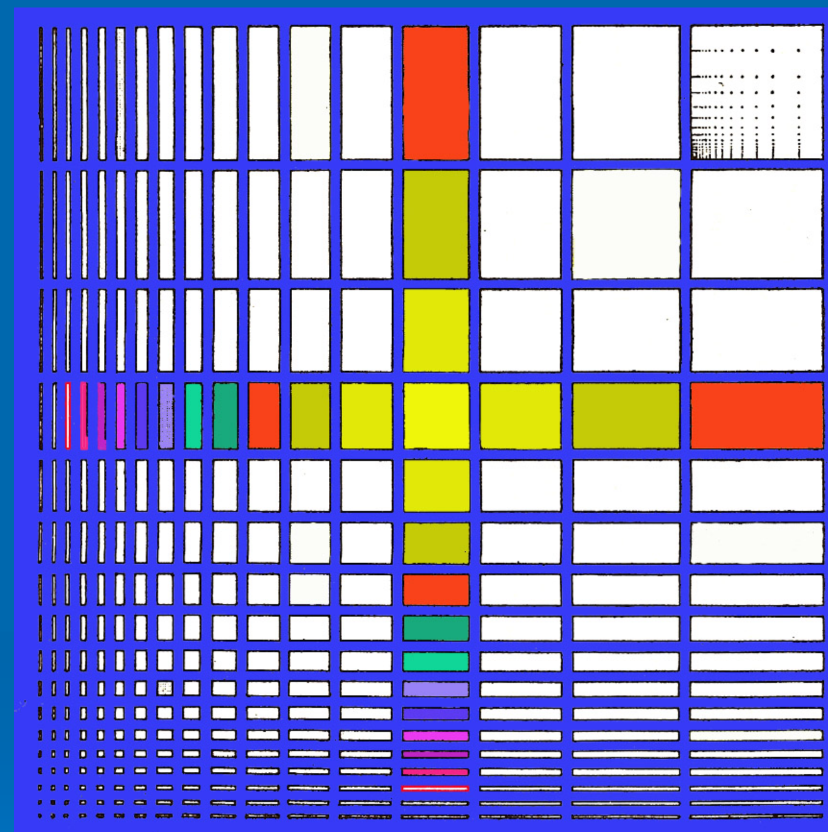
SG104.6.9.2 The Modulor Scale of Le Corbusier (2)



↑ **The Red + Blue Modulor Series showing the Golden Rectangle**

This Table of the Red and Blue Series generates extremely 'modulable' tiles and offers a Golden Ratio system of proportion which, like fractals and growth patterns in nature, is self-similar at every scale.

[◆SG104]



↑ **A Modulor Grid**

These PHI-harmonized modules can be applied to lengths, surfaces or volumes (in art, architecture etc…) which are engendered by values emanating directly from the human stature.

SG104.6.9.3 The Modulor Scale of Le Corbusier (3)

↑ <u>One of the many “panels” of the Modulor Game.</u>
This panel is based on the primary square of 2.26 m (89 inches) (1), its half 1,13 m (44.5 inches) (2) and its Golden Section 1,397 m (55 inches) (3).
(4) is the basic value of 1,828 m (72 inches). (5) is the Golden Section of (1).
(6) is the Golden Section of (2). (7) is (1) + (2). (8) is (4) + (5). (9) is (2) doubled. (10) is (6) doubled.

SG104.6.10 Golden Nuggets

Golden Ratio = Golden Mean = PHI = 1.618…
Golden Vesica (long axis = PHI, short axis =1)
Golden Triangle (base = 1, sides = PHI)
Golden Gnomon (base = PHI, sides =1)
Golden Angle = 137.5º (divergence angle in plants)
Golden Rectangle (short side = 1, long side = Φ)
Golden 3D Frame (Golden Rectangles in x, y, z)
Golden Spiral (whirling squares in Φ Rectangle)
Golden Ellipse (minor axis = 1, major axis = PHI)
Golden Tree (fractal reduction factor 1/Φ)
Golden Rhombi (5-fold Tiling 2D & 3D)
Golden String (fractal series, 0 = 1, 1 = 10)
Golden Dance - Seek Harmonic Mediation

