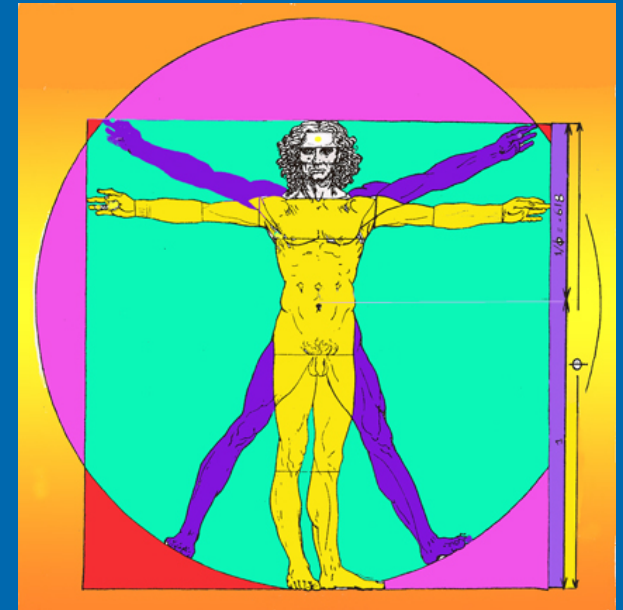
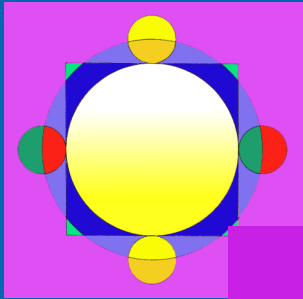


SG102 Ia. History & Traditions of Sacred Geometry

Online Module
SG 102 - Intro II

Presented by AYA
& the Sedona School
of Sacred Geometry



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SG102. Ic. History & Traditions



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images, diagrams...)

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(Jean-Yves Empeur.
Alexandria. Abrams, 2001.)

SG102. Id. Introduction to SG102 - History & Traditions

Welcome to the ancient and perennial knowledge of Sacred Geometry.

In this online educational program, Sacred Geometry will be shown to be perennial and primordial, on an archetypal or causal level, as it is ubiquitous throughout nature, from microcosm to macrocosm, including the human DNA.

There is indeed a potent case to show that the Sacred Geometry of the Golden Ratio is prior to the perceptive functions of humans, prior to human culture & history, and actually prior to the very existence of humankind.

However the understanding of the principles of Sacred Geometry and especially their applications in human cultures, whether intuitively or rationally, can be traced as a progressive curve of unveiling, throughout the course of history, the universal laws of creation and optimized transmission of energy.

In this SG102 module, we will briefly glance at some highlights of Sacred Geometry evidence in the historically recorded traditions and artifacts of the western cultures & civilizations. As we are, at the dawn of this 21st century, still barely uncovering the past of humankind, this module is only a simplified and condensed overview of the ancient lineage of Sacred Geometry.

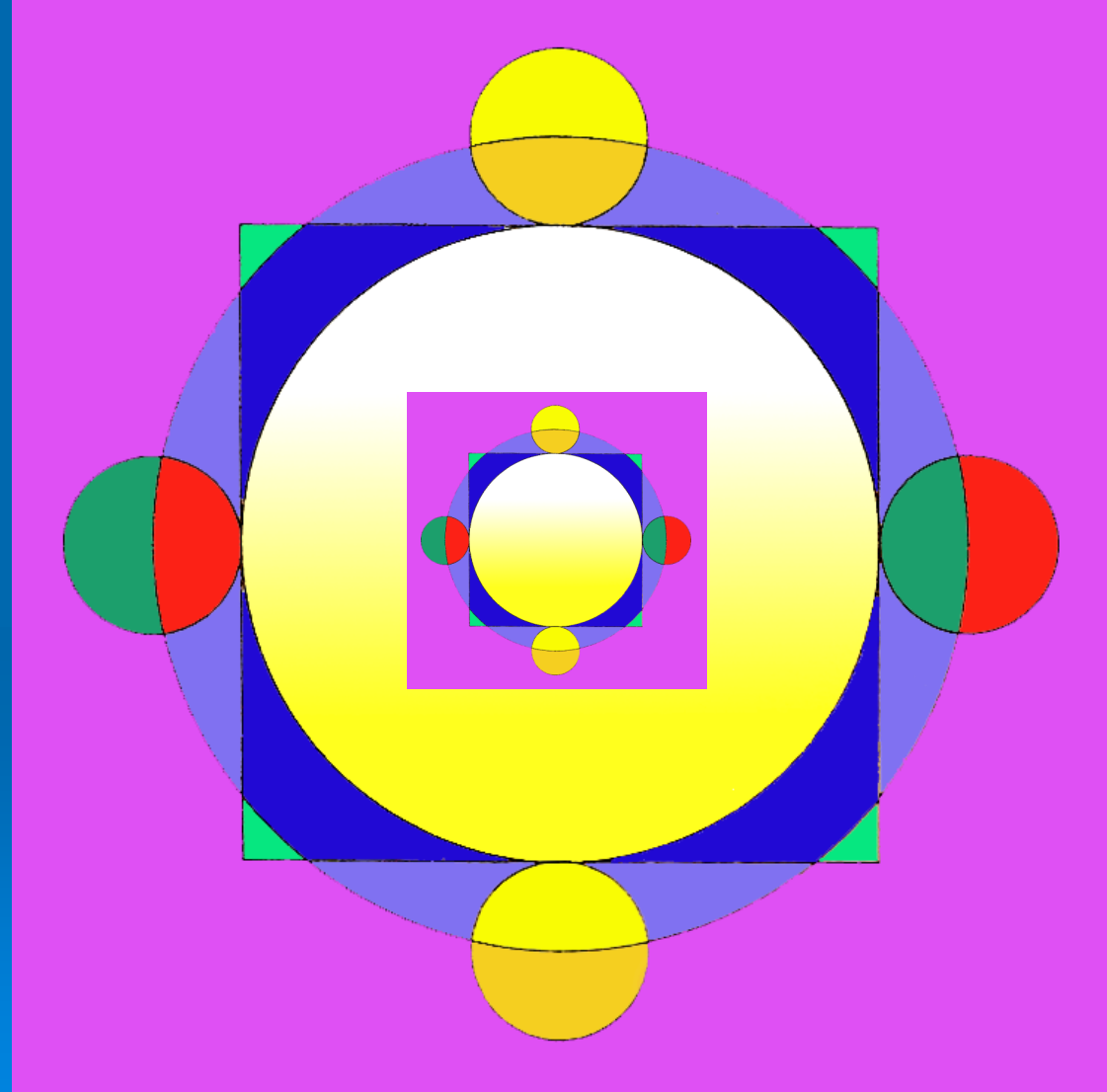
It is in fact a heartfelt invitation for much more research and many new discoveries.

Note: Sacred Geometry *rational* knowledge, in the history of western cultures, has appeared and disappeared in a cyclical dance: periods of open public emergence have alternated with periods of complete ignorance during which the knowledge has gone underground to be only carried by small esoteric groups.

But the innate *intuitive* knowledge of Sacred Geometry has been guiding human awareness and weaving itself into human cultures all along.

SG102.1 - Chapter 1 Proto Sacred Geometry

“Heavenly Temple” by Aya.
Based on the *New Jerusalem* diagram by John Michell.



SG102.1.1.1 Egypt (1) Great Pyramid (1)

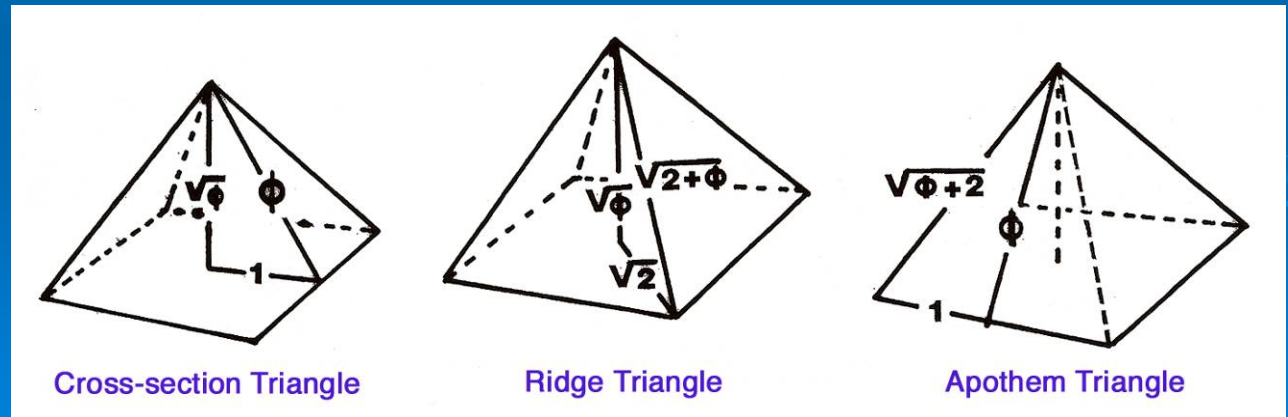
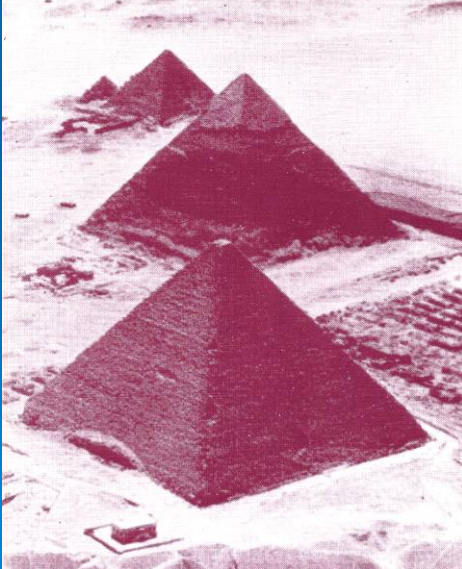
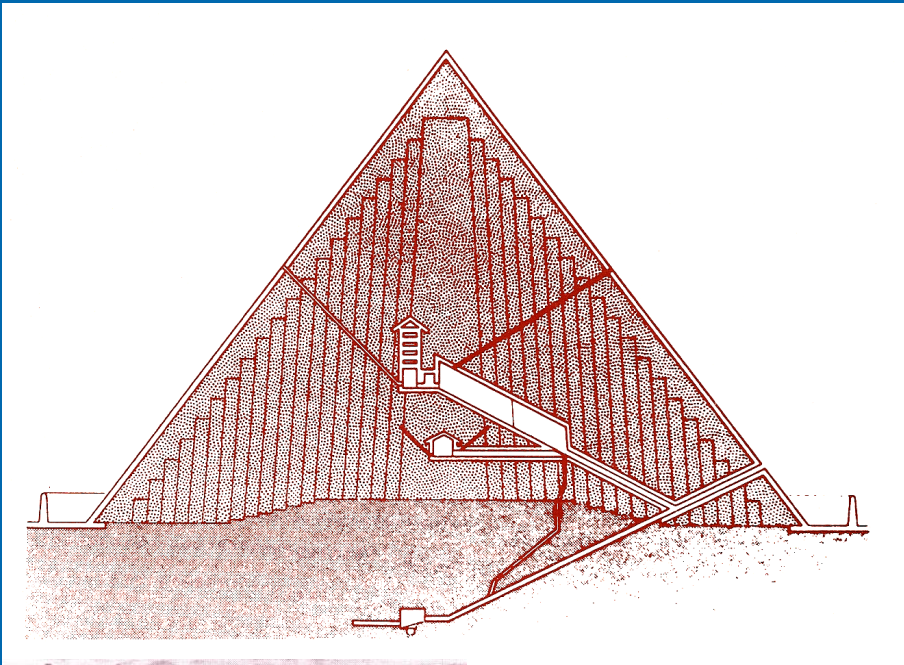
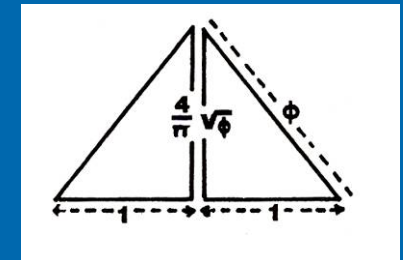
Harmonics of the Great Pyramid

Height = 280 cubits
 Side = 440 cubits
 Apothem = 356 cubits

[1 Egyptian cubit = .5236 m = pi / 6]

Apothem (Greek *apo* = from + *thema* = to lay down):
 distance from apex down one face to center of base side.

$$\begin{aligned} \text{Apothem} \\ / \text{ half base} \\ &= 356 / 220 \\ &= 1.618 = \text{Phi} \end{aligned}$$



Cross-section Triangle

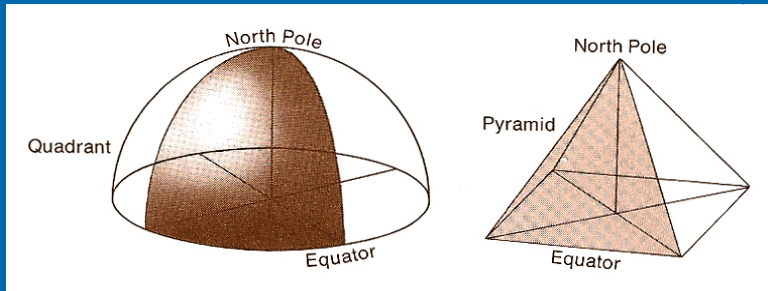
Ridge Triangle

Apothem Triangle

↑ Phi in the Great Pyramid.

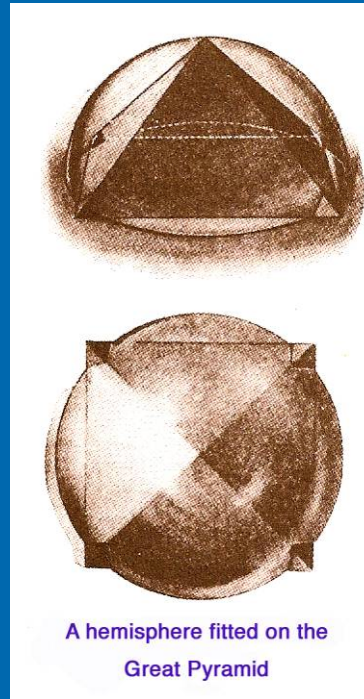
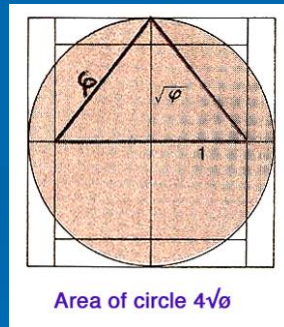
The cross-section triangle is also called Kepler's Triangle [◆SG104.4]

SG102.1.1.2 Egypt (2) Great Pyramid (2)

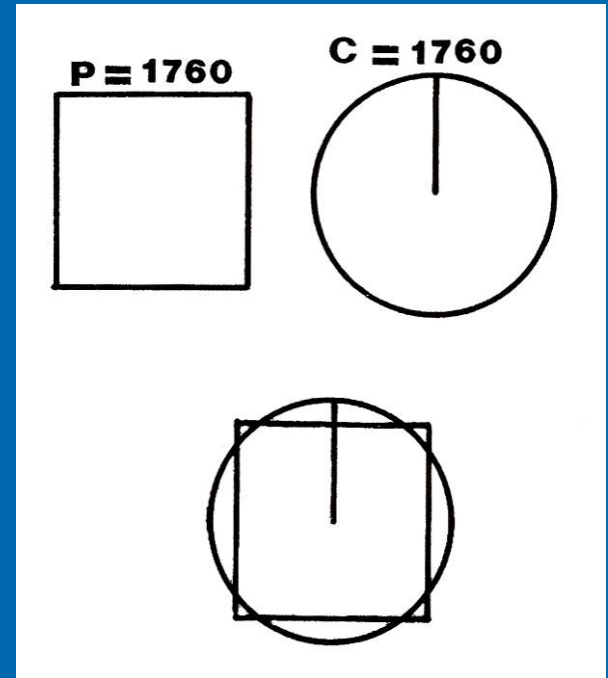
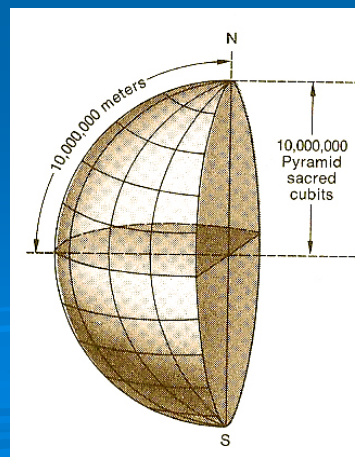


The Great Pyramid is a Geodetic Marker: each face is designed to represent one curved quadrant of the Northern Hemisphere.

Flat area of half circle = spherical area of quadrant 90° = 2 √Phi



A hemisphere fitted on the Great Pyramid



The Pyramid accomplishes the Squaring of the Circle and the Spherizing of the Cube.

The square of the base perimeter = the circumference of the circle with radius of pyramid's height.

Fractional values of π & Phi

$$\pi = 22/7 = 3.1428$$

$$\pi = 4/\sqrt{\text{Phi}} = 3.1446$$

$$\sqrt{\text{Phi}} = 14/11 = 1.2727$$

$$\pi = 6\text{Phi}^2/5 = 3.1415$$

$$\text{Also Phi} = 1.4\pi/e = 1.618$$

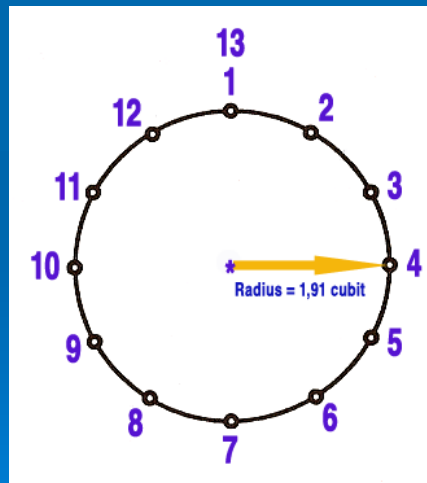
SG102.1.1.3 Egypt (3) The Harpedonaptae & the Egyptian Cubit

In ancient Egypt, aside from practical land surveying, there was a sacred aspect to geometry: the laying out of temples and sacred buildings. It was all performed by experts known by the Greek name of *harpedonaptae* (or harpedonapts). These surveyors strictly adhered to the principles of sacred geometry, using only a compass and a straight line in the form of the 13-knots cord with 12 intervals of one Egyptian royal cubit each. The rope was used for various configurations and especially for the Pythagorean (or 'Egyptian') Triangle 3-4-5. [◆SG101.5]

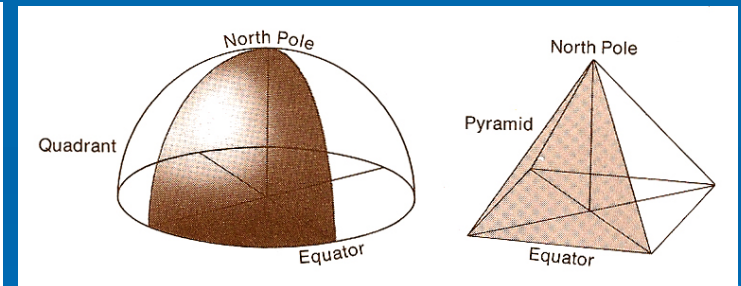
The Egyptian cubit = 1.72 foot or .5236 meter. When turned into a circle, the 13-knots rope has a radius of 1.91 cubits or one meter (.5236 x 1.91 = 1). Since 1 meter = 1/100,000th part of a quarter of the earth's meridian, this means that the Egyptian cubit and the 13-knots rope are based on the measurement of the earth's circumference. When we realize that the Great Pyramid is a reduced model of the earth's dimensions by a factor of 43,200 (which is half the number of seconds in a day), we can better appreciate the level of knowledge applied by the pharaoh's *harpedonaptae*.

Interestingly enough, Napoleon Bonaparte established the metric standard after his scientific expedition returned from the Egyptian campaign, having extensively studied the metrology of ancient Egyptian monuments.

Apparently, Napoleon used the sacred radius measure of the ancient Egyptian royal cubit circle for his length standard, many thousands of years later.



The 12 cubits rope has a radius of 1 meter



The Great Pyramid is a mini-scale model of the earth. (1/43,200)

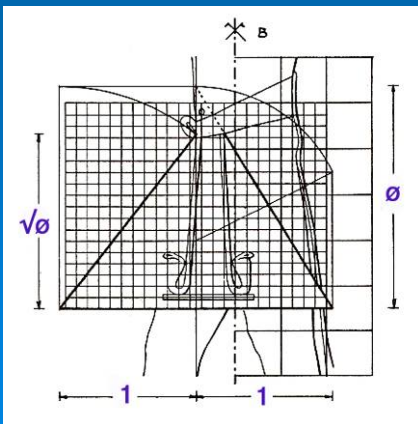
↑ Peter Tompkins. *Secrets of the Great Pyramid*. Harper, 1971.

SG102.1.1.4 Egypt (4) The Temple in Man

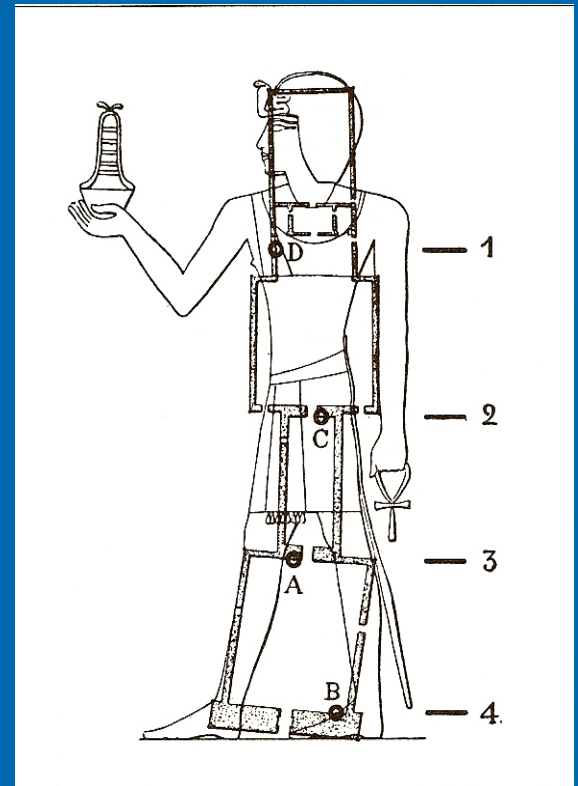
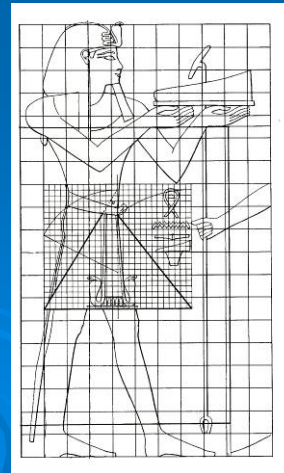
R. A Schwaller de Lubicz, an Alsatian mathematician & philosopher, has offered, after a protracted on-site study of the Temple of Luxor (between 1937 and 1952) a pioneering reinterpretation of ancient Egypt - which is now known as the *Symbolist School Interpretation*.

The rigorously documented work of Schwaller de Lubicz is an elaborate statement disproving the commonly accepted belief that our modern world is more advanced than the ancient world. Schwaller de Lubicz's work validates the ancient tradition insisting that there had once been a great doctrine in which science, religion, philosophy and art were fused into a single great synthesis.

According to Schwaller de Lubicz, the great synthesis of Egypt is based upon an exact and complete wisdom: the understanding of the principles & laws of creation. This unified knowledge was (and still is) expressed in the Egyptian sacred sites, temples and artifacts through sacred geometry, harmony, proportion and symbolism.



De Lubicz measured scores of *Royal Aprons* (the triangular loincloth worn by the Pharaohs). The lower angles were invariably Phi and $\sqrt{\text{Phi}}$.



↑ Superimposition of the figure of the king over the diagram of the temple of Luxor.

The Temple in Man
(Title of de Lubicz's magnum opus).

The entire universe is contained in a single gesture.

SG102.1.1.5 Egypt (5) Harmonic Design

M. Gadalla is an Egyptian-American independent Egyptologist who published many books showing the depth and the scope of the spirit-based Egyptian science. Says Gadalla: *“For the ancient Egyptians, geometry was the means by which humanity could understand the mysteries of the divine order... All Egyptian art and architecture, including the representation of the human figure, followed a precise canon of proportion.”*

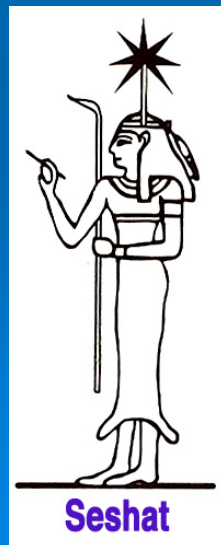
In terms of sacred architecture, the Egyptian temples were carefully laid out according to specific proportions of geometrical figures and harmonic relationships of the parts to the whole. Egyptian sacred geometers were using the Progression Series (now called Fibonacci Series) and Harmonic Root Rectangles, and chief among them the *Neb* (Golden) Rectangle.

In ancient Egyptian, *Neb* means gold, divine. How appropriate!

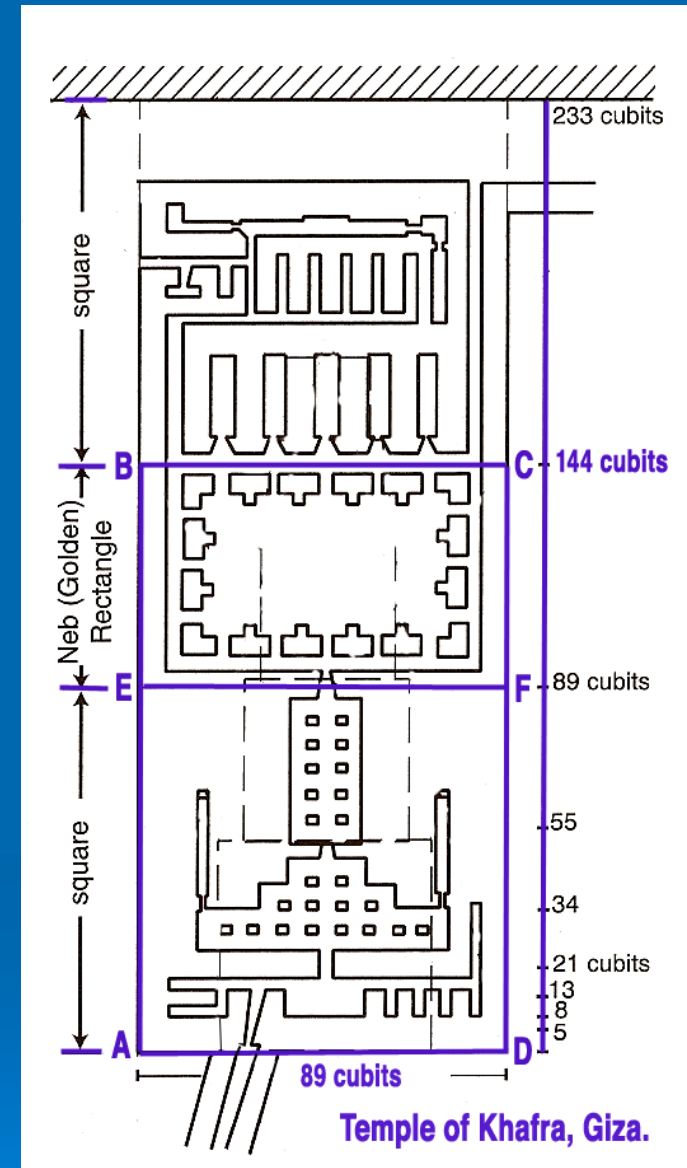
M. Gadalla explains: *“The divine significance of numbers is personified by Seshat, the Enumerator.*

The netert Seshat is also described as: Lady of Writing, Scribe, Head of the House of Divine Books (Archives), the Lady of Builders.

Seshat is considered to be the consort of Tehuti (Toth).”



Moustafa Gadalla.
Egyptian Harmony.
Tehuti, 2000.



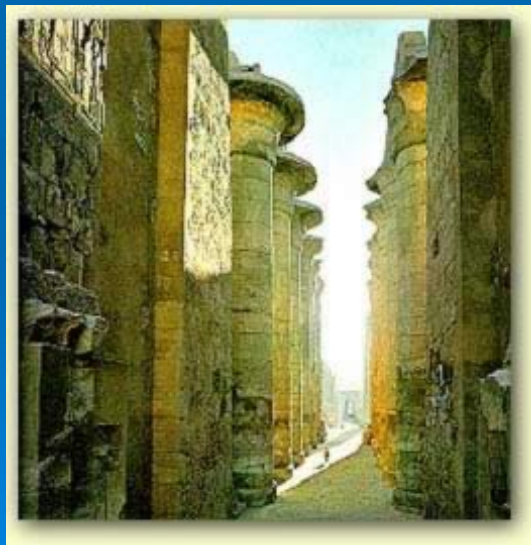
↑ ABCD and EBCF are Golden Rectangles.
Design: M. Gadalla.

SG102.1.1.6 Egypt (6) Karnak Temple

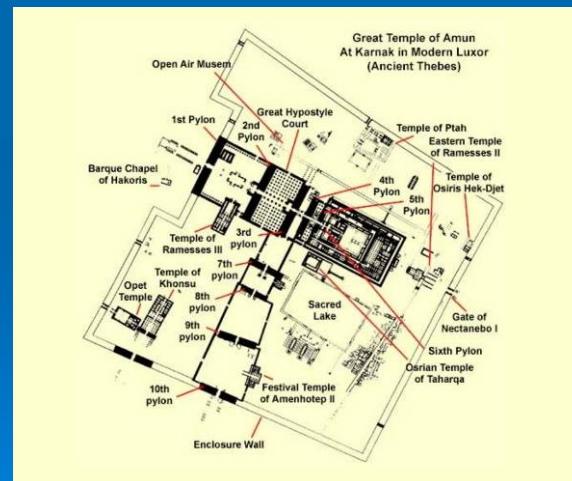
Karnak is one of the largest religious complex ever constructed in this world. The Hall is considered to be a world's masterpiece.

Begun by Ramesses I (who founded the 19th Dynasty) and continued by under Seti I (13406 - 1290), the Hall was completed by Seti I's son Ramesses II.

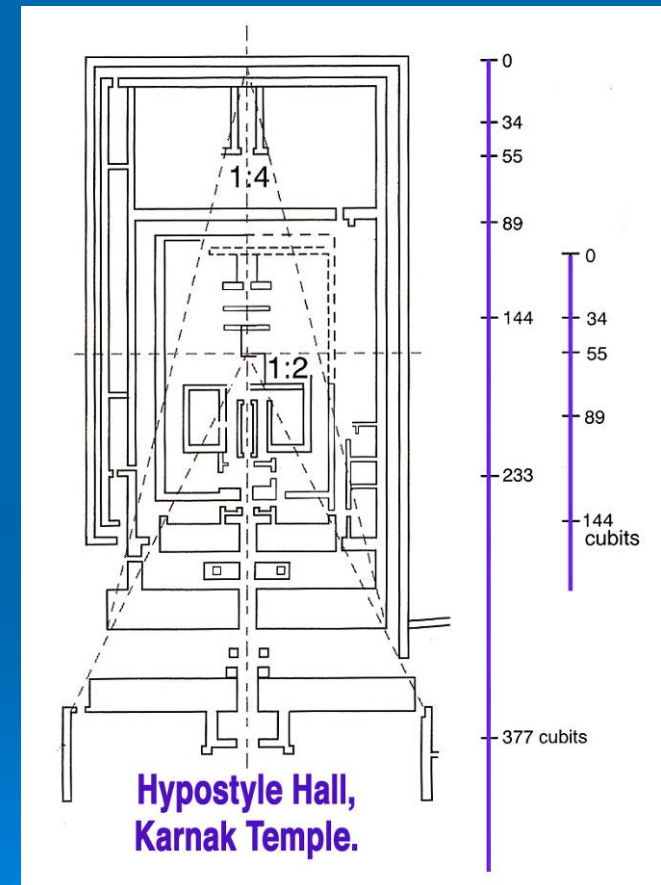
The upper portion of the Hall, as analyzed by M. Gadalla, attests to the use of the Progression (Fibonacci) Series and its related Golden harmonic proportions.



↑ View of the majestic columns.

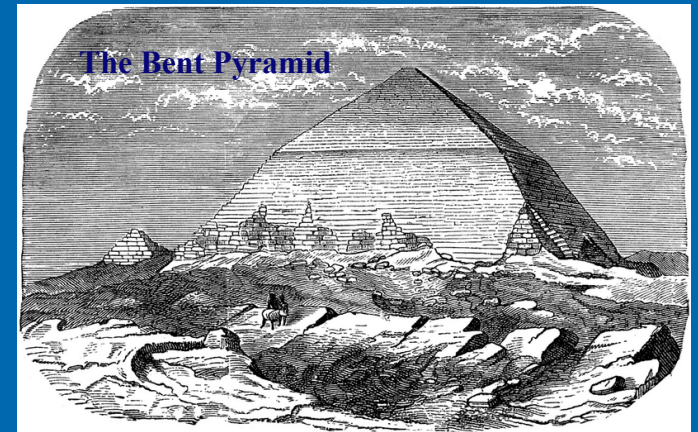


↑ Plan of the Amun Temple showing the Hypostyle Hall.

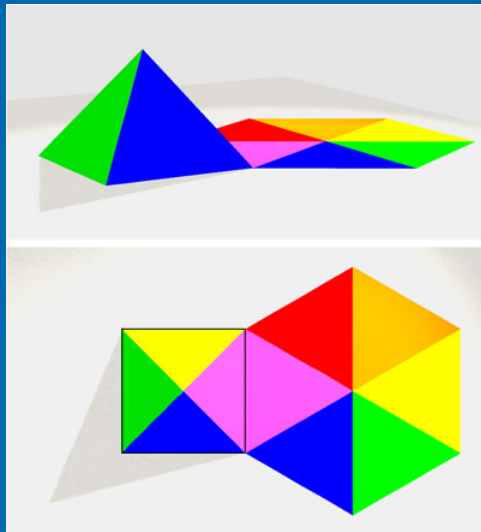


↑ Applications of the Progression (Fibonacci) Series.

SG102.1.1.7 Egypt (7) The Bent Pyramid

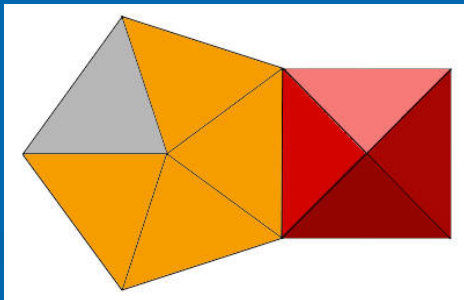


The Bent pyramid at Dahshur, Egypt, is the only pyramid with 2 slope angles. Interestingly, the lower slope matches the hexagon geometry while the upper slope matches the pentagon geometry. This is explained in details in www.world-mysteries.com and further expanded upon by Scott Onstott in his Youtube movie *Secrets in Plain Sight*.

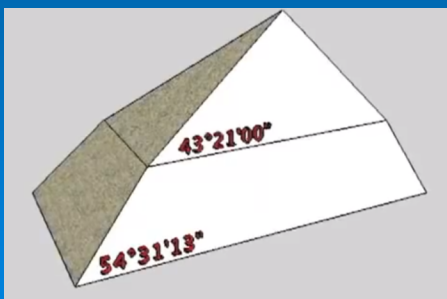


↑ Lower part matches hexagon geometry.

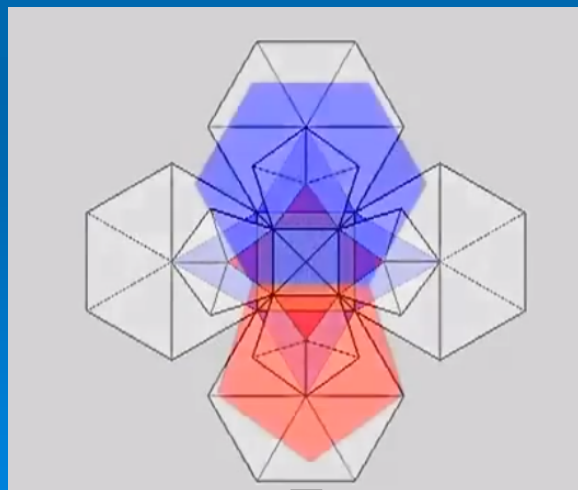
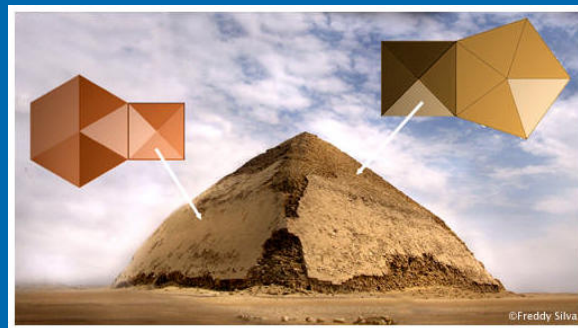
↑↓↓ [Credit](#)



↑ Upper part matches pentagon geometry.



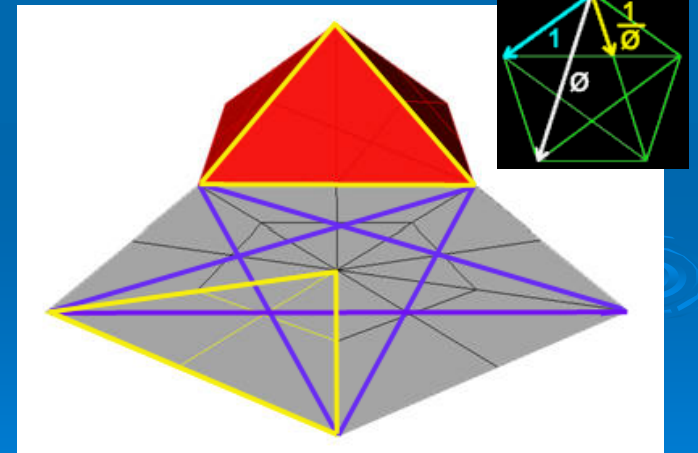
↑ The 2 slop angles of the Bent Pyramid.



↑ The coming together of hexa & penta. [Credit](#).

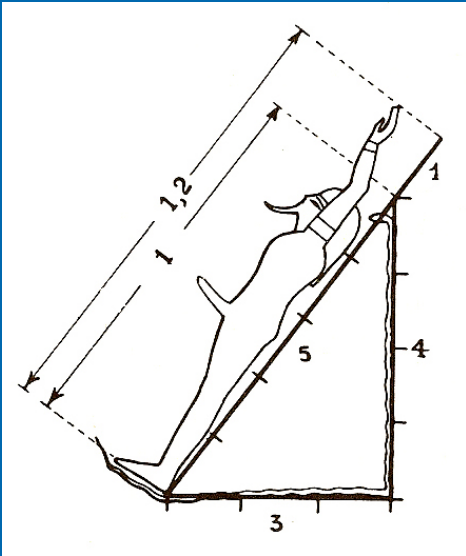
The Red Pyramid

Also located at Dahshur, the Red pyramid has the same slope angle as the upper part of the Bent Pyramid (43° 22'). Its design closely fits the pentagon geometry which is directly related to the Golden ratio [[SG202.6](#)]

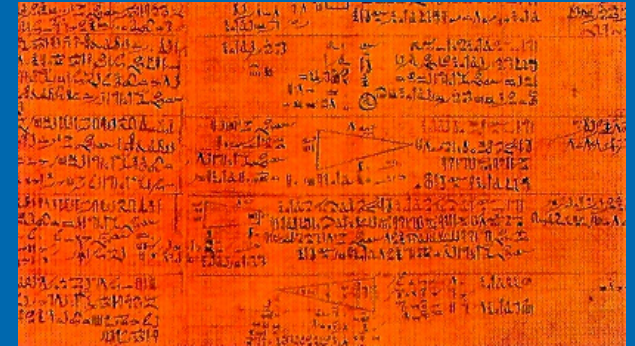
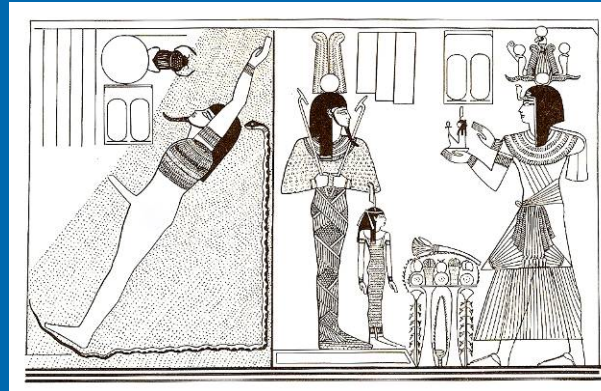
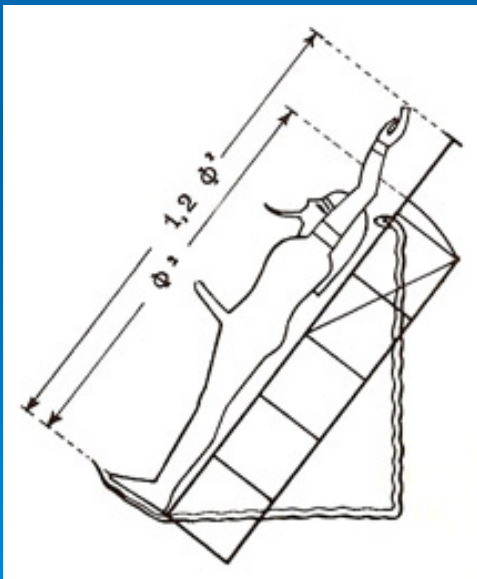


↑ The Red Pyramid can be approximated by using 4 of the 5 triangles of a pentagon. [Credit](#).

SG102.1.1.7 Egypt (8) 3-4-5 Osiris Triangle (1)



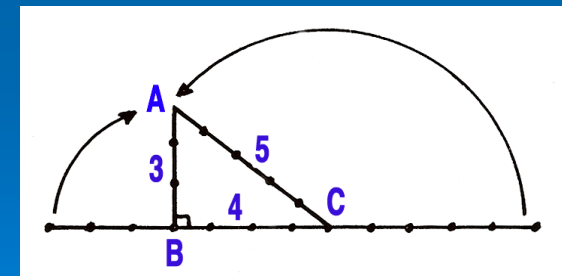
↑↓ Diagrams:
Schwaller de Lubicz
Egyptian Miracle.



↑ The *Rhind papyrus* (c. 1650 BCE, copied by the scribe Ahmes from older sources) attests, by the way of slope calculations but undeniable by the numerical value given, to the knowledge of the 3-4-5 Triangle - the Egyptian / Osiris (Pythagorean) Triangle.

← ↑ In Thebes's Valley of the Kings, to the right of the corridor of the tomb of Ramses IX, there is the figure of a royal mummy placed as the hypotenuse of the "Osiris" sacred triangle 3-4-5, whose base and height are a serpent. The arm's length is one cubit.

This represents a *Geometric Principle*. The height of the king is 5 + 1 cubit. But we know that the height of an average human body is divided by the navel into the Golden ratio which is 1 to Phi or Φ^2 . The total height is then: Φ^2 plus its 5th part $(2.618/5) = 1.2 \Phi^2 = 3.1416$. This is the value of the coefficient π (Pi).



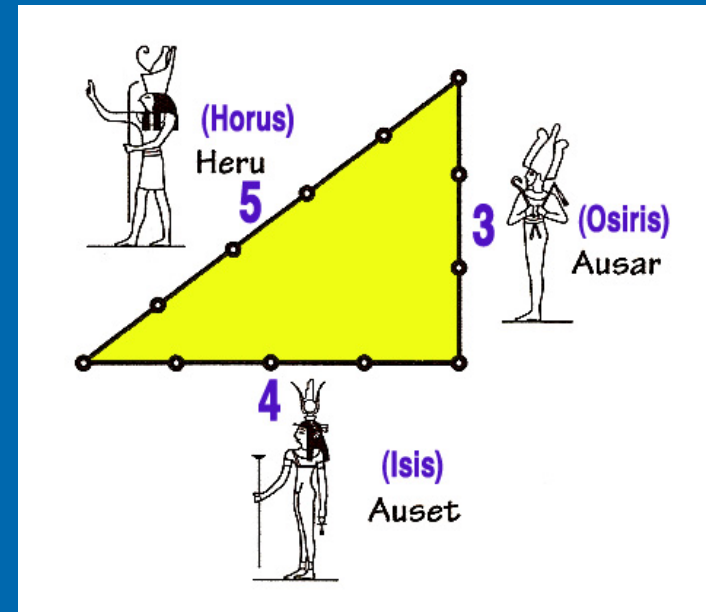
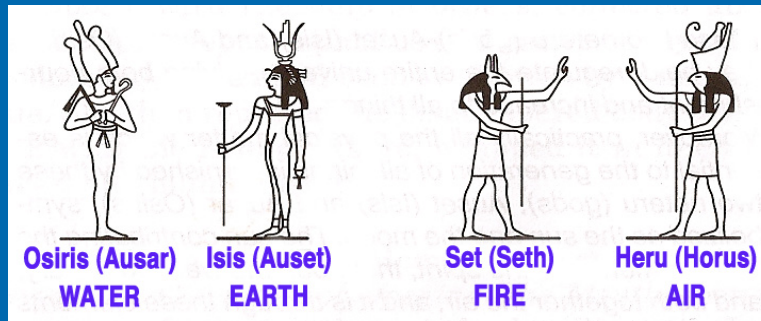
↑ Building the Osiris 3-4-5 Triangle with the 13-knots cord.

SG102.1.1.8 Egypt (9) 3-4-5 Osiris Triangle (2)

The 3-4-5 Osiris Triangle embodies much symbolism:

- The sides were associated with *Isis*, *Osiris* and *Horus* as the archetypal relationship: mother, father and son.
- Numerologically, 3, 4 and 5 are the first numbers of manifestation, beyond the two cosmic procreative parent numbers 1 and 2, the Monad and its reflection the Dyad.
- $3 + 4 + 5 = 12$. Twelve is the cosmic cycle/wheel archetype. [◆SG202]

- The 4 elements were personified by Isis, Osiris, Horus and Seth.

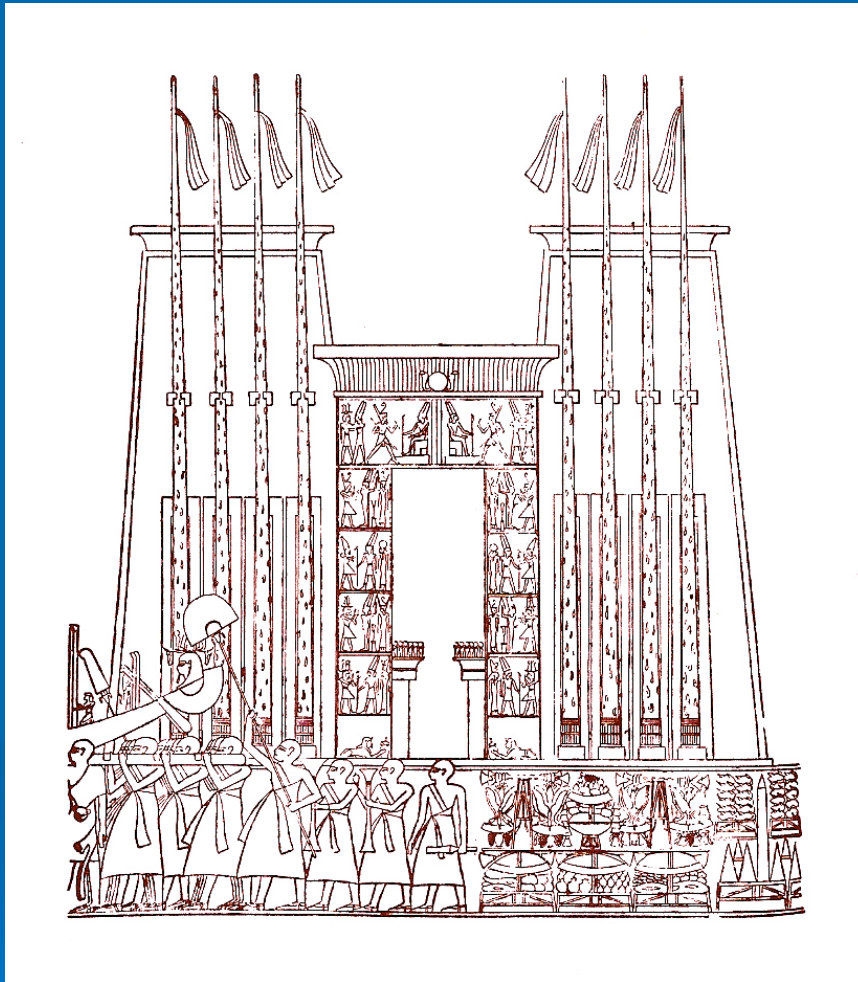


“The Egyptians hold in high honor the most beautiful of the triangles, since they liken the nature of the universe most closely to it, as Plato in the Republic seems to have made use of it in formulating his figure of marriage. This triangle has its upright of 3 units, its base of 4, and its hypotenuse of 5, whose power is equal to that of the two other sides.

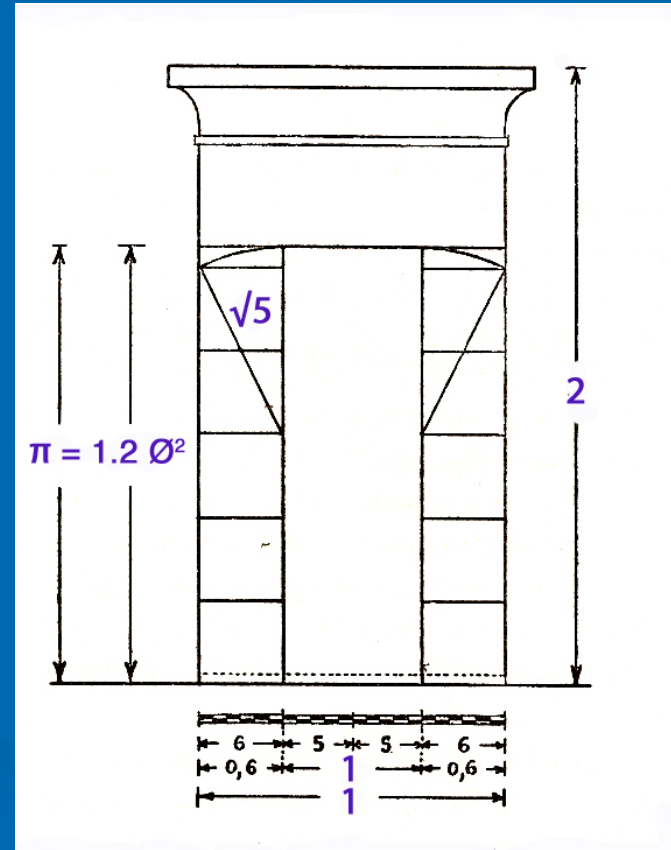
The upright, therefore., may be likened to the male, the base to the female, and the hypotenuse to the child of both, and so Osiris may be regarded as the origin, Isis as the recipient, and Horus as perfected result.

Three is the perfect odd number. Four is a square whose side is the even number Two. But Five is in some ways like its father, and in some ways like its mother, being made up of 3 and 2. And panta ('all' in Greek) is a derivative of pente ('five' in Greek), and they speak of counting as 'numbering by fives'. Five makes a square of itself.”

SG102.1.1.9 Egypt (10) The Pi / Phi Pylon



↑ On the east side of the Temple of *Luxor*, there is a relief depicting a cortege of priests exiting from the great temple of *Karnak*, carrying the king's Solar Boat.



↑ If the opening of the door equals 1, the height with the plinth equals 1.2 Phi squared or Pi.

The entire pylon is a 1 x 2 rectangle.

$$0.6 (\sqrt{5} + 3) = 0.6 \times 5.236 = 3.1416 = \Pi \text{ (uppercase } \pi)$$

SG102.1.1.10 Egypt (11) School of Sacred Science

Even educated people, in the western countries, are used to look back to the Greeks as the origin of the Western culture. This is a short-sighted view. In the ancient (Western) world, Egypt was the older fount of civilization where all the Greek luminaries went to acquire their knowledge.

“Now, let me talk more of Egypt for it has a lot of admirable things and what one sees there is superior to any other country”.

Herodotus, Greek Historian (5th century BCE).

The priestly class in Egypt began to keep written records around 4,000 BCE - most probably having inherited their knowledge from a previous civilization, as the early Egyptian culture lacks signs of gradual development but seems to have sprung up instantly. The Greeks developed their writing system more than 2,000 years afterwards.

Proclus, a Greek philosopher (412-485 CE), reports in his *Commentaries on Euclid’s Elements*:

“We shall say, following the general tradition, that the Egyptians were the first to have invented Geometry, that Thales, the first Greek to have been to Egypt, brought the theory thereof to Greece”.