Luminous
Solar
Royal
Immortal
Divine

SG104.6.11 The Golden Rule

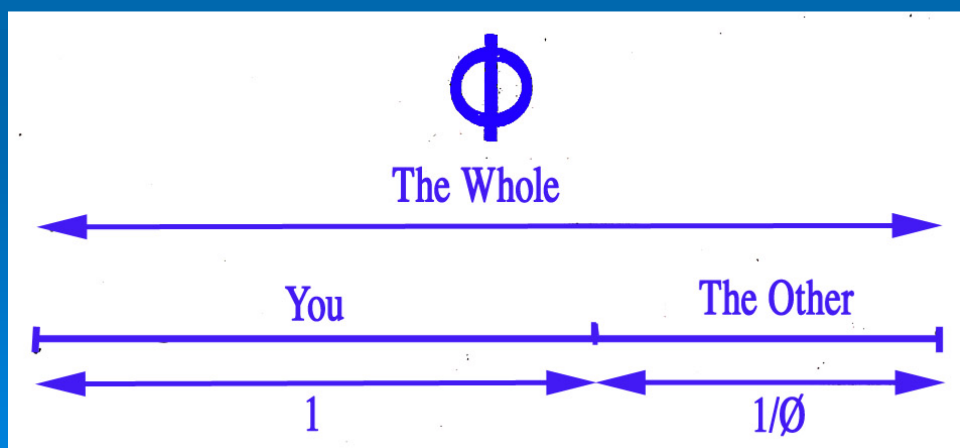
The universal Language of Life is expressed in the Sacred Dance of Creation, continuously infusing Beauty, Harmony & Love throughout the entire Cosmic Garden.

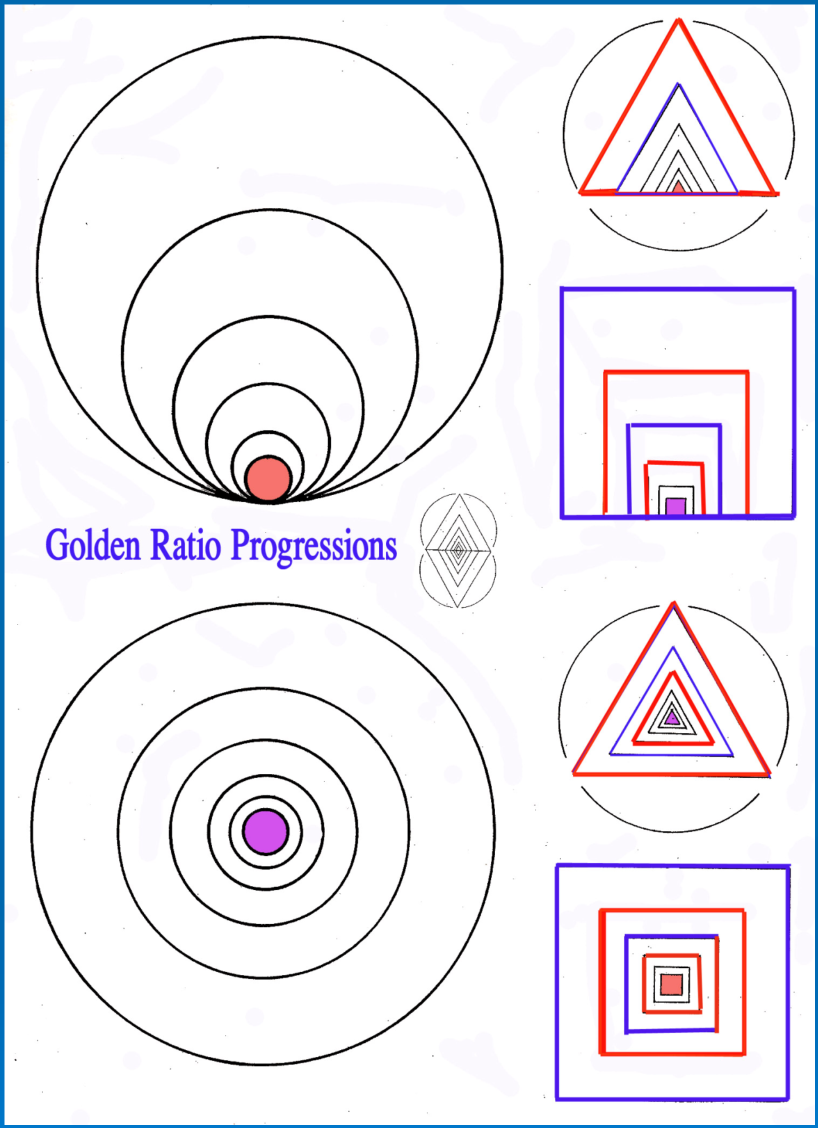
This Dance can be experienced in the magical choreography of colors, sounds, shapes, fragrances & textures… This Dance can be glimpsed in the basic building blocks of Life sharing the same frequency chords, mathematical ratios and holographic geometries found at the very root of all manifestation.