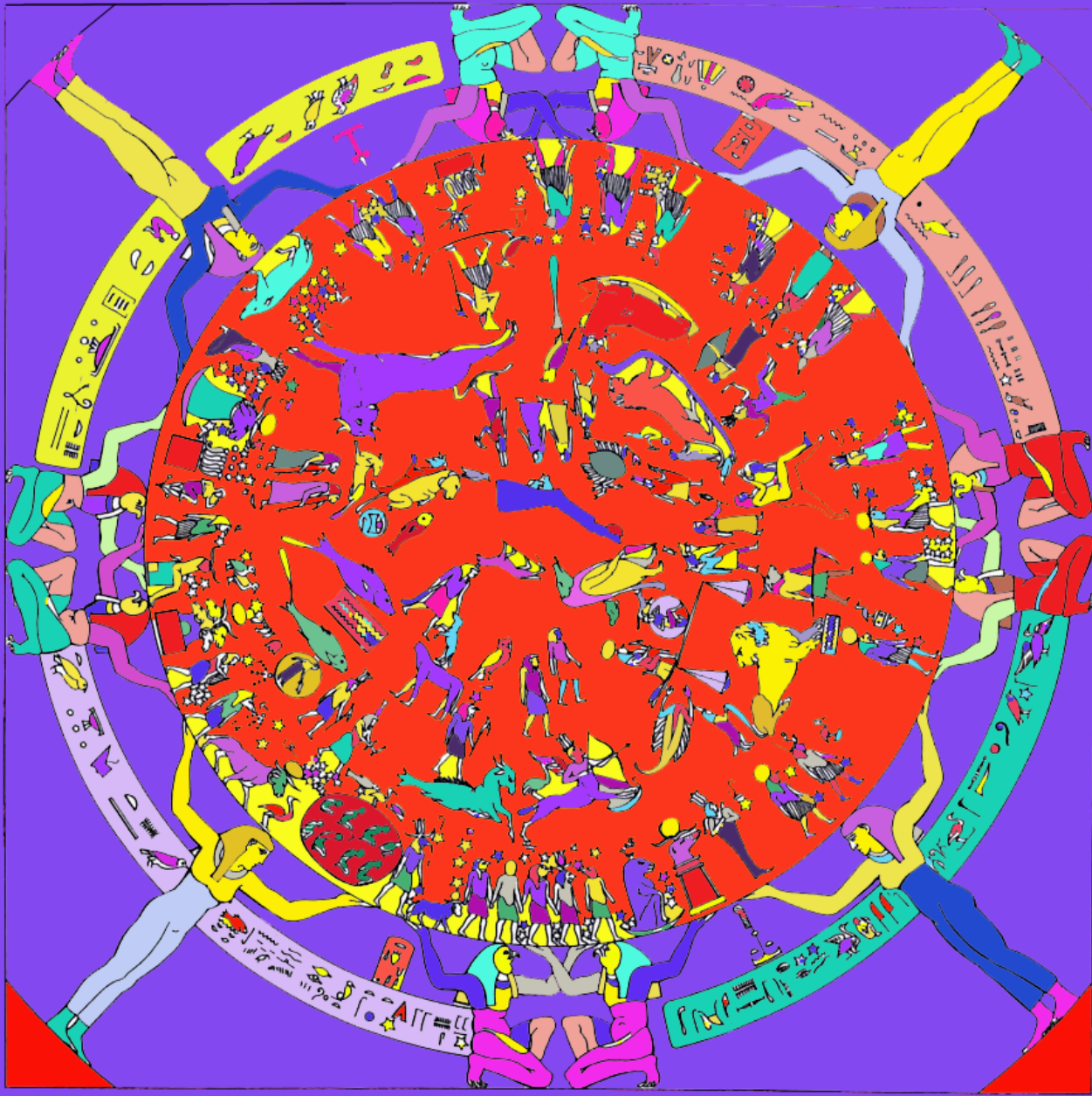
The Temple of Luxor (ancient Thebes), adjacent to the even larger temple complex of Karnak, was the ancient world’s first global university. At its prime, up to 80,000 students were receiving instruction there. All Greek “great names” are known to have studied there: Pythagoras, Plato, Aristotle, Socrates, Hypocrates, Euripides... and later Euclid & Archimedes.

The Greek St. Clement of Alexandria said that even a 1,000 pages book would not be sufficient to hold the names of all the Greeks who went to study in Egypt.



↑ Temple of Luxor (Waset).

SG102.1.1.1.11 Mandalic Interlude 1



Denderah Zodiac (circa 50 BC)

SG102.1.2.1 Ancient Indian Mathematics

Indian Mathematics has its beginnings in the Indus Valley civilization (2600-1900 BCE - use of standardized geometric weights) and the Vedic Culture (1500-500 BCE). All mathematical texts were transmitted orally (with stringent techniques for extraordinary fidelity) until 500 BCE. Thereafter the knowledge was expressed in *Sutra* (Sanskrit for “*thread*”) form, complemented by verbal (and often secret) commentaries. A Sutra is a highly compressed mnemonic formula.

- The *Sulba Sutras* (*Sulba* means “*rope*” ◆SG101.5) were appendices to the Vedas and gave rules for the sacred construction & proportions of altars. They show knowledge of the Pythagorean Theorem and proofs of it, Pythagorean Triples, approximations of squaring the circle, the first use of irrational numbers, calculation of $\sqrt{2}$, exact to 5 decimal places. The best known Sulba Sutra is the Baudhayana Sutra.

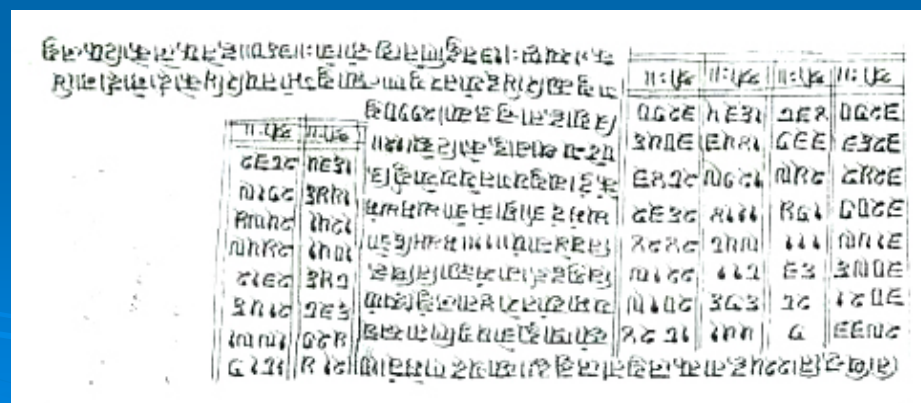
- The *Surya Prajnapti* (c. 400 BCE) is an astronomical text. It gives the length of the lunar month as 29.5161290 days - which is 20 minutes longer than than the modern measurement of 29.5305888 days.

- Jaina Mathematics (400 BCE - 200 CE) contributed the enumeration of very large numbers and infinities (they defined five classes of Infinities). They also were the first to use the *shunya* (Void) which became the zero or “*place-holder*” later passed on to the west by Fibonacci as part of the “*new*” hindu-arabic numerals. The Jain text *Shtananga Sutra* (300 BCE - 200 CE) explored the use of quadratic, cubic and bi-quadratic equations.

- Music scholar Pingala (fl. 400-200 BCE - author of the famous *Chandas Shastras*) contributed the first use of the Fibonacci sequence (some 1,500 years before the historical Fibonacci) and the first use of the Pascal Triangle (some 2,000 years before the historical Pascal). Pascal Triangle was called by Halayudha the “*staircase to Mount Meru*”.

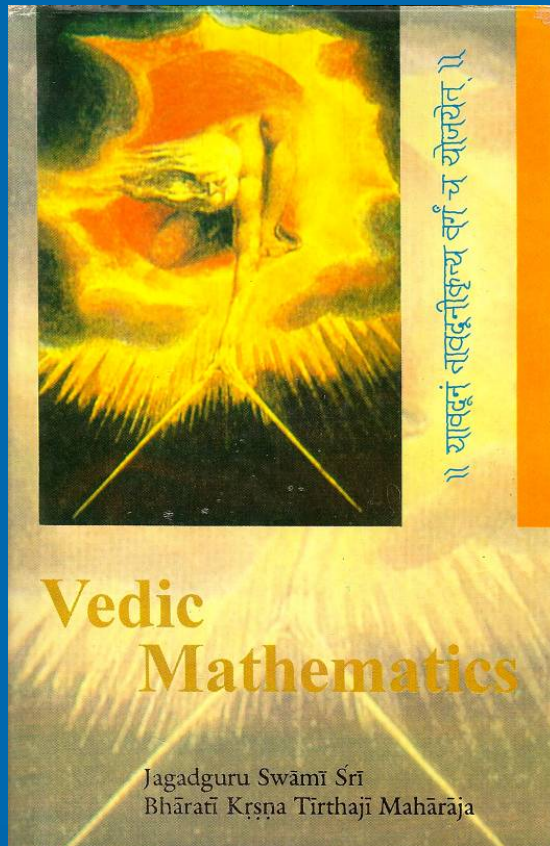
◆SG104.5] ◆SG104.5]

- The *Katyayana Sutra* (3rd century BCE) written by Katyayana, considered to be the last of the Vedic mathematicians, contains much geometric knowledge.



Indian Mathematical Sutra.

SG102.1.2.2 Vedic Mathematics & Geometry (1)



The Vedas (Sanskrit = *knowledge, sacred teaching*) are, taken collectively, the oldest texts of Indian literature, to which orthodox Hindus ascribe superhuman origins and divine authority. This large compilation (about 6 times the bulk of the Bible) was written down around 1600 BE from more ancient oral tradition.

Sri Bharati Krsna Tirthaji (1884-1960), a brilliant scholar versed in many disciplines as well as a *Shankaracharya* (the highest religious title in India), is credited to have reconstructed the ancient system of Vedic Mathematics from Vedic Scriptures that had been dismissed as non-sense.

The Vedic system he rediscovered is based on 16 sutras (The *Ganita Sutras* - *Ganita* means *mathematics*) which cover all branches of mathematics, pure & applied. Bharati Krsna wrote 16 volumes, one on each sutra but they were “unaccountably” lost and, in his final years, he wrote a single book: *Vedic Mathematics* which was published in 1985, five years after his death. Bharati Krsna believed in the spiritual & cultural harmony taught by the Vedic tradition and his ambition for humanity was a world-wide cultural & spiritual awakening.

In 1985, the first Vedic Mathematics Conference was held in India and today Vedic Math is taught and researched widely in India and, worldwide, attracts the attention of mathematicians, teachers and educators as Vedic Math methods are highly successful (and popular) with children and students.

“Vedic Mathematics is the science and technology of precision & order, which maintains the integrity of Unity and at the same time spreads diversity within it”.

(Maharishi Mahesh Yogi)

SG102.1.2.3 Vedic Mathematics & Geometry (2)

Vedic Math is highly relevant to Sacred Geometry:

- Vedic Math recognizes that significant numerical & geometrical patterns are underlying mathematical operations. In the West, Pythagoras was also teaching about visual geometric patterns, their properties and use for mathematical and philosophical knowledge. In fact, Pythagoras is reputed to have traveled to India and learned from the Vedic sages (*ancient Brahmin records are said to have preserved his name as Yavancharya or 'Ionian Teacher'*).
- These visual patterns are the way the brain and the higher mind function. Vedic Math is essentially a yoga of visual perception in 2D & 3D and the 16 Ganita Sutras are instructions for performing mental asanas of visualization and inner sight. Vedic Math, taught together with Sacred Geometry, is a great way to teach children how to keep open their inner vision or third eye. Synaesthetic resonance between left & right brains and with higher functions of consciousness brings about Wisdom (as opposed to mere learning).
- Like Sacred Geometry, Vedic Math offers the overall intention of uncovering the hidden harmony of the universe and rediscovering the laws & principles that can express and establish Peace and Harmony in human civilizations.

1	एकाधिकेन पूर्वेन Ekādhikena Pūrvena	By One More than the One Before	9	चलनकलनाभ्याम् Calana Kalanābhyām	Differential Calculus
2	निखिलं नवतश्चरमं दशतः Nikhilam Navataścaramam Daśatah	All from 9 and the Last from 10	10	यावदूनं Yāvadūnam	By the Deficiency
3	उर्ध्वतिर्यग्भ्यामं Urdhva Tiryaḡbhyām	Vertically and Cross-wise	11	व्यष्टिसमष्टिः Vyastisamastih	Specific and General
4	परावर्त्य योजयेत् Parāvartya Yojayet	Transpose and Apply	12	शेषासयङ्गेन चरमेण Śeṣāsanayāṅgeṇa Caramena	The Remainders by the Last Digit
5	शून्यं साम्यसमुच्चये Śūnyam Sāmyasamuccaye	If the Samuccaya is the Same it is Zero	13	सोपान्त्यद्वयमन्त्यं Sopāntyadvayamantyaṃ	The Ultimate and Twice the Penultimate
6	आनुरूप्ये शून्यं अन्यत् (Ānurūpye) Śūnyamanyat	If One is in Ratio the Other is Zero	14	एकन्यूनेन पूर्वेन Ekanyūnena Pūrvena	By One Less than the One Before
7	संकलन व्यकलनाभ्यां Saṅkalana Vyavakalanābhyām	By Addition and by Subtraction	15	गुणितसमुच्चयः Gunitasamuccayah	The Product of the Sum
8	पूरणापूरणाभ्यां Pūranāpūranābhyām	By the Completion or Non-Completion	16	गुणाकसमुच्चयः Gūṇākasamuccayah	All the Multipliers

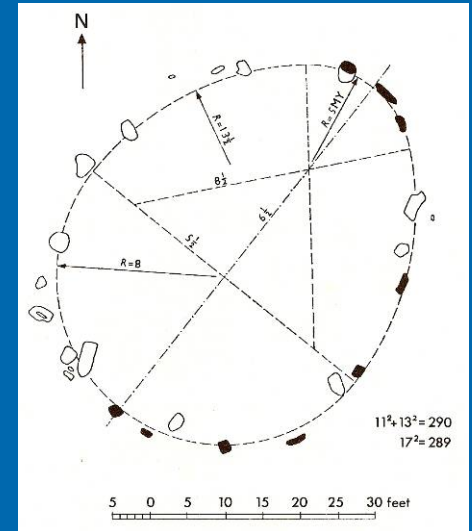
← The
16
Ganita
Sutras

SG102.1.3.1 Megalithic Sites (1) Geometric Layouts

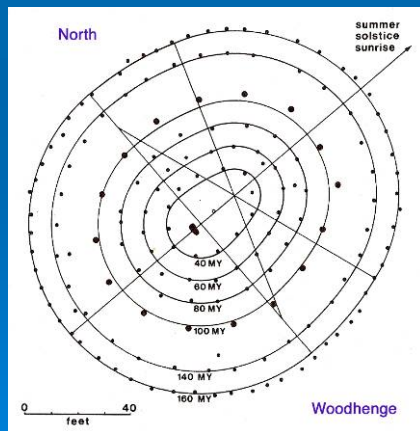
In 1967, Professor Alexander Thom, a retired Scottish engineer, published *Megalithic Sites in Britain*, a survey of more than 300 stone circles. His book was a documented revelation: our forbears had a sophisticated knowledge of geometry, astronomy, surveying and engineering techniques.

Thom showed that, far from being crude structures, these megalithic stone circles were meticulously designed according to a unified canon of geometry.

- Invariably their layout was based on *Pythagorean Triangles* or right-angled triangles where the base, height and hypotenuse are whole numbers, such as the famous 3-4-5 triangle. Their geometry also incorporates celestial sight lines.
- Their dimensions were set out in terms of a common unit of measure: the Megalithic Yard (2.72 feet).
- They were planned to emphasize integral numbers, evidently of symbolic, magical, geodetic or astronomical significance.



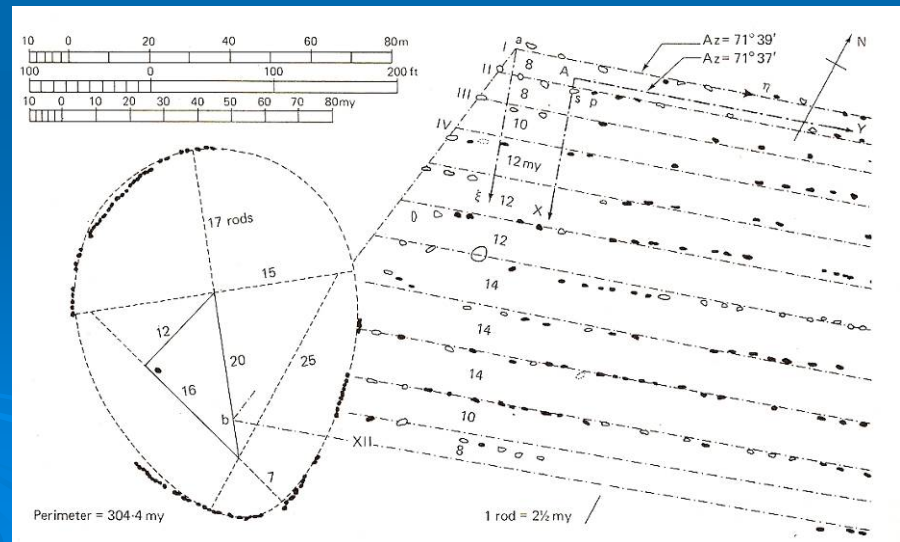
↑ Roxburghshire



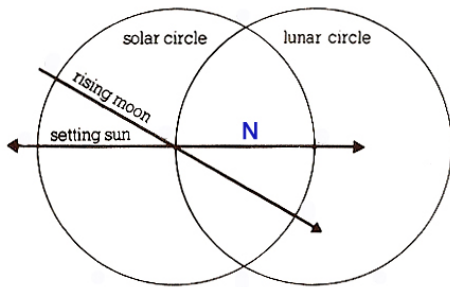
↑ Woodhenge



Carnac
(Brittany,
France)

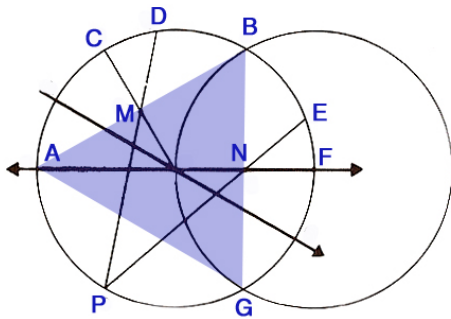


SG102.1.3.2 Megalithic Sites: Castle Rigg



The two foundation axes at Castlerigg:
sun-setting of midsummer day
& most southerly moonrise

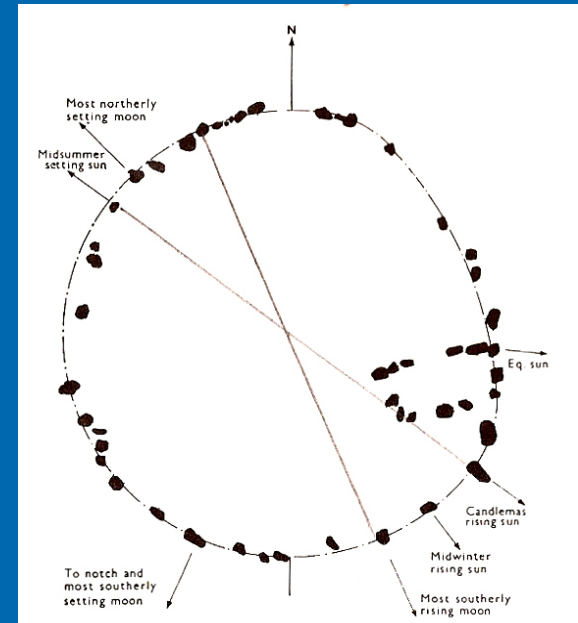
ABG forms an equilateral triangle.
N is the center of the Vesica



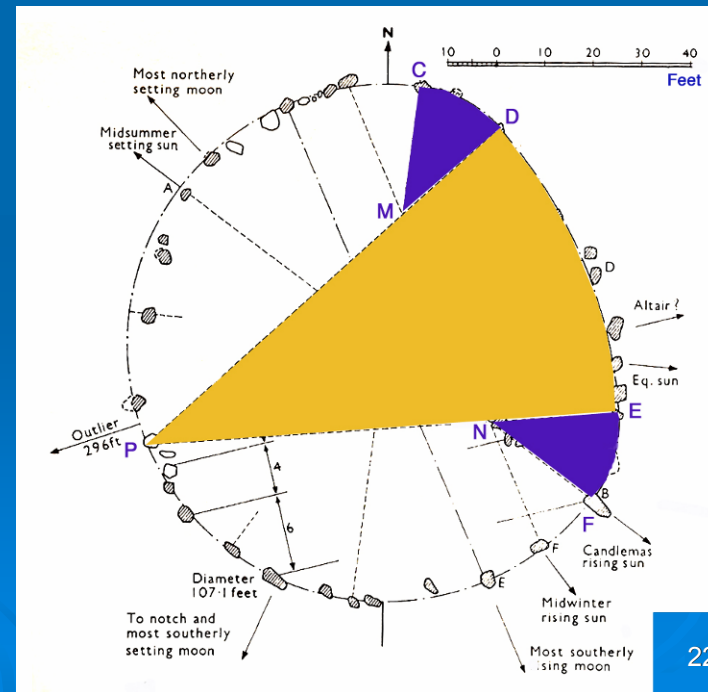
↑ Using the two key Solar & Lunar axes, the circle builders generated interlocking circles (forming a Vesica Piscis or geometric womb) and from them derived the rest of the ground plan geometry.



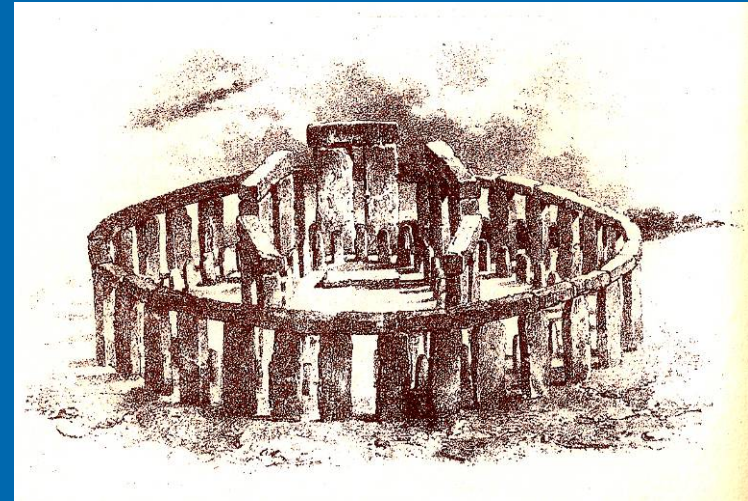
The stone circle at Castlerigg (Cumberland, England) shows 7 solar and lunar declinations. According to witnesses, experiencing on-site these celestial light alignments is an event of the 'heavenly heart'.



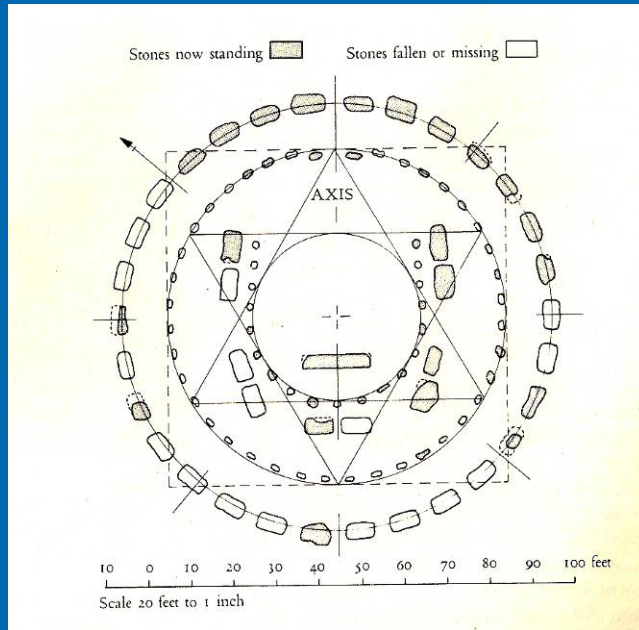
The flattened section of the ring is traced as arc CD (from center M), arc DE (from P) and arc EF (from N). The rest of the original solar circle runs from F (center of lunar circle) clockwise to C.



SG102.1.3.3 Megalithic Sites (3) Stonehenge



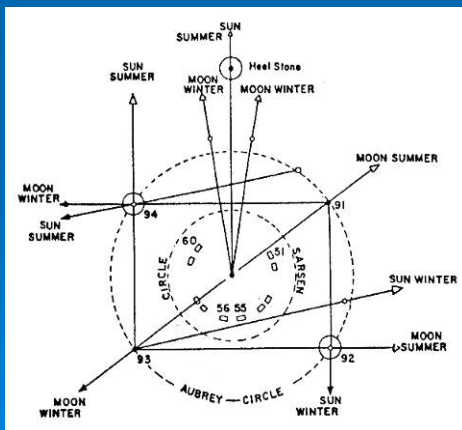
Stonehenge restored



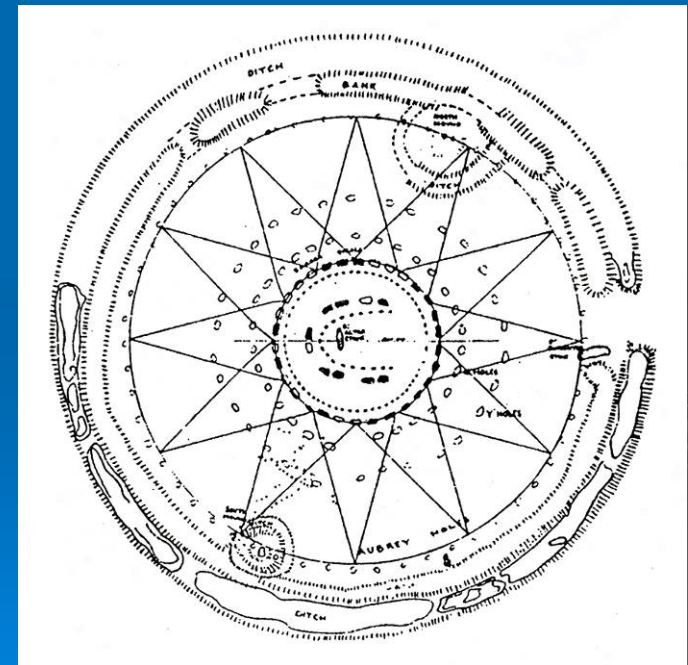
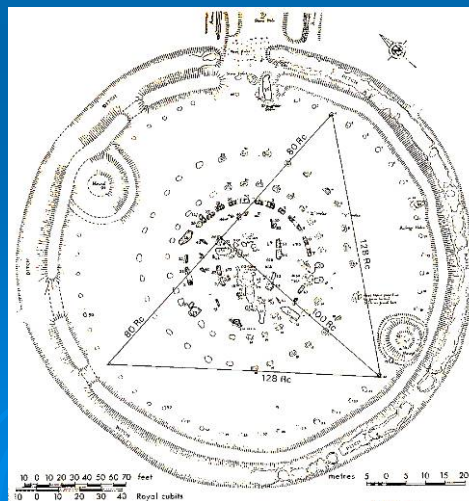
← Squaring the Circle.

The mean circumference of the outer Sarsen circle with lintels is 316.8 ft. (1/100th part of 6 miles).

A square with perimeter of 316.8 ft. would contain a circle enclosing the bluestone ring, diameter 79.2 ft.



↑ Astronomical alignments



←↑ Stonehenge geometries

SG102.1.3.4 Megalithic Sites (4) Ancient Geomancy

In Great Britain, research into ancient geomancy (the art-science of harmonizing the earth), by such pioneers as John Michell, Guy Underwood or Paul Devereux, has validated the old traditions stating that the ancient sages encoded their knowledge of the world in the dimensions of their temples. A growing evidence points to sophisticated ancient surveying techniques and possibly global mapping of the earth in view of establishing, through temples sites and geomantic landscapes, auspicious harmonic energy grids.

These grids and alignments were the most conducive to the enhancement of the currents of Life Force, the blessings of heaven upon earth, and ultimately the material & spiritual well-being of the people.

- Stone structures & earth works were placed to mark the subterranean earth currents or flows of terrestrial magnetic energies. These were the '*ley lines*', '*dragon paths*' and crossroads of old sacred cultures. The entire geographical landscaping of ancient Britain was laid out to coincide with the currents, paths, spirals & hubs of subterranean intensified magnetic influence.
- The natural earth currents were engineered (tamed) to conform to a system of harmonic universal geometry, a grid able to harness these energies for the benefit of human cultures. This is more than a one-way influence where humans only notice & seek the benefits of the earth's power spots. The human-made structures, placed upon the earth on a global scale, in turn affect the course & potency of the subterranean flows. This is the art of western *Geomancy* and oriental *Feng Shui & Vastu*.

Nowadays, we speak in terms of *earth meridians* and acupuncture techniques applied to the global icosia-dodeca planetary grid layout, as we understand that the life force that animates the human body is the same as that which flows through the veins of the earth.

- Planting upright stones, earth structures, temple architecture or labyrinths at certain influential locations allows for the introduction of sun and atmospheric energies into the earth current, a global form of alchemy or tantrism or *eco-acupuncture*.
- In opposition to the contemporary architecture & urbanism seeking convenience & profit and oblivious of the larger context of global harmonic health, ancient geomancy is a wisdom seeking to enhance the interconnectedness of the web of life and the expansion of spirit. [◆SG207.1]

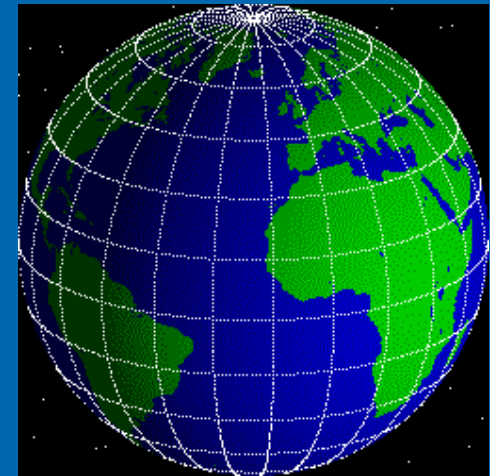
SG102.1.3.5 Megalithic Sites (5) Ancient Geodesy

Current research in ancient metrology, sacred sites and prehistoric geomancy points to the existence of a long forgotten code of knowledge, a unified number code behind all units of measure. This *canon of number* was openly applied to attract the blessings of heaven upon earth and maintain high cultural standards in ancient cultures. All the ancient units of measure relate to each other, and to the dimensions of the earth, by the same code of number as is found in every other ancient form of art & science.

Example #1: Multiples of Greek & Roman measures, in terms of the Earth's meridian circumference:

24,883,2 miles = 131,383,296 ft or $12^6 \times 44$ ft = 135,000,000 Roman feet = 90,000,000 Roman cubits = 216,000 Roman furlongs = 27,000 Roman miles = 129,600,000 Greek feet = 86,400,000 Greek cubits = 207,360 Greek furlongs = 25,920 Greek miles.

All these measures are canonical numbers, representing powers & multiples of the number 12. Note that 25,920 is also the Great Year or Precession of the Equinoxes. 1,290,000 is the Treta Yuga, in Hindu cosmology. The same units show up in space measurements & time cycles.



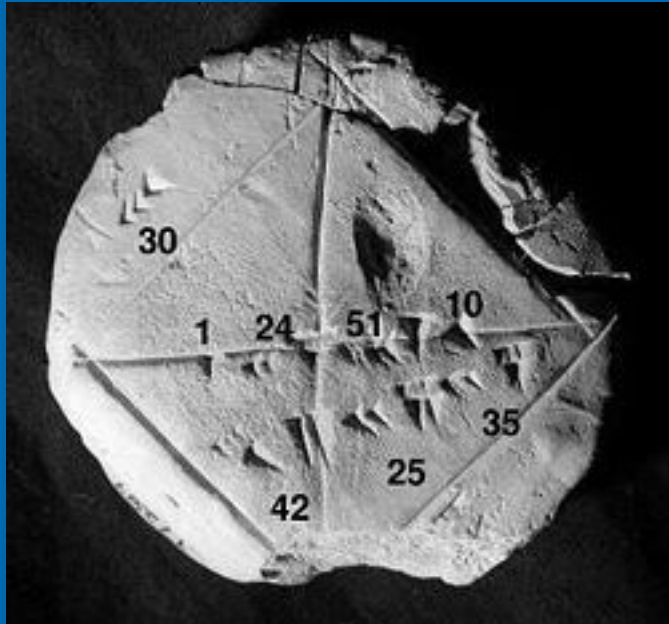
Example #2: The principal dimensions of Stonehenge represent simple fractions of the dimensions of the earth:

Earth polar radius = 20,854,491 ft = 400,000 x Stonehenge outer radius = 6,000,000 x Stonehenge lintel width.
Earth mean circumference = 131,383,296 ft = 2,520,000 x Stonehenge outer radius = 2,700,000 x Stonehenge inner radius = 37,800,000 x Stonehenge lintel width.

Example #3: The Earth's mean radius was defined in proportion to the polar axis, the earth's circumference and the units of ancient metrology as 20,901,888 ft = polar radius x 441/440 = 3958.6909 miles = mean circumference x 7/44 = $12^6 \times 7$ ft = 20,736,000 shorter Greek feet = 21,600,000 shorter Roman feet.

(All data from John Michell. *New View Over Atlantis*.)

SG102.1.4.1 Babylonian Mathematical Tablets



Babylonian clay tablet *YBC 7289* with annotations. The diagonal displays an approximation of the square root of 2 in four sexagesimal figures, which is about six decimal figures.

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = 1.41421296\dots$$

(www.en.wikipedia.org)



(www.shoyencollection.com)



(www.historyforkids.org)

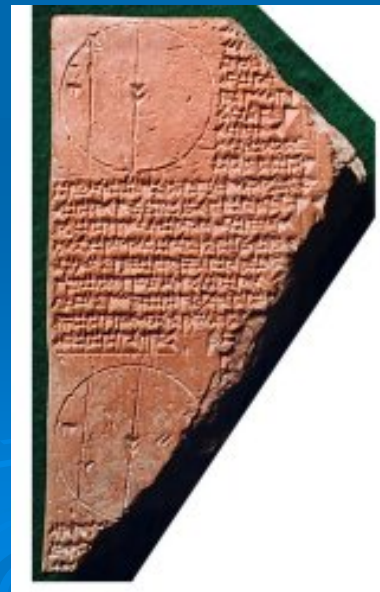


← Tablet *Plimpton #322* contains a list of 'Pythagorean Triples'. These are 3 counting numbers satisfying the Pythagoras Theorem relationship:
 $a^2 + b^2 = c^2$

(Image: Sonnabend. Maths for Elementary Teachers.)

a	b	c
120	119	169
3456	3367	4825
4800	4601	6649
13500	12709	18541
72	65	97

← First 5 rows of the Plimpton tablet



(www.biblehistory.org)

SG102 1.5.1 Neolithic Polyhedral Spheres (1)

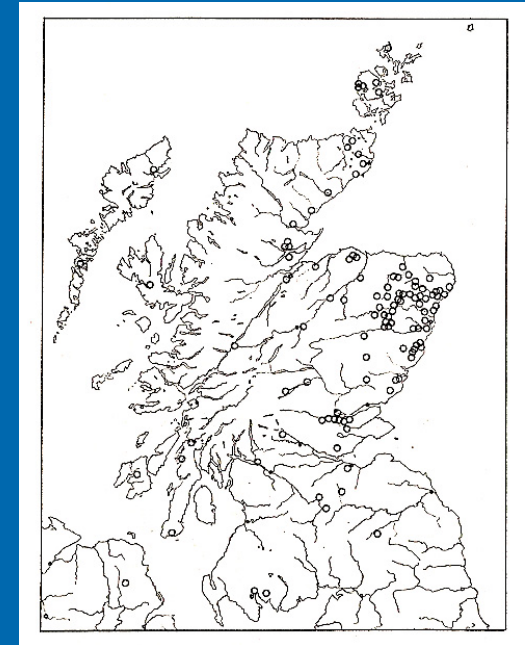
More than a thousand years before Plato described the five basic geometric forms known as the “*Platonic Solids*”, hundreds of polyhedra stone spheres displaying the same regular mathematical properties, have been found in Scotland.

These proto sacred geometry objects average 3” in diameter and are carved from soft sandstone & serpentine all the way to hard granite & quartzite. Many display spirals and cross-hatchings.

The majority of these Neolithic geometric solids is of the octohedral symmetry, the next highest percentage of the tetrahedral symmetry. In addition to the five Platonic solids, there are two semi-regular solids (Archimedean): the *cuboctahedron* (14 faces) + its dual (*rhombicuboctahedron*) and the *icosidodecahedron* (32 faces) + its dual (*rhombic triacontahedron*).

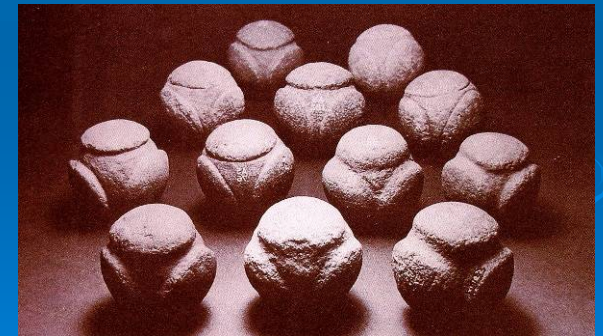


↑ A set of Scottish Neolithic “Platonic Solids”



↑ Sites of discovery in Scotland

(All pictures: Keith Critchlow.
Time Stands Still)



↑ Twelve Tetrahedral Solids

SG102.1.5.2 Neolithic Polyhedral Spheres (2)



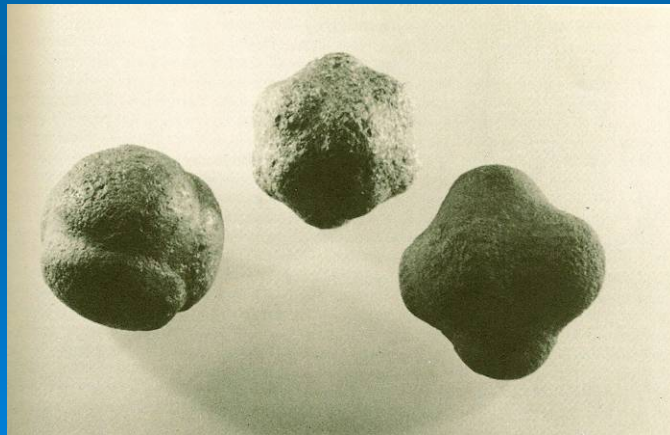
← A Neolithic Tetrahedral Sphere with vibrational waves & spirals



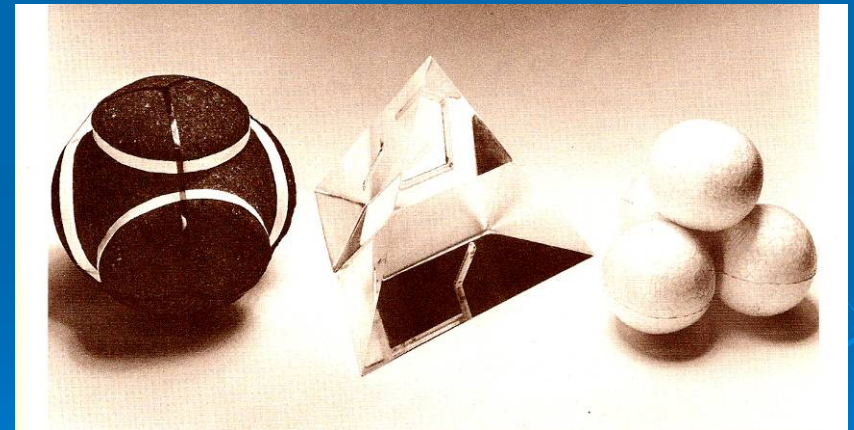
↑ Dodecahedral Symmetry in Neolithic Sphere

(All pictures: Keith Critchlow

· Time Stands Still.)

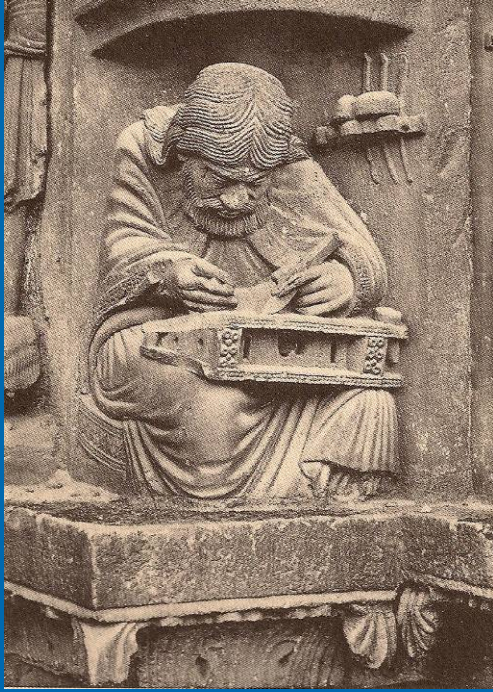


↑ Three basic Platonic Solids: Tetrahedron, Icosahedron & Octahedron



↑ Tetrahedron: Neolithic, Planar & Sphere Model

SG102.2 - Chapter 2. The Life of Pythagoras



↑ Pythagoras shown writing, in medieval fashion, with a desk on his knees.

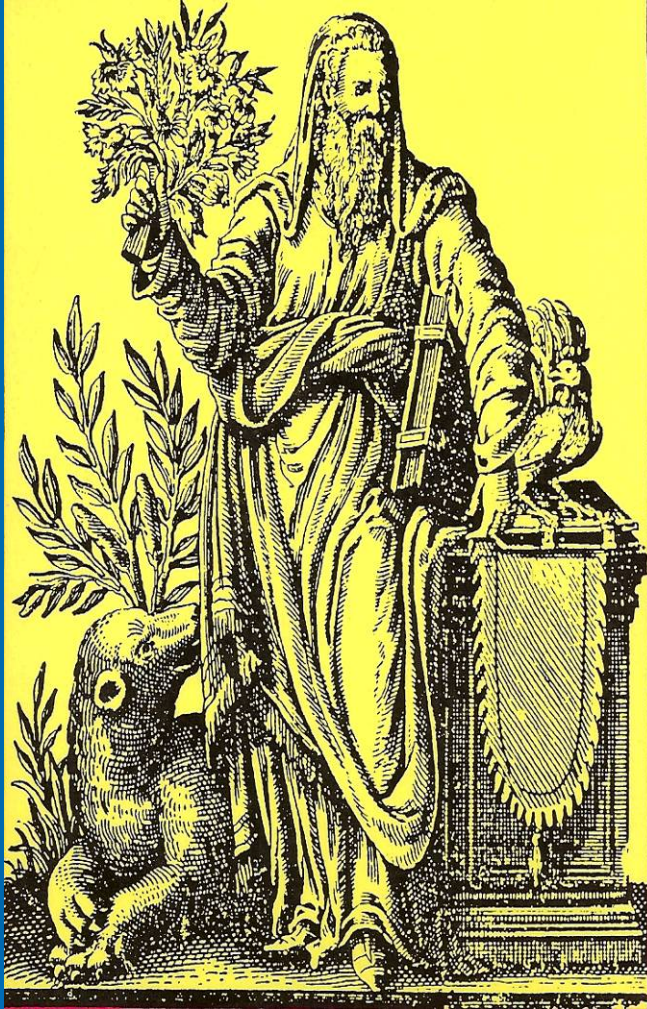
As the reputed founder of music theory, Pythagoras is associated with the figure of Music (above him, on the archivolt). Music was one of the *Seven Liberal Arts* forming the basis of medieval education.

Chartres Cathedral, France. Archivolt forming the tympanum of the Virgin Portal, West façade. c. 1145-1170.



↑ Pythagoras experimenting with stretched cords to establish the proper relationships between number ratios and sound frequencies.

SG102.2.1.1 Life of Pythagoras of Samos: The Traditions



(Image: Pythagorean Sourcebook. K. S. Guthrie,)

Pythagoras was born in Sidon (Phoenicia) in the beginning of the 6th century BCE. This was a time of world teachers: *Gautama Buddha, Zoroaster, Confucius and Lao-Tse.*

The father of Pythagoras was informed by the *Pythoess* (prophetess at Apollo) in Delphi that his wife would give birth to a son whose destiny was to become a great benefactor to mankind. Hence the child was named ‘*Pythagoras*’ (‘*predicted by the Pythia*’).

As a young man, Pythagoras won the praised olive branch at the 48th Greek Olympiad Games. After extensive travels, (all the way to India, it is reported), and initiatic instructions in many mystery schools (Egyptian, Eleusinian, Babylonian, Chaldean, Brahmanic...), Pythagoras returned to the West and established a school & spiritual community at Crotona (Southern Italy) where he taught the fundamentals of esoteric mathematics, music & astronomy, as well as ‘philosophic’ wisdom (Pythagoras is said to have coined the word ‘*philosopher*’). The School at Crotona was later burned down by fanatics.

Pythagoras didn’t leave any writings but his teachings, transmitted by an eager lineage of disciples, apologists (like Plato) and seekers, have had a prodigious influence on the western thought, culture and consciousness. Sacred Geometry is one of the legacies from the Pythagorean School.

SG102.2.1.2 Life of Pythagoras: the Historical Pythagoras

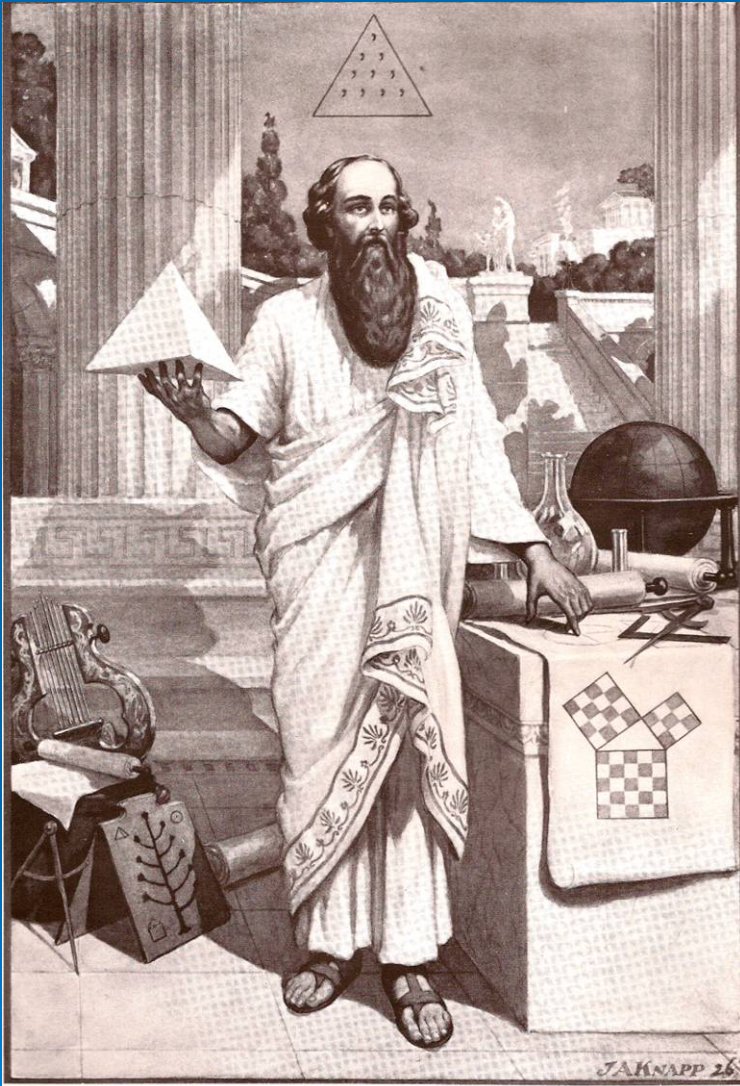


Image: M. P. Hall. *Secret Teachings of All Ages*.

The historical facts about Pythagoras are scant & clouded in legend. We have three *Lives of Pythagoras* from late antiquity: by Diogene Laertius (3rd c. CE), Porphyry of Tyre (c. 233 – c. 305 CE) and Iamblichus of Chalcis (c. 250 - c. 325 CE). These biographies, although written almost a millennium after Pythagoras, are valuable as they quote many earlier writers, often verbatim.

Represented as the greatest wise man of early Greece, Pythagoras is described as both a mystical teacher, credited with numerous supra-natural deeds, and a scientific philosopher, credited with discovering the mathematical ratios common to archetypal numbers, music and celestial phenomena.

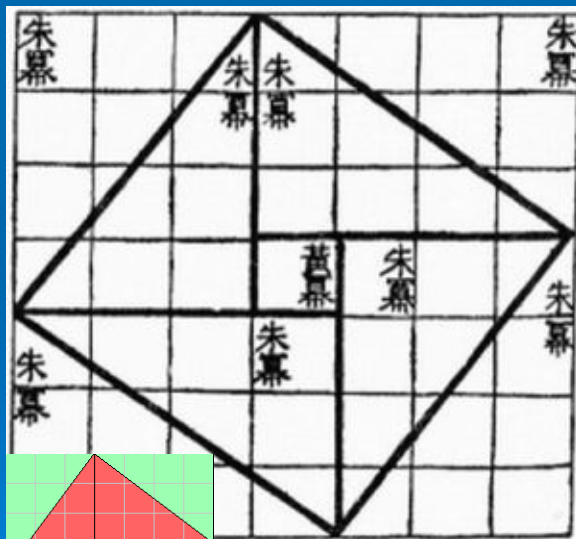
A fourth, albeit short, biographical account is the *Life of Pythagoras* by Photius (c. 809 – 891 CE), a Byzantine patriarch of the Imperial Academy in Constantinople. Photius may have preserved parts of Aristotle's lost treatise *On the Pythagoreans*.

Of the earliest Pythagorean writings, we have fragments from Philolaus of Tarentum (born c. 474 BCE), a pupil of Lysis (one of the two Pythagoreans who escaped the burning of the school at Crotona). Philolaus' texts reflect the same view as later attributed to Pythagoras by Aristotle (384 - 322 BCE).

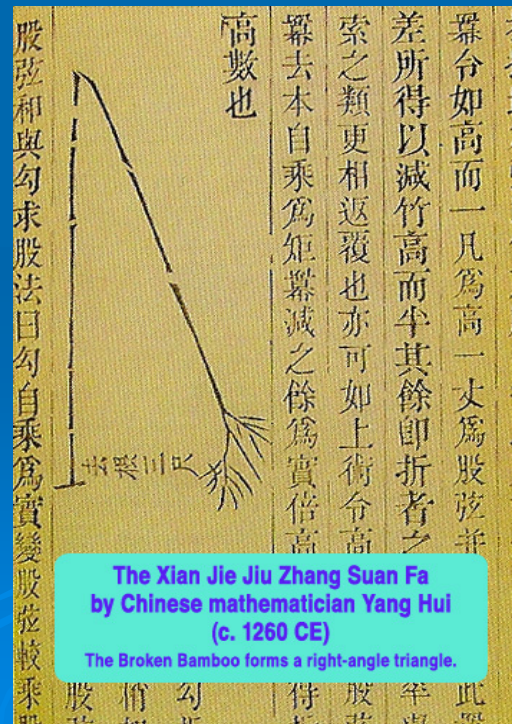
SG102.2.1.3 The Pythagorean Theorem World Wide

The so-called “*Pythagorean*” Theorem has in fact been worked out long before Classical Greece and has been known the world over: in Egypt, Babylon, India, Japan, China...

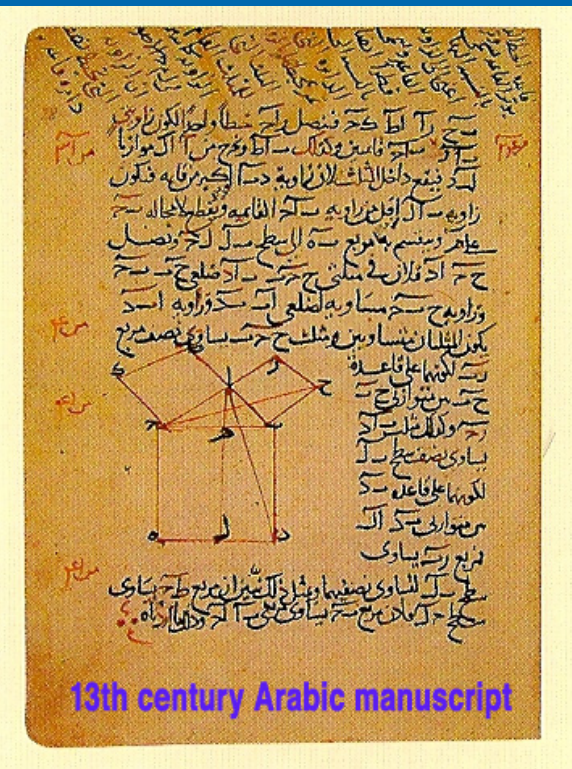
- Egyptian papyri (c. 2000 BCE) and Mesopotamian tablets (c. 1800 BCE) give problems with Pythagorean Triples.
- The *Baudhayana Sulba Sutra* (c. 800 BCE) has a geometrical proof for an isosceles right triangle.
- The *Apastamba Sulba Sutra* (c. 600 BCE) contains a numerical proof. In India, the Pythagorean Theorem is known as the *Bhaskara Theorem*.
- The Chinese text *Zhou Bi Suan Jing* (Classic of the Gnomon and the Circular Paths of Heaven) gives a visual proof. In China, the Pythagorean Theorem is called the *Gougu Theorem*.



↑ Proof from the Zhou Bi Suan Jing

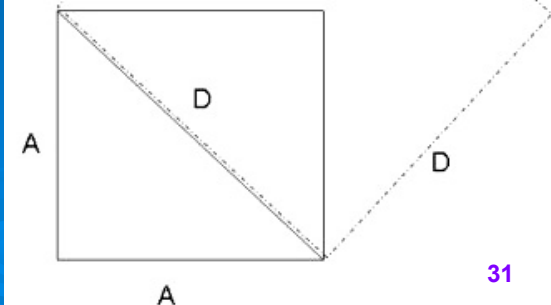


The Xian Jie Jiu Zhang Suan Fa by Chinese mathematician Yang Hui (c. 1260 CE)
The Broken Bamboo forms a right-angle triangle.



13th century Arabic manuscript

"The rope which is stretched across the diagonal of a square produces an area double the size of the original square."
(Baudhayana Sulba Sutra)



SG102.2.2.1 How Pythagoras Healed by Music & Words (1)

Quotes from Iamblichus 'Life of Pythagoras' (1)

Iamblichus (c. 250 - c. 325 CE) was a Neo-platonic philosopher who attempted to write a 10 volumes *Encyclopedia of Pythagorean Thought*. The first volume of this unfinished corpus is an extensive '*Life of Pythagoras*'. Here are some quotes relevant to the use of music as a healing modality.

“Pythagoras conceived that the first attention that should be given to men should be addressed to the senses, as when one perceives beautiful figures and forms, or hears beautiful rhythms and melodies. Consequently he laid down that the first erudition was that which subsists through music’s melodies and rhythms, and from these he obtained remedies of human manners and passions, and restored the pristine harmony of the faculties of the soul.