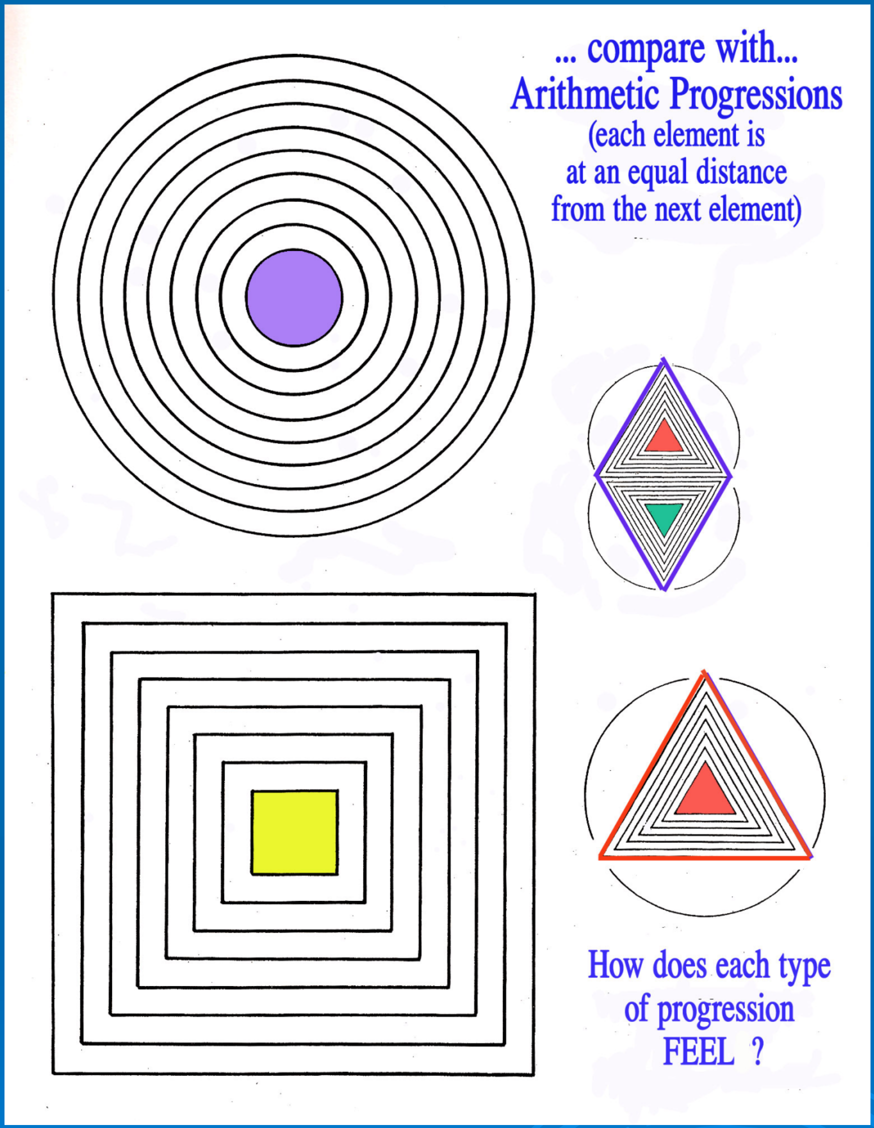
Verifiable in the physical domains of the Cosmos as the "Golden Ratio" PHI, this Sacred Dance really is the Golden Relationship & Golden Rule:

Give to others
The same Love, Beauty & Harmony
That You and Others
Are receiving from the Whole

Love,
like PHI,
is a
tri-unity Dance
within the Same
One Self:
I
the Other
the Whole