For his disciples, he arranged and adjusted what might be called ‘preparations’ and ‘touchings’, divinely contriving mingling of certain diatonic, chromatic and enharmonic melodies, through which he easily switched and circulated the passions of the soul in a contrary direction, whenever they had accumulated recently, irrationally, or clandestinely - such as sorrow, rage, pity, over-stimulation, fear, manifold desires, angers, appetites, pride, collapse or spasms. Each of these he corrected by the rule of virtue, attempering them through appropriate melodies, as though some salutary medicine.

In the evening, likewise, when his disciples were retiring to sleep, he would thus liberate them from the day’s perturbations and tumults, purifying their intellective powers from the influxive and effluxive waves of corporeal nature, quieting their sleep, and rendering their dreams pleasing and prophetic. But when they arose in the morning, he would free them from the night’s heaviness, coma and stupor through certain peculiar chords and modulations, produced by either simply striking the lyra, or adapting the voice.

SG102.2.2.2 How Pythagoras Healed by Music & Words (2)

Quotes from Iamblichus 'Life of Pythagoras' (2)

From the same source (Iamblichus), here is another passage describing the healing powers Pythagoras extended through his words and presence.

“According to credible historians, his (Pythagoras) words possessed an admonitory quality that prevailed even with animals.

The Daurian bear, who had severely injured the inhabitants, was by Pythagoras detained. After long stroking it gently, feeding it on maize and acorns, and compelling him by an oath to leave alone living beings, he sent it away. It hid itself in the mountains and forest, and was never since known to injure any irrational animal.

At Tarentum, he saw an ox feeding in a pasture, where he ate green beans. He advised the herdsman to tell the ox to abstain from this food. The herdsman laughed at him, remarking that he didn't know the language of oxen; but that if Pythagoras did, he had better tell him so himself. Pythagoras approached the ox's ear and whispered into it for a long time, whereafter the ox not only refrained from them, but never even tasted them. This ox lived a long time at Tarentum, near the Temple of Hera, and was fed on human food by visitors till very old, being considered sacred.

Once happening to be talking to his intimates about birds, symbols and prodigies, and observing that all these are messengers of the Gods, sent by them to men truly dear to them, he brought down an eagle flying over Olympia, which he gently stroked, and dismissed.”

(From 'The Life of Pythagoras' by Iamblichus, quoted in The Pythagorean Sourcebook, K. S. Guthrie).

SG102.2.3 The Aphorisms of Pythagoras

The historiograph Iamblichus gathered an anthology of symbolic sayings of Pythagoras. Aphorismic statements was one of the favorite methods of instruction used in the Pythagorean School at Crotona.

Here is a brief selection:

"Declining from the public ways, walk in unfrequented paths

*Assist a man in raising a burden,
but do not assist him in laying it down*

*Govern your tongue before all things,
following the gods*

The wind blowing, adore the sound

Wear not a narrow ring

One, Two, Three, Four

Do not urinate against the sun

Receive not a swallow into your house

Never sing without harp-accompaniment

Having departed from your house, turn not back"

SG102.2.4.1 Pythagorean Heritage in the West (1)

In his *Pythagoras and the Pythagoreans* (2001), Charles Kahn, Professor at the University of Pennsylvania, gives us an overview of the current scholar research about the Pythagorean Tradition, outlining the “reverberation” of Pythagorean concepts throughout antiquity and the Renaissance, all the way to modern developments in science. Says Kahn: “*The notion of Cosmic Harmony expressed in musical ratios and conceived as astral music is one of those ideas of genius that have remained amazingly fruitful over the centuries*”.

Joscelyn Godwin, in his Foreword to the “*Pythagorean Sourcebook*” (1987), calls Pythagoras the “*very midwife of our epoch*”.

Pythagoras and Plato. Although current scholars point to a re-interpretation of the Pythagorean doctrines by Plato (427-347 BCE) and his followers, Plato himself can be viewed as the most important Pythagorean thinker in the western tradition. It has been noted that Plato’s Academy was inspired by its parent Pythagorean School. And the main aspects of the Pythagorean teachings are transmitted and amplified in Plato’s writings: the *Phaedo* sings the immortal destiny of the soul and the precious ‘recollection’ of prenatal knowledge; the *Timaeus* (“*The single most important text for the future of the Pythagorean Tradition*” - Kahn) shows the role of Number as a cosmic paradigm and template of creation (“*God Geometrizes*”). Moreover, throughout Plato’s works, music and harmony are credited with direct social and moral education, in pure Pythagorean fashion.

Plato’s friend Archytas of Tarentum (Plato visited him in 388 BCE) was a student of Philolaus, an eminent statesman and an outstanding mathematician who solved the geometrical (‘Delian’) problem of how to double the volume of the cube. In music and harmonics, Archytas set standards for later mathematical precision. The Pythagorean Tradition was then sporadically picked up by the Hellenistic “*mathematikoi*” who were concerned with scientific philosophy: Aristoxenus, Speusippus, Xenocrates...

SG102.2.4.2 Pythagorean Heritage in the West (2)

The Neo-Pythagorean Revival. The first sign of public renewed interest in Pythagorean thought was from Cicero (106-43 BCE) mentioning his friend Nigidius Figulus and the revival of Neo-Pythagorism.

The next phase concerns the so-called '*Neo-Pythagoreans*' at the beginning of our era: Posidonius, Eudorus,, Philo of Alexandria, Apollonius of Tyana (who saw himself as a reincarnation of Pythagoras), Plutarch of Chaeronea (well known for his *Lives* and for being a priest of Apollo at Delphi). In the second century, we have Theon of Smyrna (floruit c. 125 CE) who wrote the *Mathematics Useful For Understanding Plato* and . Nichomachus of Gerasa (active c. 140-150 CE) whose *Introduction to Arithmetic* became a classic until the Renaissance and who also wrote a *Manual of Harmonics* and a *Theology of Arithmetic*.

With Numenius of Apamea in Syria (fl. c. 160) and Plotinus (c. 205-270), Pythagoreanism and Platonism are oriented towards systems of cosmic theology. Finally Porphyry (233-305) and Iamblichus both wrote essential *Lives of Pythagoras*.

Pythagoras in the Renaissance. The humanist philosophers of the 15th century's Renaissance, such as Marsilio Ficino (1433-1499 / Ficino was the founder of the Florentine '*Academy*'), Pico della Mirandola (1493-1494) and the German Scholar Johann Reuchlin were the first Europeans to translate and open to the West the full literature of the Pythagorean & Platonic Traditions. These humanists, in the words of Charles Kahn, '*restored the image of Pythagoras as omniscient sage familiar from late antiquity*'. Nicholas of Cusa (1401-1464) developed a universal system based on mathematical ratios, thus prefiguring one strand of the coming mathematical science of nature.

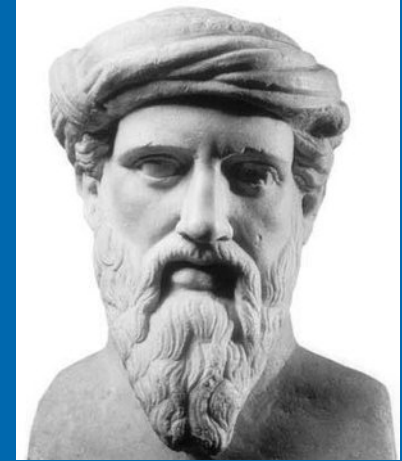
Bridging forth into modern science, astronomers Copernicus (1473-1543), Galileo (1564-1642) and most eminently Johannes Kepler (1571-1630) invoked the Pythagorean School as a precedent for heliocentric astronomy. Kepler went on to dedicate his life to decipher the geometry and harmony of the '*Spheres*'. us Robert Fludd (1574-1637) drew many cosmological diagrams about the Cosmic Monochord.

SG102.2.4.3 Pythagorean Heritage in the West (3)

Modern & Contemporary Pythagoreanism. Thomas Taylor (1758-1835 - *Theoretic Arithmetic of the Pythagoreans*, 1816) and Fabre d'Olivet (1767-1825) were the two great 'Romantic' Pythagoreans. At the end of the 19th century, the Theosophical Society and various esoteric groups gave Pythagoreanism a new impetus. And, in the 20th century, the German 'Harmonic School' with Hans Kayser and others accumulated a lot of data on the presence of Sacred Geometry and musical proportions in Nature, while, in the US, Manly P. Hall's *'Secret Teachings of All Ages'* devoted three chapters to Pythagoras.

On the scientific side, it has been pointed out that the pioneers of Quantum Physics were acquainted with Pythagorean musical ratios: Max Planck (1858-1947) developed his Quantum Theory by observing overtones on a monochord. Another founder of Quantum Theory, Werner Heisenberg (1901-1976) said that insights into the Law of harmonic relationships in "*one of the strongest impetus in human science*". Astronomers have been looking for and finding confirmation of harmonic musical ratios in the solar system, thus vindicating Kepler (although not in the exact same terms). The 'Platonist' mathematician Alfred N. Whitehead described Pythagoras as the first thinker to "*divine the importance of number*".

In the musical domain, the limitations of equal temperament, the standardization of the concert pitch "A" and the exposure to non-western musical modes have brought about a renewal of interest and research into '*Pythagorean Tuning*' (exact classical ratios of octave, fifths and fourths) and '*Just Intonation*'. (See the Lambdoma Keyboard ◆SG103.3 and SG201)

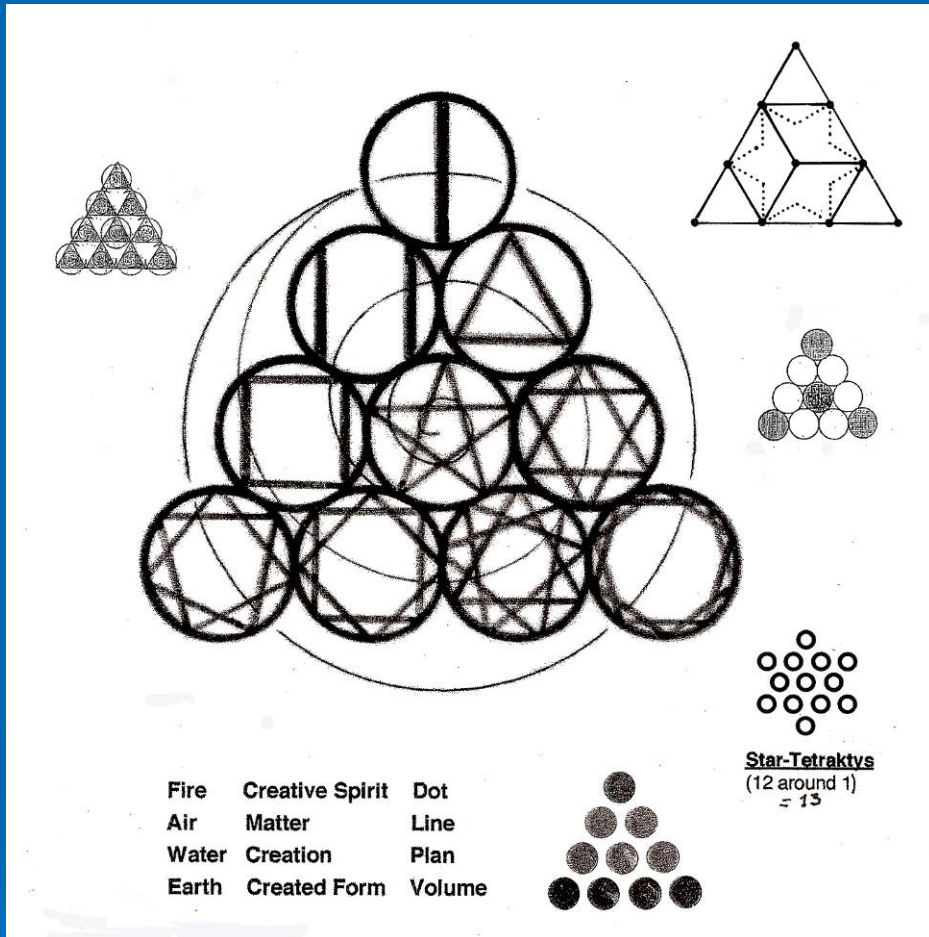


Pythagoras

*"There are some things
so serious
we have to laugh
about them".*

Niels Bohr

SG102.2.5 The Holy Tetraktys



$1 + 2 + 3 + 4 = 10$ or Decad.

For the Pythagoreans, the *Tetraktys* symbolizes the Perfection of Number: Unity starting at One, proceeding through 4 levels of manifestation, and reuniting with Unity ($1 + 0 = 1$)

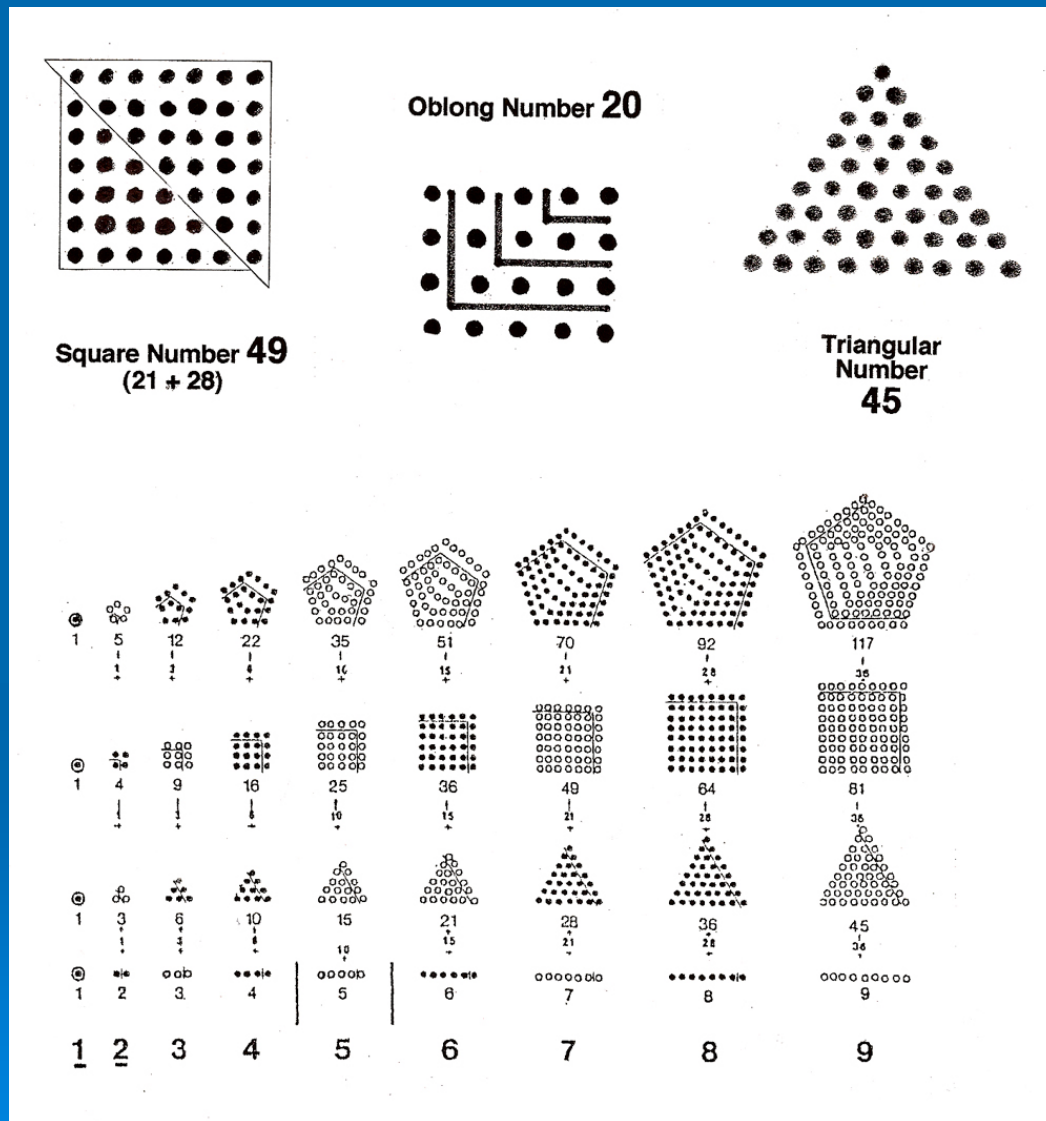
As **Geometry**, the *Tetraktys* represents the point (1), the line (2), the area (3) and the volume (4).

As **Music**, the *Tetraktys* contains the mathematical harmonic ratios of the musical scale: octave (1:2 or *Diapason*), perfect fifth (2:3 or *Diapente*), perfect fourth (3:4 or *Diatessaron*) and double octave (4:1).

As a **Triangle Number**, the *Tetraktys* shows the dynamic quality of triangular growth. It incorporates both the Odd (unlimitedness) and the Even (limitedness), whereas Square Numbers are exclusively composed of odd integers and Oblong Numbers of even integers. Since the universe is a sacred dance of Limited & Unlimited, the *Tetraktys* was called '*Kosmos*' (world order or adornment), '*Ouranos*' (Heaven), '*Pan*' (the All) and '*Pure Harmony*'.

As a **Cosmogram**, the *Tetraktys* came to be an inclusive paradigm and diagram of the universal 4-level pattern of cosmic manifestation: 4 elements, 4 dimensions...

SG102.2.6.1 The Shape of Numbers (1)



For the Pythagorean School, Numbers are live entities with personalities, family identities and an intimate world of relationships, likes & dislikes.

Triangular Numbers can be portrayed as a triangle of evenly placed dots or pebbles. Each Triangular Number is half an Oblong Number.

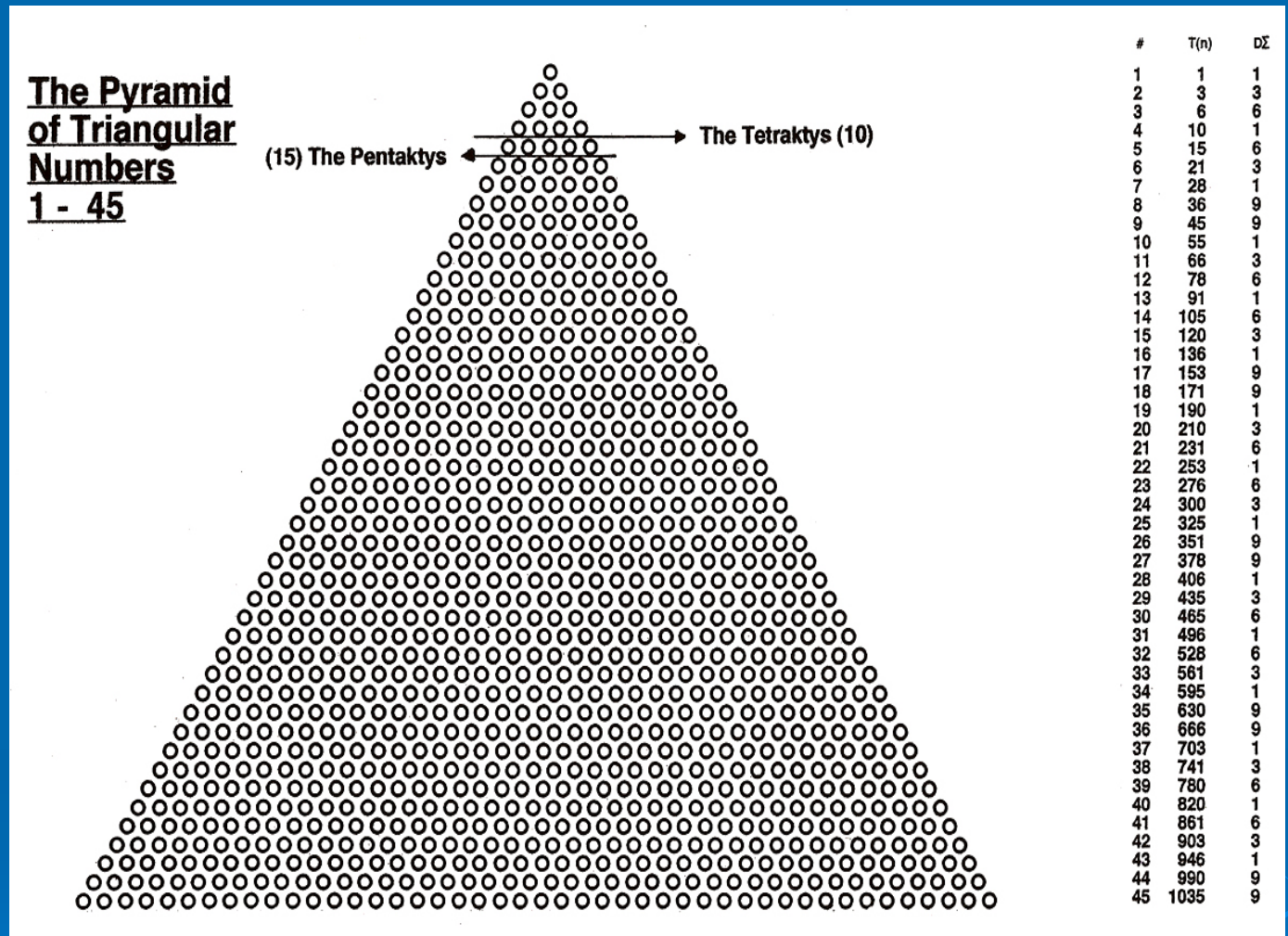
Every **Square Number** is the sum of two successive Triangular Numbers. Thus 49 is made up of 21 (6th Triangular) and 28 (7th Triangular).

Oblong Numbers are formed by the addition of successive even natural numbers.

See [\[◆SG202\]](#): Archetypal Numbers.

SG102.2.6.2 The Shape of Numbers (2)

→
The
Pyramid
of
Triangular
Numbers
From 1 to 45

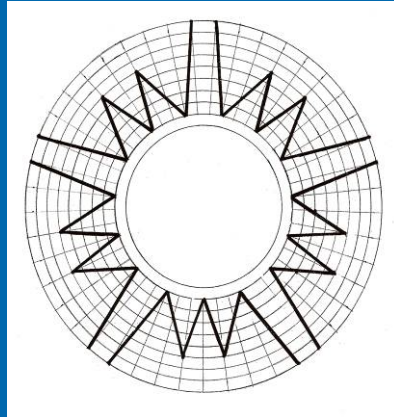
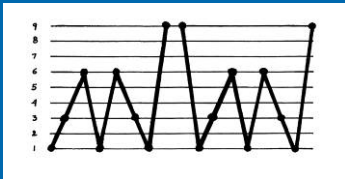


Notes: the top 10 numbers are the *Tetraktys*. The top 15 numbers are the *Pentaktys*. The formula to find any Triangular Number is: $[n(n+1) / 2]$. Example: Triangular Number 9 is $T9 = (9 \times 10) / 2 = 45$

Notice the pulse-9 frequency of the Digit Sum: 1-3-6-1-6-3-1-9-9

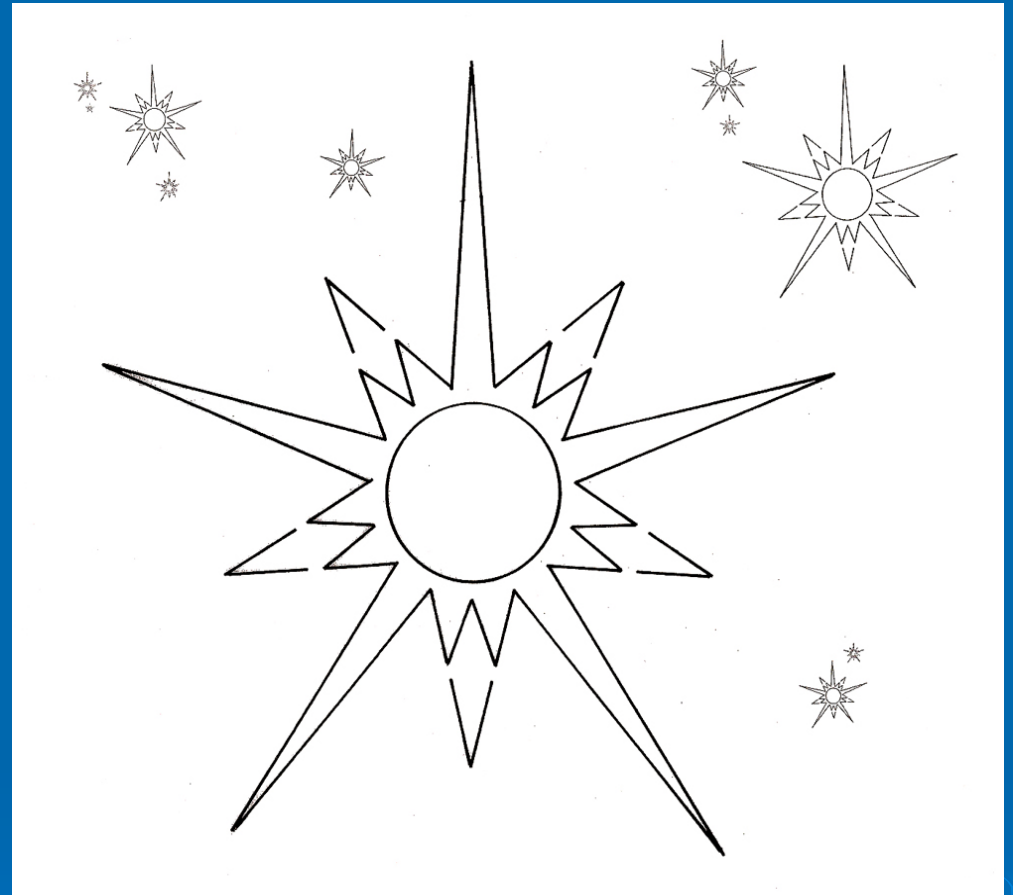
SG102.2.6.3 The Shape of Numbers (3)

The Pulse- Mandala of Triangular Numbers



If you can imagine the numbers 1 - 9 to be musical notes on a 9-lines staff, two Digit Sum pulses [2(1-3-6-1-6-3-1-9-9)] of the Triangular Numbers look like the wave pattern above-left. If mandalized for the 5 pulses of all 45 numbers, we get the above-right *Pulse-Mandala*.

If we fully extend the rays, we get the Star Pattern on the right.

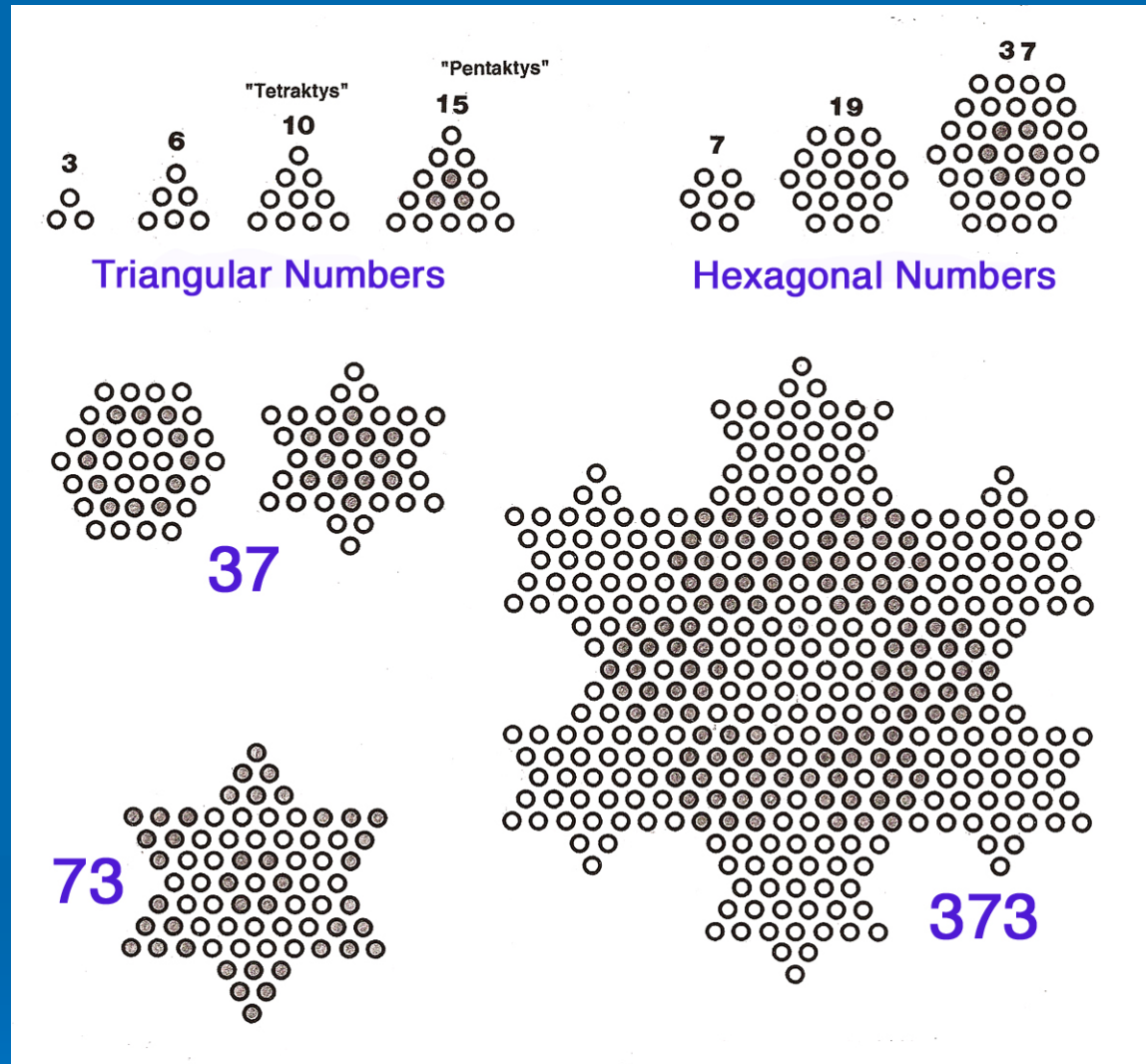


The Pulse Mandala of Triangular Numbers embodies the Pythagorean concept of numbers as belonging to families of shapes with distinct relationships. We can thus glimpse the aura or energy-field of a specific family: the Triangular Numbers. Try the Pentagonal Numbers...

The Pythagorean description of the Music of the Spheres is a direct synaesthetic experience of the complex music of wave/frequency inter-relationships, the cosmic orchestra...

... and one dances in harmony with the universe...

SG102.2.6.4 The Shape of Numbers (4)



Star Numbers

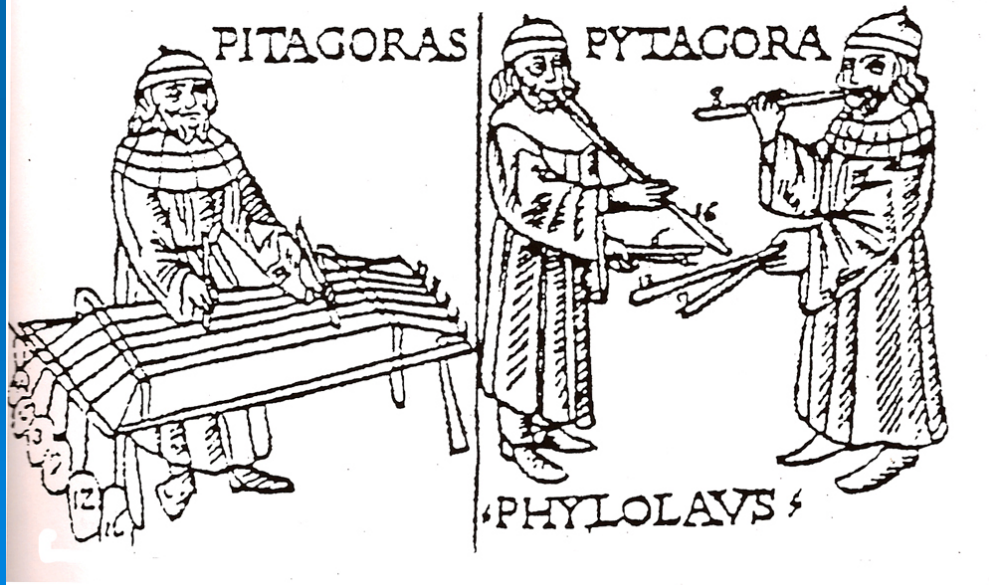
Playing with figured numbers can be lots of fun, for the kids in all of us. You can use colored dots, coins, marks on the sand, pebbles, tokens, golf balls... New relationships between numbers pop up suddenly and the Number Field jumps ALIVE.

For instance, combining Triangular Numbers and Hexagonal Numbers creates Star Numbers. Below are some of the steps leading to the beautiful Star Number **373**. (After Bonnie Gaunt who calls it the 'Logos Snowflake' - Genesis One , 2003)

- ➔ **37** is a go-between number. It belongs to two families: Hexagonal & star.
- ➔ StarNumber **73** contains 6 Triangular Numbers 6 around the Hexagonal Number 37
- ➔ Star Number **373** contains 6 Star Numbers 37, 6 Hexagonal Numbers 19, around the core of a seventh Hexagonal Number 37.



Pythagoras experimenting with Musical Ratios

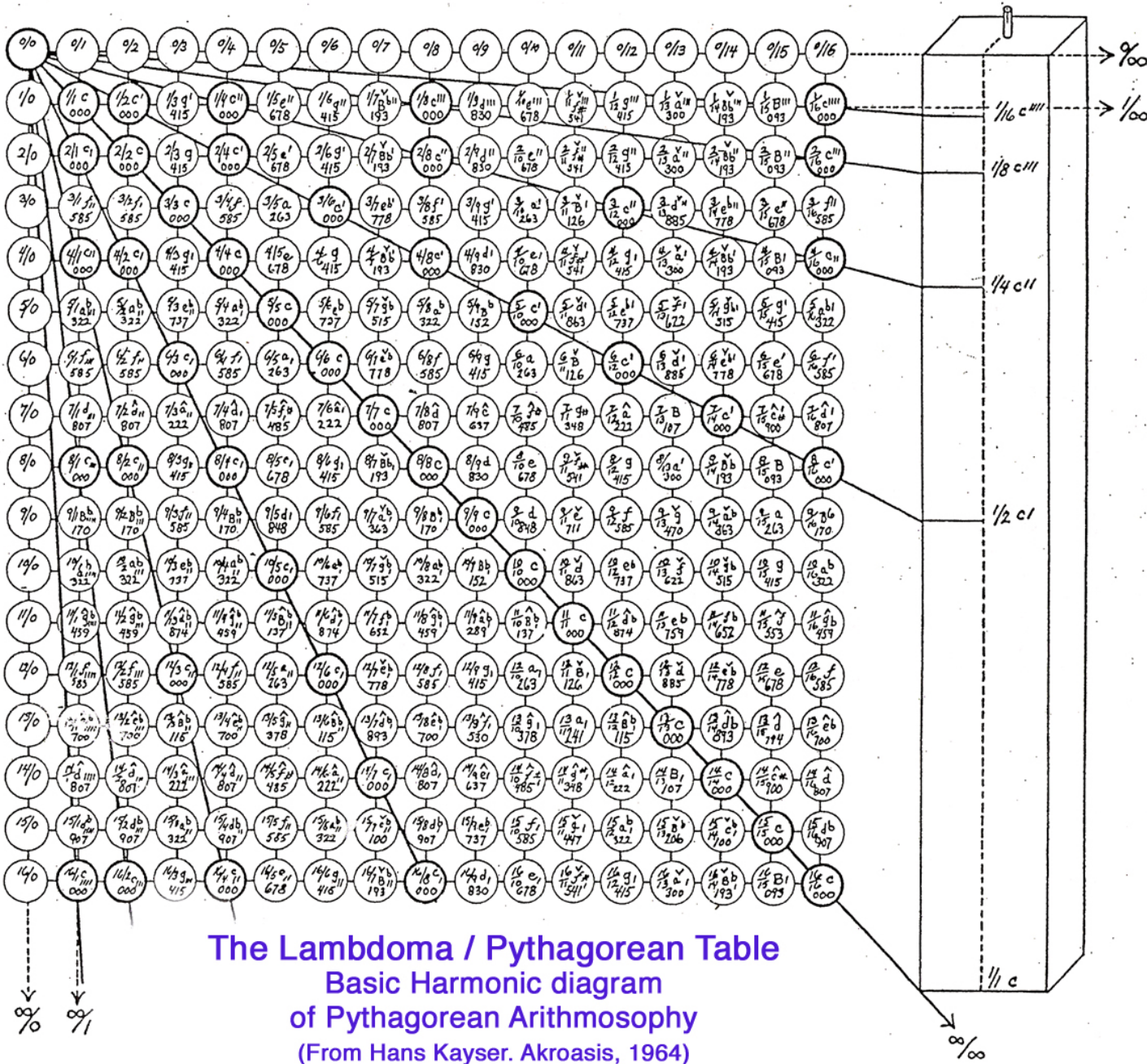


SG102.2.7 The Father of Musical Harmony

Pythagoras is credited with establishing and teaching the relationship between number ratios and sound frequencies. He is shown here experimenting with bells & water glasses (top right), stretched strings (bottom left) and various sized pipes (bottom right).

Jubal is a Biblical character said to be the 'ancestor of all who played the lyre and pipe'. He is portrayed here overseeing an experiment with the sounds of hammers of various weights.

← Pythagoras. *Medieval woodcut.*



The Lambdoma / Pythagorean Table
 Basic Harmonic diagram
 of Pythagorean Arithmosophy
 (From Hans Kayser. Akroasis, 1964)

SG102.2.8.2 The Lambdoma & Pythagorean Table (2)

Note: on the right stands a monochord with 5 octaves (C to C''') obtained by halving the string length.

See ◆ SG201

SG102.2.9 The Comma of Pythagoras

In music, octaves (1:2) and fifths (2:3) do not match: they are mathematically incommensurable. These two ratios are antagonistic, like the square and the pentagon or the $\sqrt{2}$ geometries and the Phi geometries. It takes ascending through 12 fifths to reach about the same note as when ascending the octaves. And, even then, there is a slight discrepancy: 12 consecutive fifths reach slightly further than 7 octaves. The note heard at 7 octaves differs from the note heard at 12 fifths by a small amount called the '*Comma of Pythagoras*'.

Comma of Pythagoras = 1.0136

Mathematically speaking, the ratio 2:1 (Octave) doubled 7 times = 128 or 2^7 . In other words, a note 7 octaves higher than the original note has a frequency 128 times greater. On the other hand, the Fifth has the ratio 3:2 = 1.5 and, when expanded 12 times, we have $1.5^{12} = 129.75$. So there is a difference of $129.75 / 128 = 1.0136$. This is the Comma of Pythagoras. It is also worth 24 cents [8424 (12 fifths) - 8400 (7 octaves)]

Robert Temple, in his book *The Crystal Sun*, a masterful exposure of optical technologies in the ancient world, makes very interesting comments about the Comma and explains that it was known, with great exactitude, to the Pythagorean School.

"A value of the Comma computable to an astonishing 9 decimal places appear in the form of an arithmetical fraction preserved in the ancient Greek Pythagorean treatise Katatome Kanonos (Division of the Canon). There we are told that the number 531,441 is greater than twice 262,144. Twice 262,144 = 524,288... If we carry out the division, we obtain the number 1,013643265, namely, the Comma of Pythagoras expressed to 9 decimal places."

(New edition of the *Sirius Mystery* by Robert Temple)

Note: the identical number to 9 decimal places is given in '*Math and Music*', published in 1995!!!!

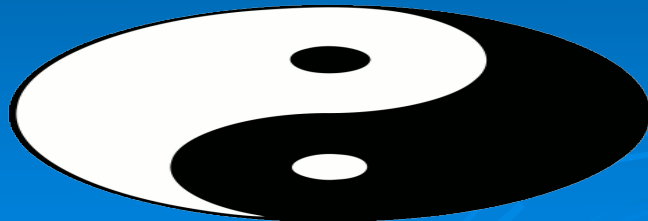
SG102.2.10 Cosmic Harmonia

The Pythagoreans were questing for the Larger Whole that could reconcile the antagonistic parts (the octave & the fifth) and account for the Comma of Pythagoras within the intuited larger multi-dimensional context of 'Cosmic Harmonics'. It was a quest similar to the scientific search for the Grand Unified Theory.

The Pythagoreans called this primordial principle Harmonia, after *Harmonia*, the daughter born of the Goddess of Love *Aphrodite* (Venus) and the God of War *Ares* (Mars).

A solution was found by the Chinese, in the 16th century: they invented the system of 'equal temperament'. Adopted by Bach, this compromise is now universally accepted. Equal temperament solves the problem of the incompatibility between octave and fifth by cheating in a 'distributed' way and shaving a portion of each note to create 'semitones'. Therefore, on a modern piano, notes sound slightly 'flat' and contemporary music is now globally 'flat'. Of course this compounded by the fact that electronically synthesized sounds lack the extra depth of harmonic overtones. On the other hand, equal temperament has the advantage that music can be transposed from key to key without having to retune the instruments.

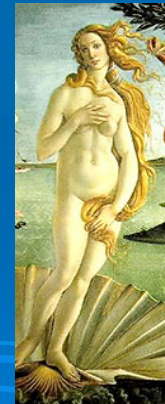
The perennial search is on again, at the cutting edge of human consciousness, for the principles & laws of Universal Harmonics, for the magical fruit of happy love between Venus & Mars, for the Tao uniting Yin & Yang.



Ares/Mars
God of War,
Father of Harmonia



Harmonia



Aphrodite/Venus
Mother of Harmonia

SG102.3 - Chapter 3. Classical Antiquity



Golden Rectangles in the Parthenon

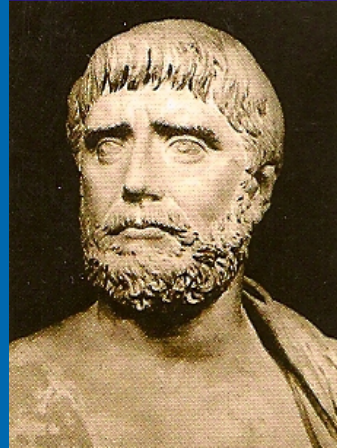
SG102.3.1. Early Greek Mysticism

Before Aristotle's empirical logic of linear cause & effect, the experience of complementary oneness between the spiritual & material worlds was essential to the early Greek mystical philosophers.

"All things are full of souls & spirits".
Heraclitus (6th century BCE).

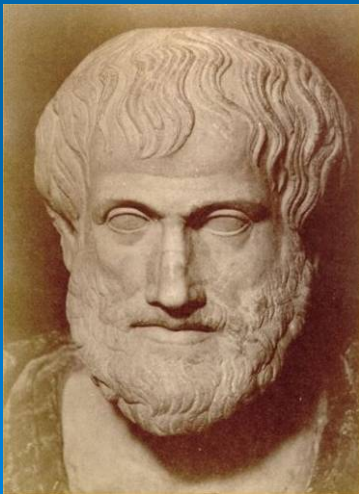
"All things are full of Gods".
Plato (5th century BCE)

***"Soul is intermingled
in the whole universe".***
Thales of Miletus



Thales of Miletus was one of the Seven Wise Men of Antiquity, a group of Greek Sages (*Sophoi*) from the 7th & 6th centuries BCE.

He used his knowledge of geometry (he is credited with the discovery of five geometry theorems), to measure the Egyptian pyramids.



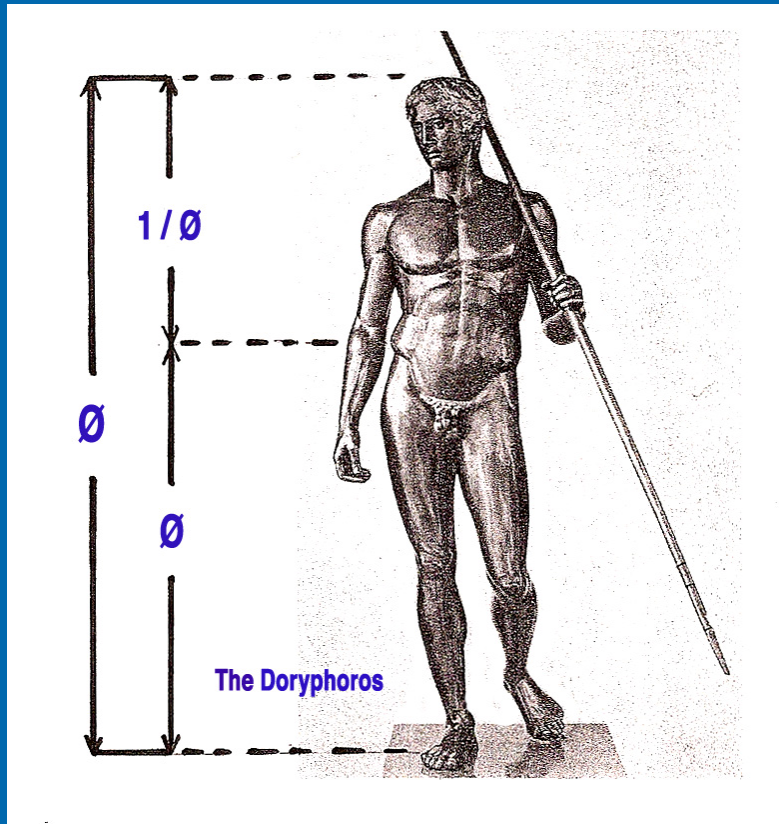
Heraclitus, according to the few fragments we have, taught that even though everything is constantly changing there is an underlying Logos (proportion) to it all.

***"Listening to the Logos rather than to me, it is wise to agree
that all things are in reality one thing and one thing only".***

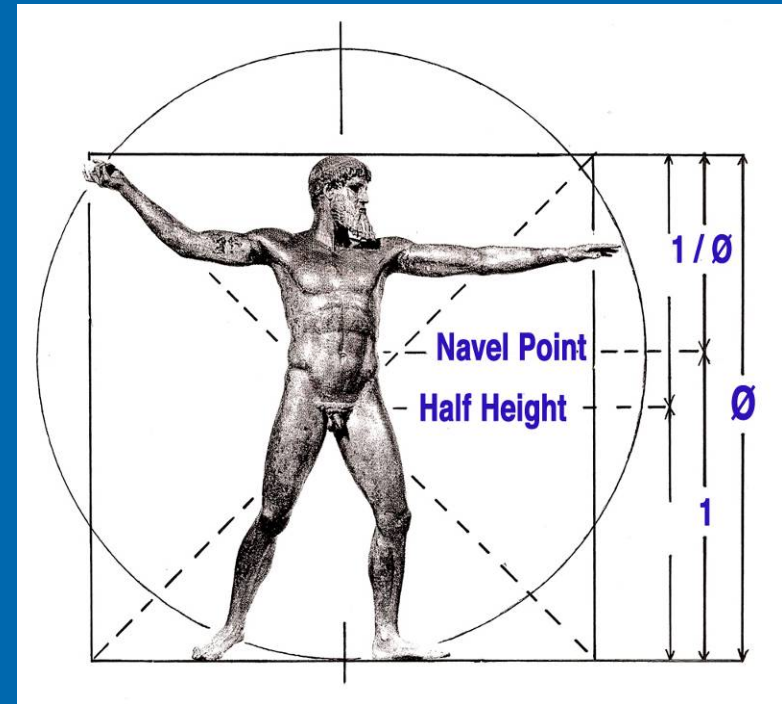
Proportion (Logos) was an expression of Harmonic Oneness and Symmetry (*analogia*) where differences were unified rather than polarized.

Heraclitus and the early Greeks understood proportion to be an experiential relationship where the components are integral parts of the whole, in a musical dance of perfection, thus anticipating what we now call the *Golden Ratio*.

SG102.3.2. Phi in the Greek Statuary



↑ The “*Doryphoros*” (Spear-Bearer) by Polykleitos, c. 450/440 BCE. Munich.



↑ Statue of Zeus, c. 460 BCE, exemplifying the Canon of human beauty: the navel marks the Golden Section.

“Polykleitos wrote a treatise to explain his meticulous system of proportions as exemplified in the Doryphoros, but the book is lost... Polykleitos called both the Doryphoros and the book his ‘Canon’ (measuring rod or standard)... The Greek system of human proportion, like that of Polykleitos, derive from their love of mathematical relationships.”

(F. Hartt. Art, A History of Painting, Sculpture, Architecture. 1985)

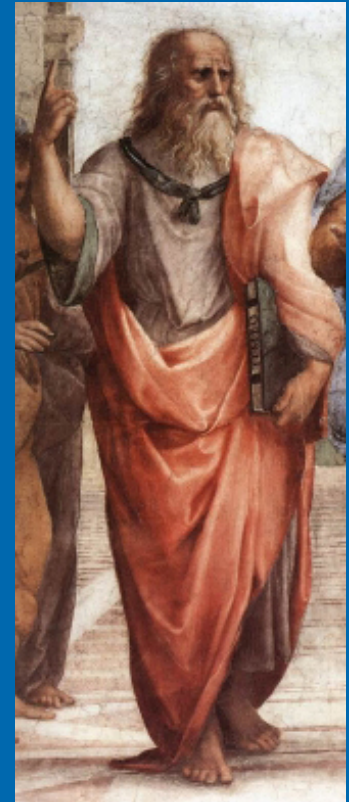
SG102.3.3.1 Plato: His Life

Plato (c. 427-347 BCE) was born to an aristocratic family in Athens. His real name was Aristocles. “*Plato*” (meaning *broad*) is a nickname referring to his physical appearance as a wrestler.

Plato traveled extensively. According to his biographer, Olympiodorus, Plato studied sacred science in Egypt with the priests of the Temple of Luxor (*an ancient world university, housing 80,000 students at its zenith*). He also went to Phoenicia to acquire the science of the Persian Magi. At home he was initiated into the Pythagorean system. In Sicily, where he was attempting to convince the ruler, Dionysios, to devote himself to the welfare of his subjects, Plato was sold off as a slave, only to be purchased back by a compassionate Greek compatriot.

Back home in Athens, Plato founded (387 BCE) the Academy whose frontispiece proclaimed the famous requirement:

“Nobody enters here unless he be a geometer”.

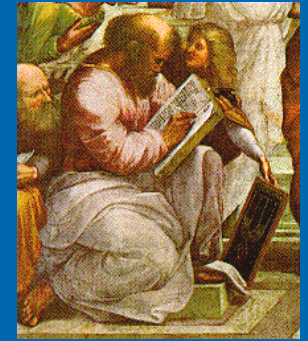


At the Academy, Plato taught a comprehensive curriculum, based on his Philosophy of Number. Of number and geometry, Plato says:

“No other branch of education has such a vast range of applications as mathematics; but its greatest advantage is that it wakes up the individual... making him quick to learn, sharpening his memory and wits and leading him beyond his normal capacity by divine art”.

“There is faculty in the mind of each one of us which these geometrical studies purify and rekindle after it has been ruined and blinded by other pursuits, though it is more worth preserving than any eye since it is the only organ by which we perceive the truth.”

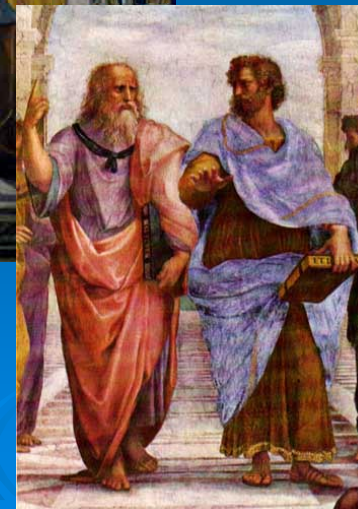
SG102.3.3.2 The School of Athens by Raphael



Pythagoras



Euclid



← Plato & Aristotle

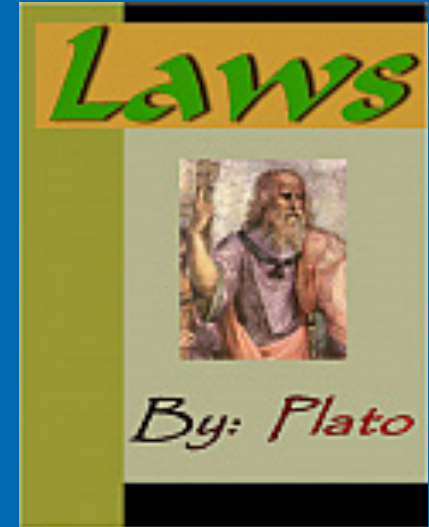
'Scuola di Atene' by Raffaello Sanzio (1483-1520) is a live window into the classical father figures of the western culture. The painting itself embodies several golden Rectangles and phi-related angles.

SG102.3.3.3 Plato: Philosophy of Number (1)

Behind the usual label of being a philosopher, Plato was actually an initiate. His job was to give hints & glimpses about the ancient Canon of Number & Proportion without divulging it openly, because it had been entrusted to him under an oath of secrecy.

In his *Laws*, Plato states that the Egyptian priesthood has in their possession a system of lawful proportions and harmonies that had sustained for millennia the high cultural standards of their civilization. He states that the sacred ratios of numbers were studied by all of those who became priests or practiced any of the arts influencing society, like music, architecture or politics.

In the *Timaeus*, Plato describes the geometric creation of the world and mentions the five regular solids (*now known as the 5 'Platonic' solids*) as the basis for the harmonic structure of the universe. Plato informs us of a certain proportion (*now called the Golden Ratio*) is the most binding of all mathematical relationships and the key to the physics of the cosmos.



Note: the Golden Ratio plays an essential role in the measurements of the Platonic Solids.

“In all his cosmological allegories, Plato uses the same set of numbers & geometrical diagrams, applying them to music as well as astronomy, and thus illustrating his belief that number is the ‘natural bond’ holding together the entire universe.

Plato’s source was the ancient Canon of Number which he either learnt wholly from certain teachers or partly reconstructed himself. His own concern was to renew the influence of the Canon and make it once more effective as an instrument for universal harmony”.

(John Michell. *Dimensions of Paradise*)

SG102.3.3.4 Plato: Philosophy of Number (2)

A perennial question of philosophy is the relationship of the One to the Many or the Whole to the Parts or, simply put, Spirit to Matter. Plato gives us hints via geometry and number.

In *Metaphysics* [987b 19-22], Plato speaks about the principles of the Great and the Small in their relationship to the One: “As matter, the Great and the Small were principles; as substance, the One; for from the Great and the Small, by participation in the One, come the Forms, the Numbers”.