SG104.6.12 Golden Ratio Progressions

Phi Geometry / Growth versus Arithmetic Scaling

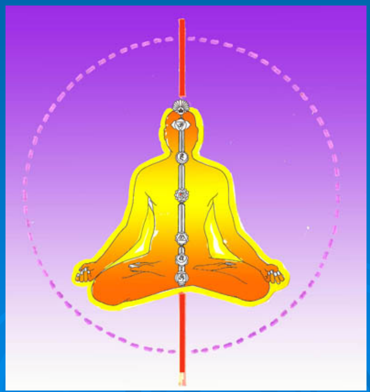
SG104.Ca Conclusion

Fibo, PHI & Cosmic Harmony

Congratulations! You have just got a glimpse of the Pulse harmonizing the Cosmos.

PHI, as the Cosmic Mediator, is able to unite the different parts of a whole so that each part retains its full identity and yet blends into the larger whole. With PHI, you never leave Unity: you always keep a direct harmonic scale-invariant line to Unity and self-integration. With PHI, you are always home!

In terms of contemporary science, PHI is the edge and bridge between chaos and order. When PHI steps into a non-linear situation, it creates the initial island of self-organization and stability. A chaotic/disharmonious process that gets involved in the spiraling "*attractor*" vortex of PHI will go in the direction of increased self-reference or self-similarity, coherence and all-inclusiveness with Unity.



Joy to your Heart!
Infinity to your Soul!

SG104.Cb Online SG School Curriculum: Intro & Intermediate

Sacred Geometry Introductory Level: 8 Modules

SG 101	Intro I	Sacred Geometry: Universal Order & Beauty
SG 102	Intro II	History & Traditions of Sacred Geometry
SG 103	Intro III	Sacred Geometry: A Grand Tour
SG 104	Intro IV	PHI: the Golden Ratio & the Fibonacci Series
SG 105	Intro V	The Golden Rectangle & Golden Spiral
SG 106	Intro VI	Pentagons, Pentagrams & the Penta-Modules
SG 107	Intro VII	The Five Platonic & 13 Archimedean Solids
SG 108	Intro VIII	The Vesica Piscis: Cosmic Womb of Creation

Sacred Geometry Intermediate Level: 8 modules

SG 201	Interm I	The Monochord, Music & Cymatics
SG 202	Interm II	The Power of Archetypal Numbers
SG 203A	Interm IIIA	Sacred Geometry Resurgence in Science - Part 1
SG 203B	Interm IIIB	Sacred Geometry Resurgence in Science - Part 2
SG 204	Interm IV	PHI in the Human Body, Biology & DNA
SG 205A	Interm VA	The SG of Nature - Part 1: Plants & Phyllotaxis
SG 205B	Interm VB	The SG of Nature - Part 2: Animals & Minerals
SG 207	Interm VII	SG in Architecture, Sacred Sites & Green Design

SG104.Cc Online SG School Curriculum: Advanced

Sacred Geometry Advanced Level: 8 modules

SG 301	Adv I	Golden Cosmos: Planets, Stars & Cosmology
SG 302	Adv II	SG in Art, Culture & Creativity
SG 303	Adv III	Universal Symbols: Primordial Knowledge
SG 304	Adv IV	Labyrinths: a Mini-Pilgrimage to Self
SG 305	Adv V	Mandalas & Yantras: Sacred Vortices
SG 306	Adv VI	Languages & Gematrias: Sacred Communication
SG 307	Adv VII	Sacred Geometry in the Healing Arts
SG 308	Adv VIII	Harmony on Earth: Science & Consciousness of Harmony

Upon completion of each level (Introductory, Intermediate & Advanced), a Certificate of Graduation from the Sedona School of Sacred Geometry will be presented to Certification Students.

Postgraduate seminars on current Sacred Geometry research, discoveries & updates will be offered in harmonic time.