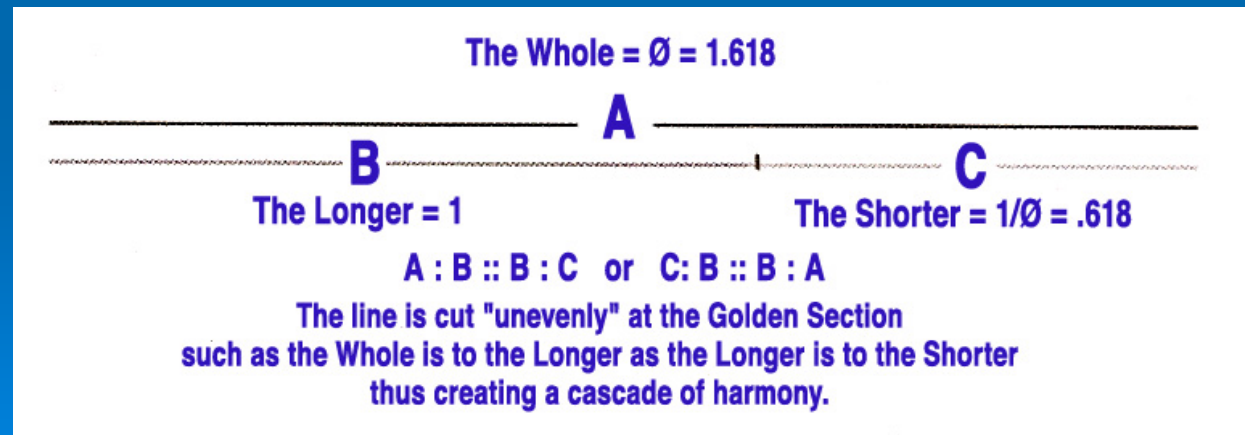
In the *Timaeus* [31b-32a], Plato explains that a continuous geometric proportion is the best of all cosmic bonds. Through continuous proportions, the World Soul binds harmoniously the intelligible world of principles above (including pure mathematics) and the sensible world of material objects below.

Note: A ratio is the relationship between two numbers. A proportion is the relationship between two ratios and can be discontinuous (involving 4 terms) or continuous (involving 3 terms) such as: $A : B :: B : C$ (A is to B as B is to C)

In the *Republic* [509d], Plato asks the reader to “take a line and divide it unevenly”.

From these various hints, we are subtly and insightfully led to the only way to form a proportion from a simple ratio, the unique “Golden Section” such as:

*the Whole to the Longer
= the Longer to the Shorter.*

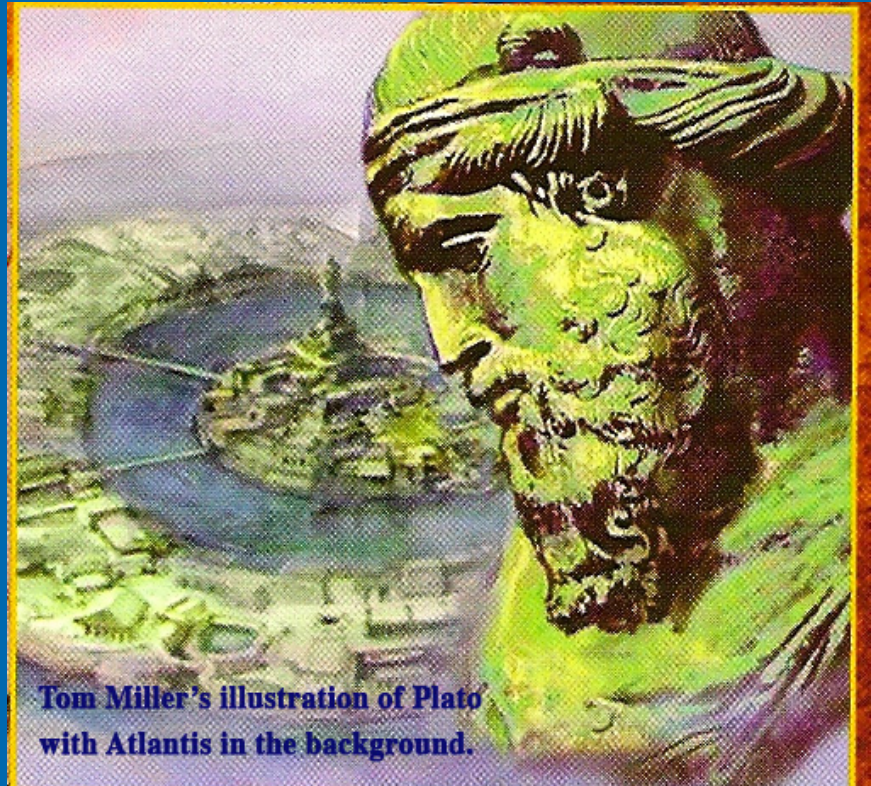


SG102.3.3.5 Plato's Cities

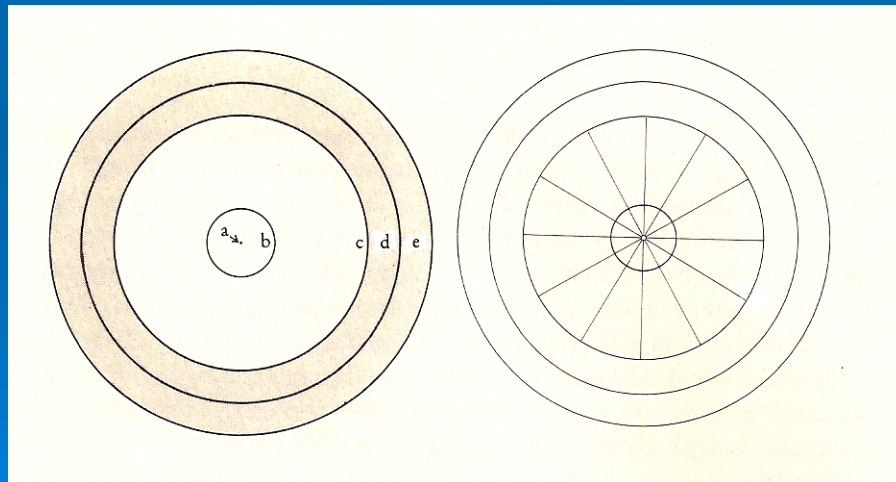
A main theme of Plato's teachings is the Ideal City, represented as a cosmological diagram. The descriptions of the two ideal cities of Magnesia (*Laws*) and Atlantis (*Critias*) are replete with geometrical allegories, mathematical clues and number ciphers pointing to a code of numbers behind the structure of the universe but never revealing it explicitly.

According to John Michell, Magnesia's plan is based on the number 12 (the cosmological Heavenly City) and exemplifies 5040 as a top canonical number.

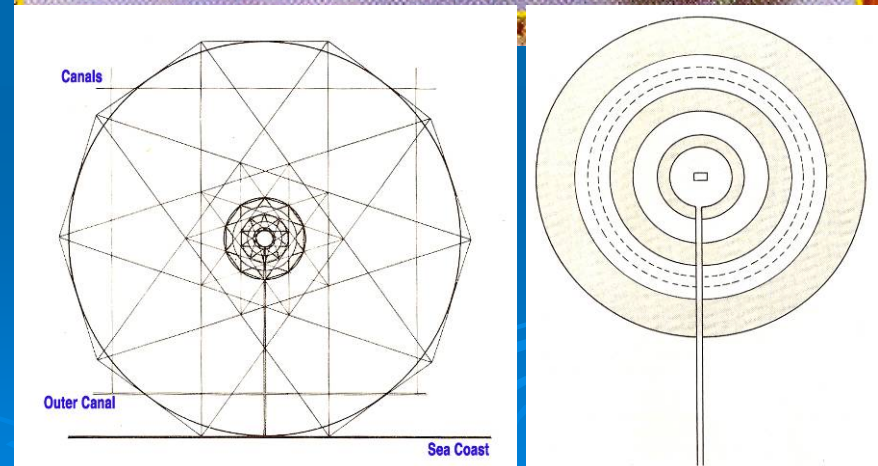
The layout of Atlantis is the 10-fold decagon, with less divisors but displaying the Golden Ratio.



Tom Miller's illustration of Plato with Atlantis in the background.

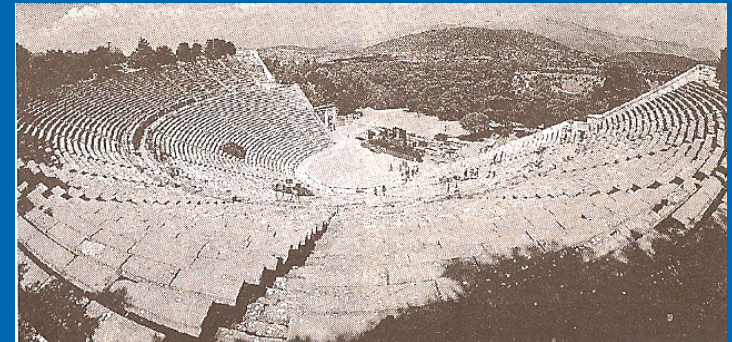
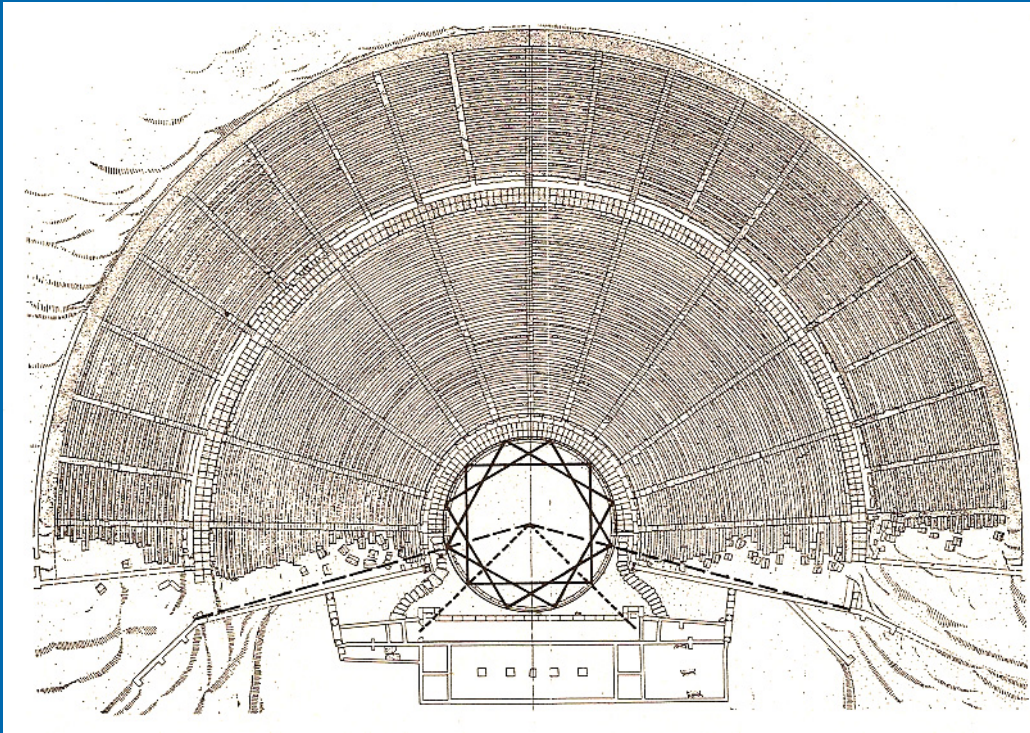


↑ 12-based layout of Magnesia



↑ 10-based layout of Atlantis

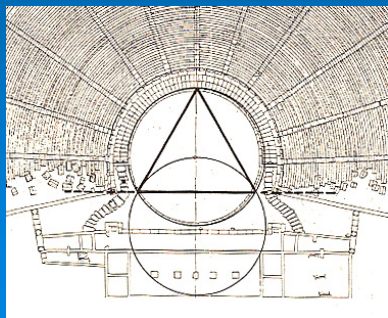
SG102.3.4 Greek Theater of Epidaurus



↔↕ 4-fold symmetry at Epidaurus (end of 4th century BCE).

The two levels of the *Epidaurus* theater relate proportionally according to the Golden Section as 1 : Phi. If the width of the higher level equals the side of a regular pentagon, then the width of the lower level is that same pentagon's diagonal, forming a Golden Triangle.

Also, the number of rows in each auditorium level is 21 and 34, respectively. Note the Fibonacci numbers!



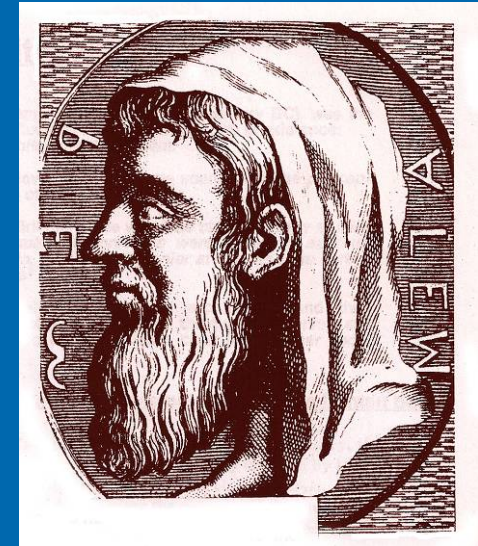
← 3-fold symmetry: the base of an equilateral triangle inscribing the orchestra circle marks the boundaries of the *theatron* at its uttermost edge. Note the Vesica Piscis!

(Diagrams & data from Rachel Fletcher. *The Power of Place*, 1991)

SG102.3.5.1 Euclid's Elements

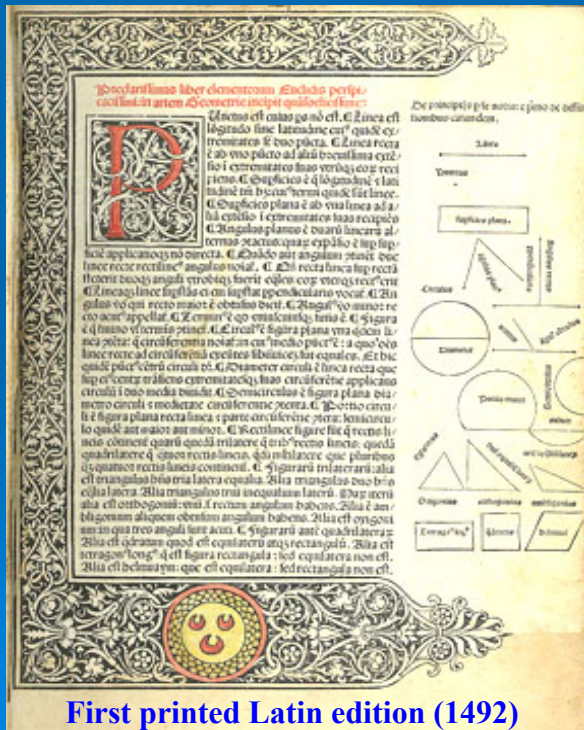
The '*Elements*' by Greek mathematician Euclid of Alexandria (325-265 BCE) are a vast synthesis of classical Greek knowledge of mathematics & geometry. The second most published book in history (Bible is first), the '*Elements*' have been vastly influential for over 2300 years and are still used to teach geometry.

Abelard de Bath (c. 1070-1145), a Benedictine monk traveling in Spain disguised as a Muslim, acquired an Arabic copy of the *Elements* and translated it into Latin. This was to be the basis for all successive European editions until the 1570 English translation by Billingsley, famously prefaced by Dr. John Dee.



Definition of the Golden Section: *“A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment so is the greater segment to the smaller”*

(*Elements*, Book VI, Def. 3)



First printed Latin edition (1492)

- I. Equilateral Triangle & Theorem of Pythagoras
- II. Geometric Algebra
- III. Properties of the Circle
- IV. Regular Polygons
- V. Ratios & Proportions

- VI. Geometry & Similitude
- VII, VIII, IX. Theory of Numbers
- X. Quadratics
- XI. Geometry in Space
- XII. Volumes of solids
- XIII. The Five Platonic Solids

↑ The 13 books of the *Elements*.
Note that Book 13 is about the 5 Platonic Volumes.

SG102.3.5.2 Euclid



We know little of Euclid's life, except that he taught in Alexandria.

“(Euclid) lived in the time of the first Ptolemy; for Archimedes, who followed closely upon the first Ptolemy makes mention of Euclid, and further they say that Ptolemy once asked Euclid if there were a shortened way to study geometry than the ‘Elements’, to which he replied that there was no royal road to geometry”.

Proclus (5th century CE)

Euclid was younger than Plato but in sympathy with this philosophy since he ended the *Elements* (Book XIII) with the construction of the 5 so-called Platonic Solids. Euclid wrote a dozen scientific books, of which only 4 survived, Among them *Optics* contains some early studies on perspective.

Stobaeus (Greek historian - 5th century CE) reports this anecdote: *‘Someone who had begun to learn geometry with Euclid, when he had learnt the first theorem, asked Euclid: ‘What shall I get by learning these things?’.* Euclid called his slave and said: *‘Give him three pence since he must gain out of what he learns’.*



SG102.3.6 The Circle Geometries of Archimedes

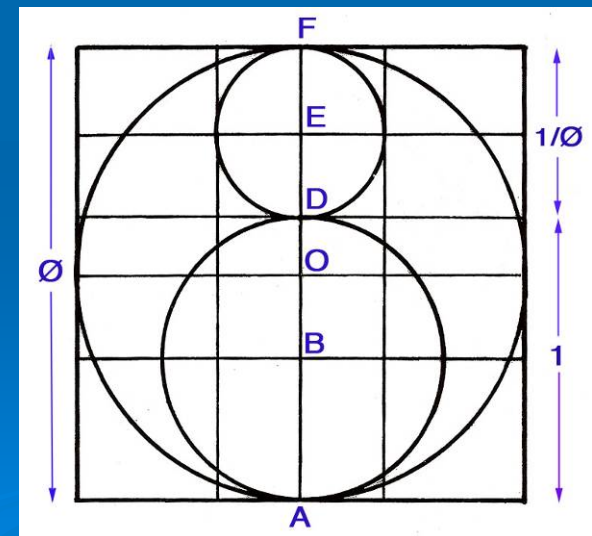
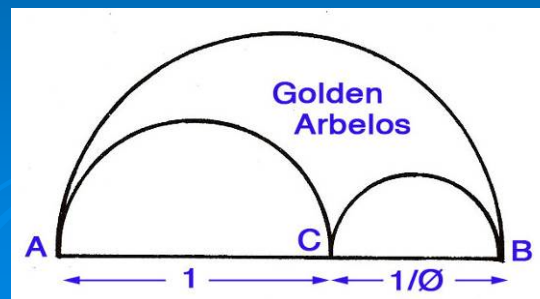
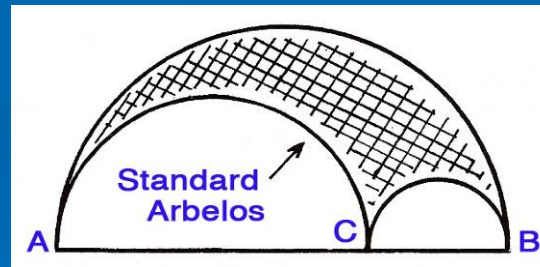
Archimedes (ca. 287-212 BCE) is considered one of the foremost mathematicians of all times and the greatest creative genius of the Mediterranean ancient world, renowned for his mechanical inventions. He was killed by a Roman soldier as the Roman troops finally entered the city of Syracuse (Sicily) they had besieged for 3 years.

Archimedes requested that his tomb be inscribed with a figure of a sphere and a cylinder to commemorate his discovery that the volume of a sphere is $\frac{2}{3}$ the volume of the circumscribed cylinder.

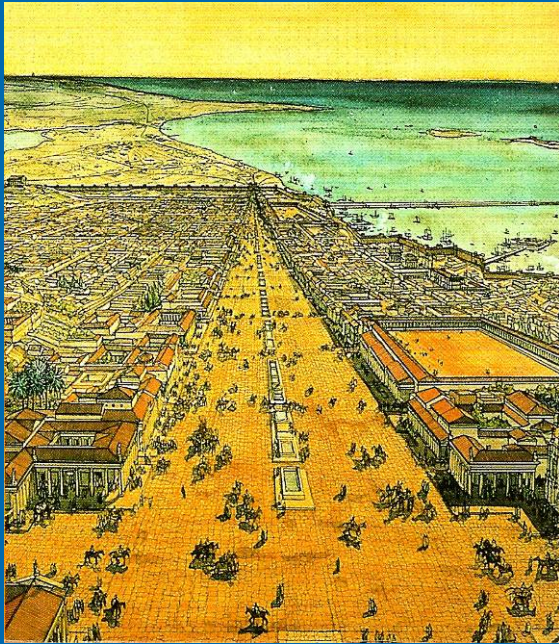
Although most of Archimedes' technological achievements are lost, but some of his books have survived.

In his *Book of Lemmas*, Archimedes studies the mathematical properties of certain geometric figures involving circles: the *Arbelos* and the *Salinon*.

The Arbelos →
(Greek *arbelos* = shoemaker's knife - because the figure resembles the blade of a cobbler's knife). Properly, the area between the 3 circles AC, AB and CB. [◆SG104.4]

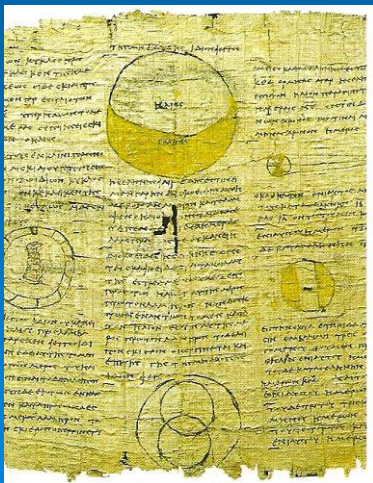


SG102.3.7 Hellenistic Alexandria



Ancient Alexandria.

The *Canopic Way* ran from the eastern Gate of the Sun to the Gate of the Moon. It was lined with arcades.



Papyrus fragment from
a treatise on astronomy
by Eudoxus of Cnidos

Alexander founded the city of his name in 331 BCE and died in 323. The famous Alexandria Library was conceived by Ptolemy I (Alexander's general who took over Egypt in 319 BCE) as a place to gather all the world's knowledge. The project was carried on by Ptolemy's successors.

The *Mouseion* was the Research Institute, next to the Library, where scholars from all over the world dedicated themselves to gathering & translating ancient manuscripts and to expanding all knowledge. The prime versions of the Greek classics were eagerly sought after and studied.

Thus *Alexandria* became the intellectual and cultural capital of the world. All ethnic groups, philosophies, faiths and beliefs were blending and enriching each other.

In this "Alexandria Renaissance" context, the first wave of Neo-Pythagoreans flourished: Posidonius, Eudorus of Alexandria, Philo of Alexandria, Alexander Polyhistor, Sextus Empiricus...

Some "Pseudo-Pythagorean" writings of this period reflect an initiatory knowledge which integrates the scientific, numerical approach with the metaphysical & mystic cosmologies. The universe was considered to be a vast system of harmonic, mathematical relationships whose resonances are tuned like music instruments. Light, color, sound & numbers are aspects of the One, the Universal Mind diffused throughout the world.

Theon of Smyrna (now *Izmir*, Turkey) was a Greek philosopher (c. 70 - c. 135) who wrote several mathematical commentaries on the Pythagorean & Platonic schools. The only book that survived is *Mathematics Useful to Understand Plato*. In his treatise, Theon describes the properties of the 10 numbers (Monad to Decad), the 4 Divine numbers (1 to 4), the Pentad, and the relationship Ennead and the Decad.

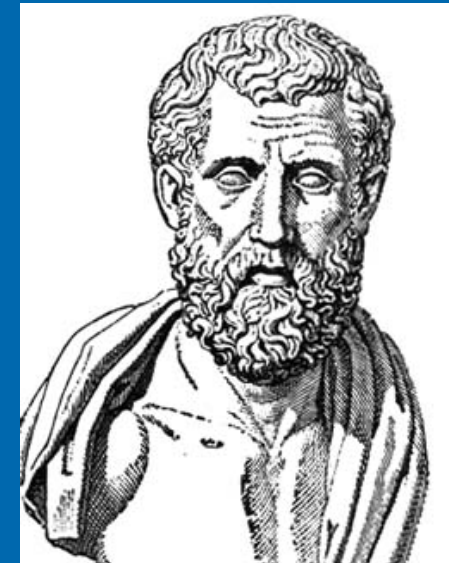
Theon refers to his section on music as “*a dissertation on the harmony of the world*”. For the Pythagoreans, Divinity is numbers and harmony. The symphony of the world, says Theon, “*has the greatest power, being truth in reason, felicity in life and harmony in nature; and this harmony, which is diffused throughout the world, will not be found unless it is revealed first through numbers.*”

In the section on astronomy, Theon explains that the Earth is a sphere and that the planets, Sun, Moon, and the sphere of fixed stars are all set at intervals congruent with an octave.



← Claudius Ptolemy (87-150), the celebrated astronomer and geographer, is known to have used several of Theon’s observations in his work.

SG102.3.8.1 Neo-Pythagoreans (1) Theon of Smyrna



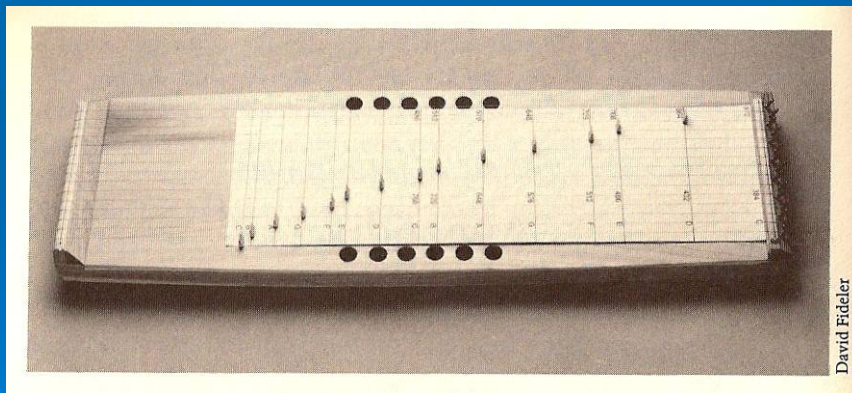
Note: the modern Egyptologist & mathematician R. Schwaller de Lubiez found in Theon’s work the clues that enabled him to uncover the ancient Egypt system of symbolic wisdom. Thus, remembering that Theon derived his views from Pythagoras & Plato and that Pythagoras was initiated in Egypt, we have here a completed loop of history.

SG102.3.8.2 Neo-Pythagoreans (2) Nicomachus

Nicomachus was a Syrian Greek who, in his time, was to arithmetic what Euclid was to Geometry. Born in Gerasa (now *Jerash*, Jordan) c. 60 CE, Nicomachus brings together Pythagorean mathematics, philosophy and harmonics in a language that everyone can understand.