Questions: phi@schoolofsacredgeometry.org

SG104Cd StarWheel Blessing

SW#33. "Lion Path". www.starwheels.com

SG104.Ce
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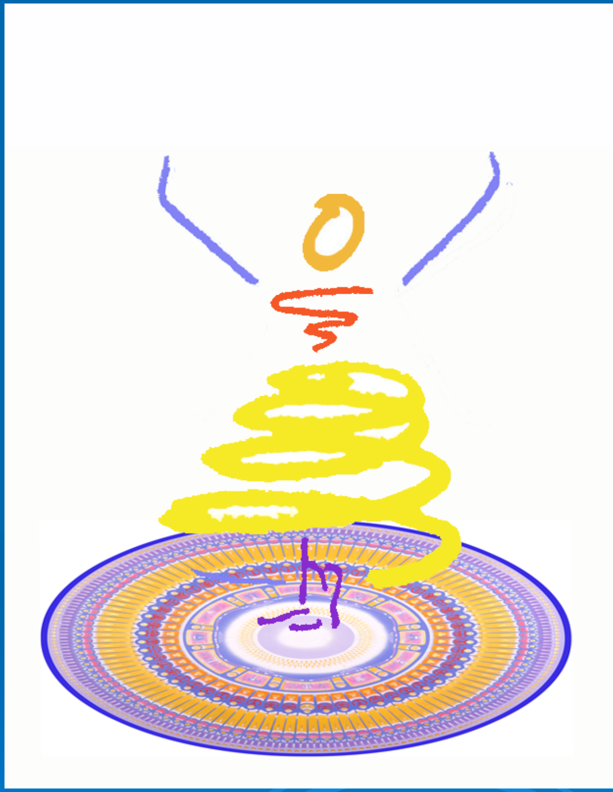
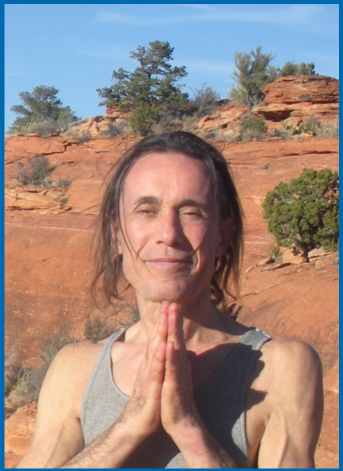
FB Group: Sedona School of Sacred Geometry



Φ celebration

SG104.Cf About Aya



A native of France, Aya is a visionary artist and celebration yogi who has dedicated his life to serve humanity and to develop sacred arts education. In his late 20's, Aya realized that his professional life in the French diplomatic service was not fulfilling his heart's desires; he quit everything to go on an extended vision quest. His path took him around the world to visit a variety of sacred sites & cultures and to receive inspiration from many teachers.

In 1985, in Santa Monica, CA, Aya was gifted with a spiritual vision prompting him to create a series of 108 airbrushed neo-mandala paintings: the "StarWheels". The StarWheels, a happy family of vibratory flowers for the Earth, are looking for sacred spaces to be graced with their presence... (www.starwheels.com / www.starwheelmandalas.com)

Moving to Sedona, Arizona, in 1997, Aya has been involved with sacred arts classes & events, mandala creation, Sedona guided tours, labyrinth making and Sacred Geometry teaching. Aya has presented several StarWheel art exhibits, has sponsored community awareness events at the Sedona Library, has developed, in collaboration with Gardens for Humanity, the Peace Garden arboretum at the Sedona Creative Life Center, was a speaker at the Sacred Geometry Conference (Sedona, 2004), co-designed several labyrinth sites (The Lodge at Sedona, Magos' Ranch...), and was on the management team of the Raw Spirit Festival in 2006 - 2008.

Realizing that Sedona was progressively becoming a global spiritual university for many seekers from around the world, Aya founded in 2005 the Sedona School of Sacred Geometry. The school is offering online access to Sacred Geometry PDF modules, with 17 modules completed so far. In the school's website, Aya states: *"We are living at the extraordinary and exciting times of a global transformation to a higher order of human consciousness... Sacred Geometry is the expression and resurrection of our deep innate wisdom, now awakening from a long sleep: seeing again the all-encompassing, fractal-holographic unity of nature, life and spirit... The keyword is HARMONY."* (www.schoolofsacredgeometry.org)

Aya's visionary dream, supported by his non-profit educational organization, the StarWheel Foundation, is the co-creation of an international eco-village "The School of Celebratory Arts" - a green environment encouraging young people of all nations to develop their creative consciousness and thus contribute to a new, spirited, life-respecting global civilization on Earth. (www.starwheelfoundation.org).

Since 2012, Aya is dancing the body divine, after his re-discovery of Yoga, Partner Yoga and AcroYoga. Aya is currently the AcroYoga.org Jam coordinator for Sedona and a teacher of yoga swing asanas.

Blessings in Anjali!