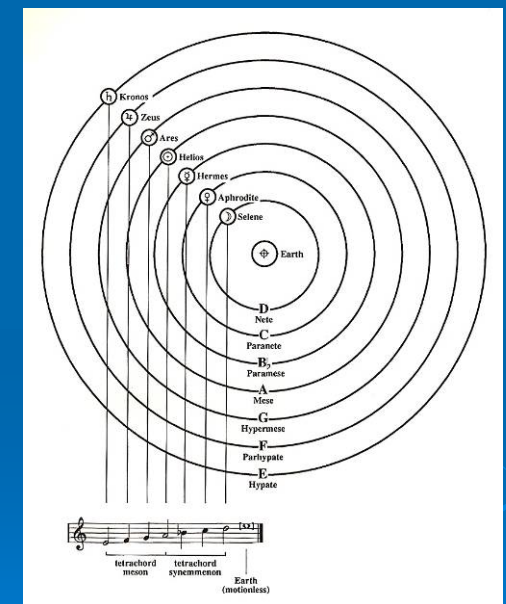
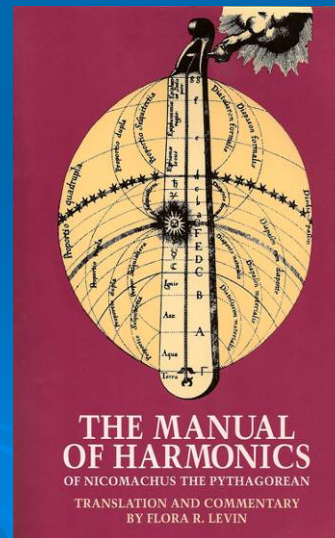
In his *Manual of Harmonics*, Nicomachus, expounding on Pythagorean teachings, describes the musical scale as the model of the harmonic order behind the structure of the cosmos and human existence.

Through proportion and harmony,
the musical scale bridges the gap between two extremes.
By incorporating the cosmic patterns of harmonic symmetry,
the musical scale demonstrates how the phenomena of nature
are interconnected through reciprocity
and how all parts of the whole are harmoniously related.



David Fideler

↑ 15 strings *Polychord* tuned to the regular diatonic scale described in Plato's *Timaeus*, spanning two octaves.



↑ The '*Music of the Spheres*', according to the *Manual*

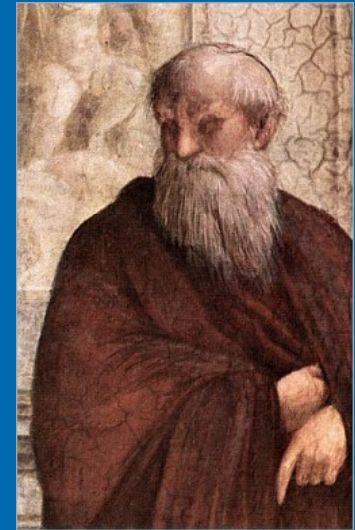
1
 1 2
 1 2 3
 1 2 3 4
 1 2 3 4 5
 1 2 3 4 5 6
 1 2 3 4 5 6 7
 1 2 3 4 5 6 7 8
 1 2 3 4 5 6 7 8 9

The Triangular
 Ennead

SG102.3.9 Neo-Platonicians

In the 3rd century CE, the Neo-Platonists re-introduced into mathematics the mystical & contemplative dimensions that had been cut out by Euclid in his *Elements*.

Plotinus (204-270 CE), one of the most influential philosophers in antiquity after Plato, constructed in his *Enneads* a great synthesis absorbing the Neo-Pythagorean tradition into Neo-Platonism.

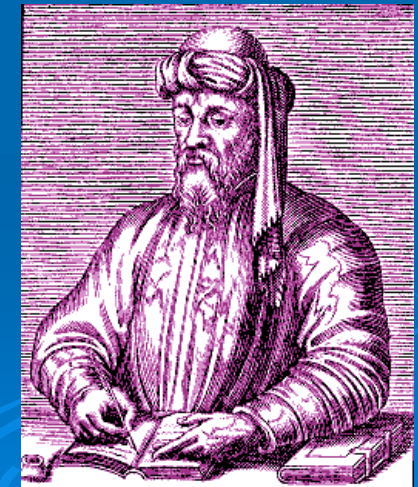


Plotinus
 (The School of Athens
 by Raphael)

Following Plato’s quote “*Nine concerns the Absolute*”, Plotinus describes the Ennead of 9 numbers as the numerical codification of the Cosmos. Elaborate systems of Enneadic tables & grids, using natural as well as “shaped” and “sacred” numbers, were used to read correspondences between the Divine worlds and the worlds of space-time.

Porphyry (meaning “*purple*”) of Tyre (233-309 CE), the disciple and editor of Plotinus, wrote a classic *Life of Pythagoras* couched in rationalistic terms. Porphyry held that the ancient Indians, Egypt, Hebrews and other peoples had possessed the same original wisdom that was represented in Greece by Pythagoras.

Iamblichus (c. 250-325 CE), the pupil & rival of Porphyry, also wrote an extensive *Life of Pythagoras*, which was an introduction to his Encyclopedia of Pythagorean School (mostly lost). Pythagoras is represented as the paragon of the Divine Sage.



Porphyry

SG102.3.10.1 Vitruvius (1)

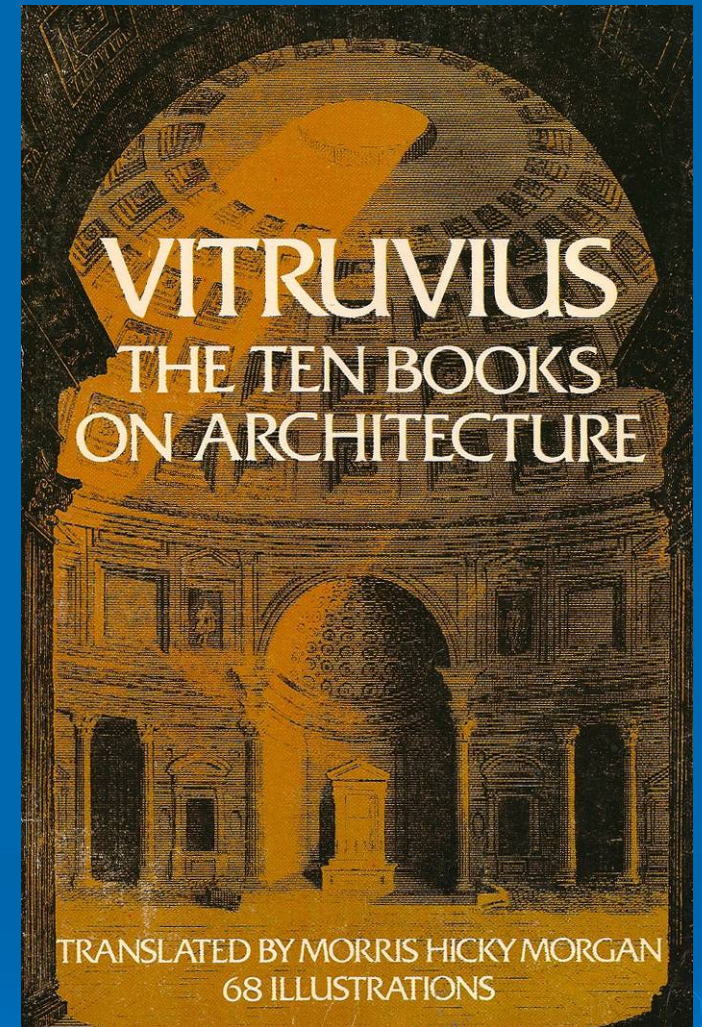
Marcus Vitruvius Pollio, a contemporary of Cicero (1st century BCE), is the author of the oldest and most influential work on architecture: *De Architectura Libri Decem*.

In this book, Vitruvius recorded the Golden Canon of human proportions known to earlier civilizations, like Classical Greece. This Canon came to be known as the “*Vitruvian Canon*”, later to be illustrated by Da Vinci & Dürer.

Vitruvius’ fame soared with the *princeps* edition of 1486 and the invention of printing. According to the Encyclopedia Britannica, he was “*the chief authority studied by architects, and in every point his precepts were accepted as final. Bramante, Michelangelo, Palladio, Vignola, and earlier architects were careful students of Vitruvius.*”

While extensively describing all the basics of architecture and their relationship with the proportions of the human body, the musical intervals and irrational numbers, Vitruvius also digresses in a charming manner about anecdotes of his times.

Vitruvius on Symmetry: “*The design of a temple depends on symmetry, the principles of which must be most carefully observed by the architect. They are due to proportion, in Greek analogia. Proportion is a correspondence among the measures of the members of an entire work, and of the whole to a certain part selected as standard.*”

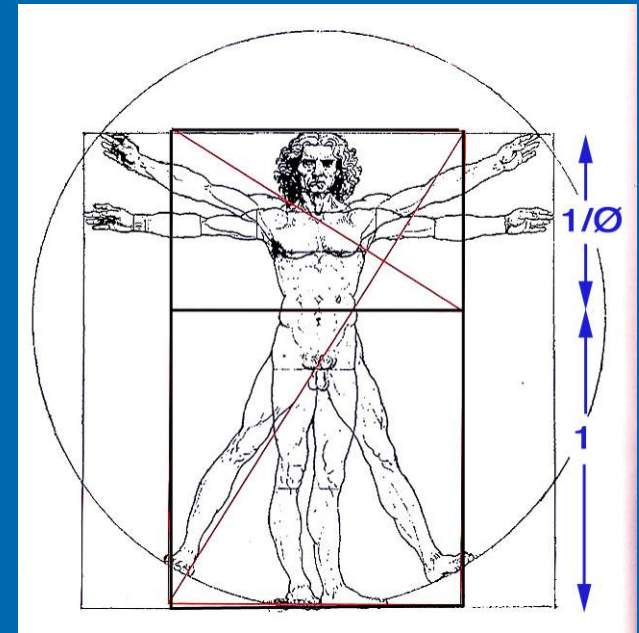


SG102.3.10.2 Vitruvius (2)

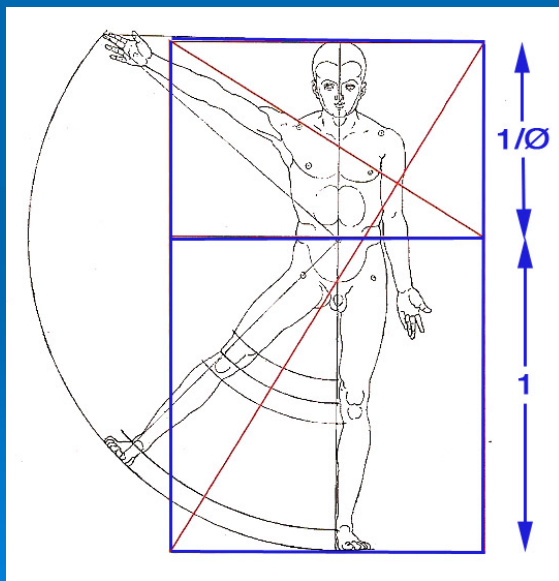
Vitruvius on the Human Body. “... *In the human body the central point is naturally the navel. For if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centred at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle therefrom.*

And just as the human body yields a circular outline, so too a square figure may be found from it. For if we measure the distance from the soles of the feet to the top of the head, and then apply that measure to the outstretched arms, the breadth will be found to be the same as the height...”

(Ten Books on Architecture. Dover, 1960. Book III, Chap.1)



↑ ‘Human Figure in a Circle,
Illustrating Proportions’
Leonardo Da Vinci, 1485-1490



↑ ‘Man inscribed in a Circle’
Albrecht Dürer, after 1521

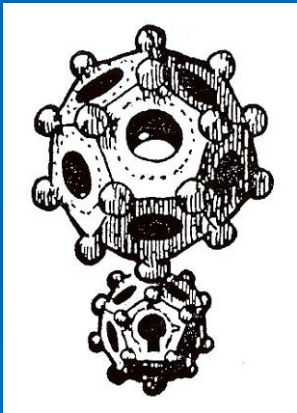
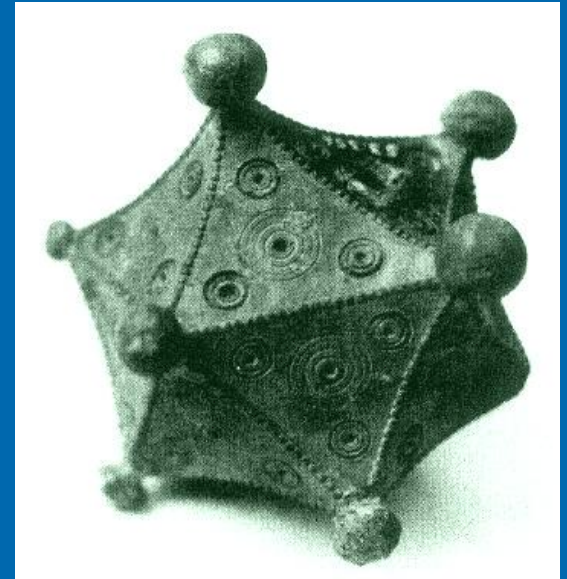
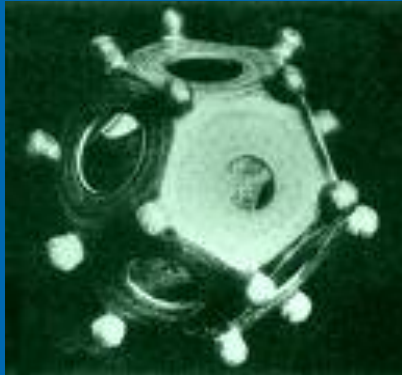
“Nature has designed the human body so that its members are duly proportioned to the frame as a whole...”

Vitruvius

In the *Vitruvian Canon*, the human form is divided in half at the sex center, and by the Golden Ratio at the navel. This proportioning system was used in Classical Greece and was found again in the Renaissance by artists such as Da Vinci & Dürer, under the tutelage of Luca Pacioli, the author of ‘*De Divina Proportione*’ (1509).

Both Da Vinci’s and Dürer’s drawings clearly apply the canon of proportions described by Vitruvius. [◆SG204]

SG102.3.11 Roman Dodecahedra



Gold and bronze
“balls” found
in Vietnam.
Recognized in Asia
as sacred Taoists tools
used in acupuncture.

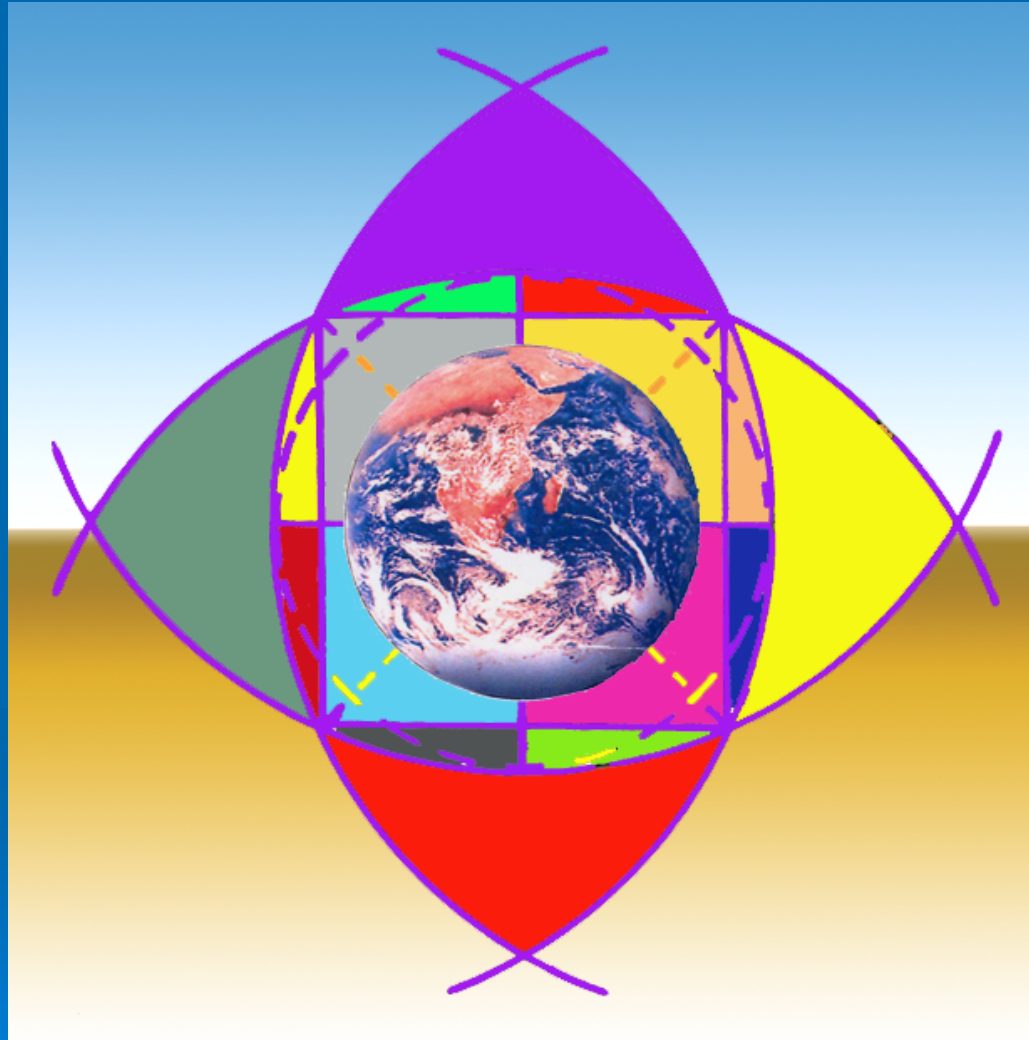
Over 100 bronze dodecahedra, dating from the Roman times (2nd - 4th century CE), have been found in various parts of Europe from England to Hungary to Italy, with most found in Germany & France.

They are usually hollow spheroids, whose vertices are marked by knobs & faces by circular holes (sometimes accented by concentric circles). Holes have different sizes. The spheroids have various sizes, from 4 cm to 11 cm.

An icosahedron (top right) has also been reported.

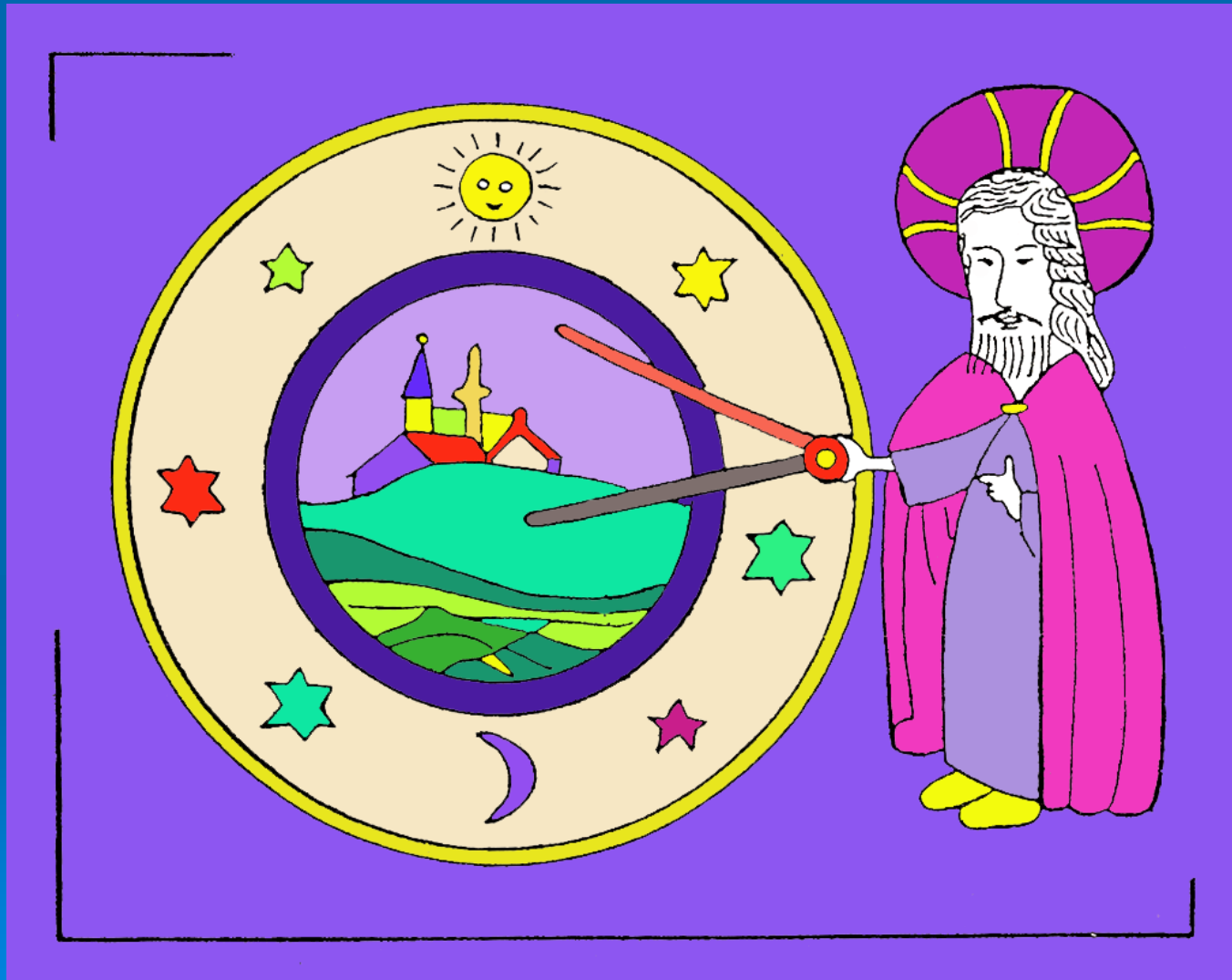
The function or use of the dodecahedra remains a mystery. No mention of them has been found in contemporary accounts or pictures of the time.

Sacred Geometry Worldwide

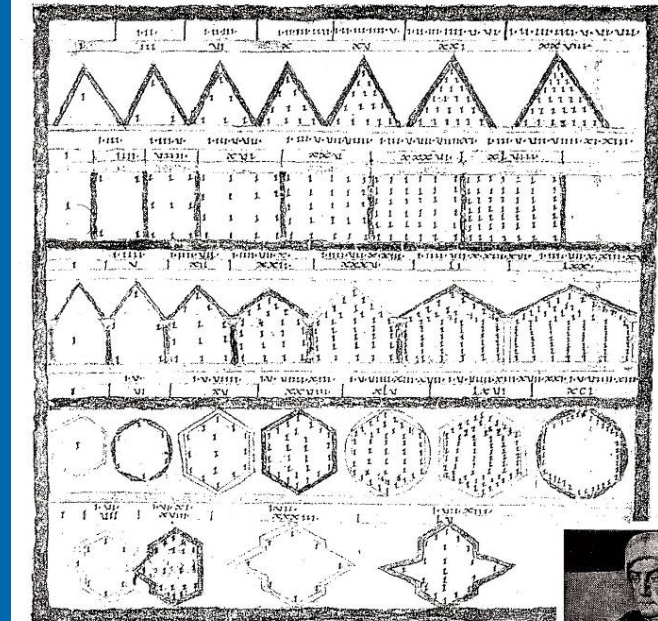


The study of the history and traditions of Sacred Geometry *worldwide* will be the theme of post-graduate presentations, seminars and research projects.

SG102 - Chapter 4 The Middle Ages in the West



SG102.4.1 Boethius



Page from *Arithmetica* by Boethius
showing 5 families of
Figurative Numbers

Right: Portrait of Boethius
by Just of Gand (15th c.)



↑ Boethius discusses Music Theory with students. c. 1405. In the 500s, Boethius wrote a treatise entitled ‘*Concerning Music*’. It was so authoritative that it continued to be used in universities for 1000 years. Boethius points to a geometric diagram of musical ratios that seemingly floats up in the air (i.e. a visualization).

Anicius Manlius Torquatus Severinus Boethius was a translator of the Greek Classics and a prolific writer & teacher. He is considered to be the last of the Roman scholars and the first of the Medieval scholastics. He attempted to reconcile ancient Greek philosophy and the Christian doctrines. Put to death by Theodoric, the emperor of the western empire, for his political & religious beliefs, he was later canonized as St Severinus.

Boethius explained that the spirit soul and human body obey the same sacred principles of proportion that can be found in the cosmos and in music. We are happiest when we flow with these principles as they create a harmonic resonance within our beings.

SG102.4.2.1 Islamic Golden Age

The Islamic Golden Age, also known as the *Islamic Renaissance*, extended from the 8th century to the 15th century. During this period, artists, philosophers, mystics, engineers & poets not only preserved the texts and traditions of antiquity but added unique and powerful contributions.

An influential Quran’s hadith states: “*The ink of the scholar is more holy than the blood of the martyr*”, thus elevating knowledge above politics and religion.

Through this Golden Age, the Muslim world became the unrivaled intellectual center for science, philosophy, education, architecture and the arts.

In Baghdad, the seat of the *Abbasid Caliphate*, caliph Al Mamoun established a grand project called the “*House of Wisdom*”, to be the first large scale library and university since the burning of the Library at Alexandria. There, brilliant scholars like Al Kindi, Al Khwarizmi (c.783-850), the Banu Musa Brothers and many other luminaries worked to translate into Arabic the classical texts from antiquity and to lay the foundations of many new branches of knowledge, which in turn led to the revival of learning in Western Europe.

Mathematics and Geometry, as applied to Islamic *aniconic* (without image) art & architecture, were specially flourishing.

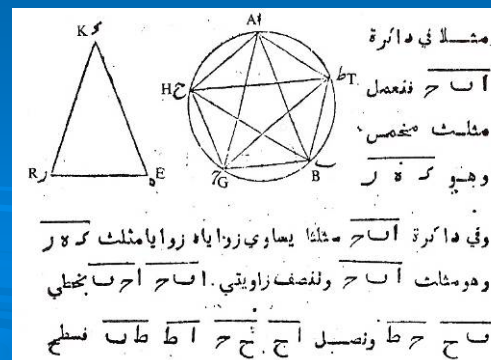
Mid-13th century use of the Golden Triangle by Nasir ad-Din at-Tusi →



Al Khwarizmi, the founder of Algebra



↑ Al Kindi, the father of Islamic philosophy and pioneer scientist

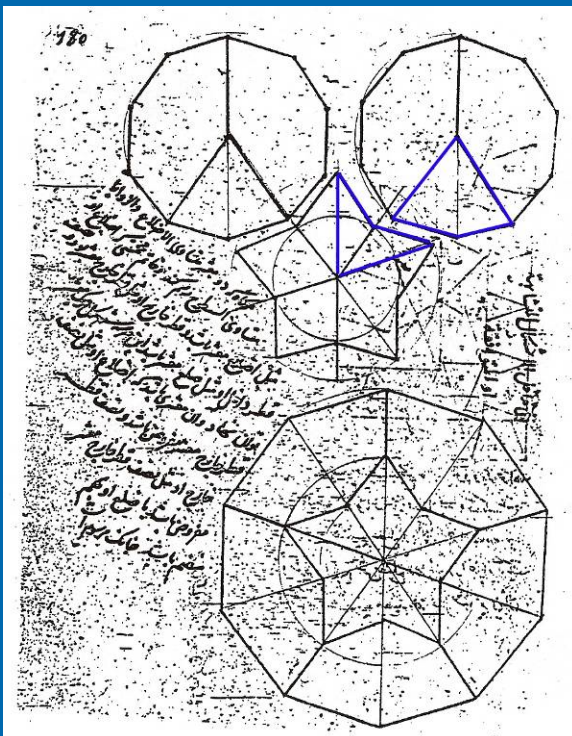


SG102.4.2 Islamic Pentagonal Seal

In the 70s, in the *Bibliothèque Nationale* in Paris, an anonymous manuscript was found inserted in the Persian translation of a treatise by Abu'l-Wafa'al Buzjani *The Book of What Artisans Need from Geometry Problems*.

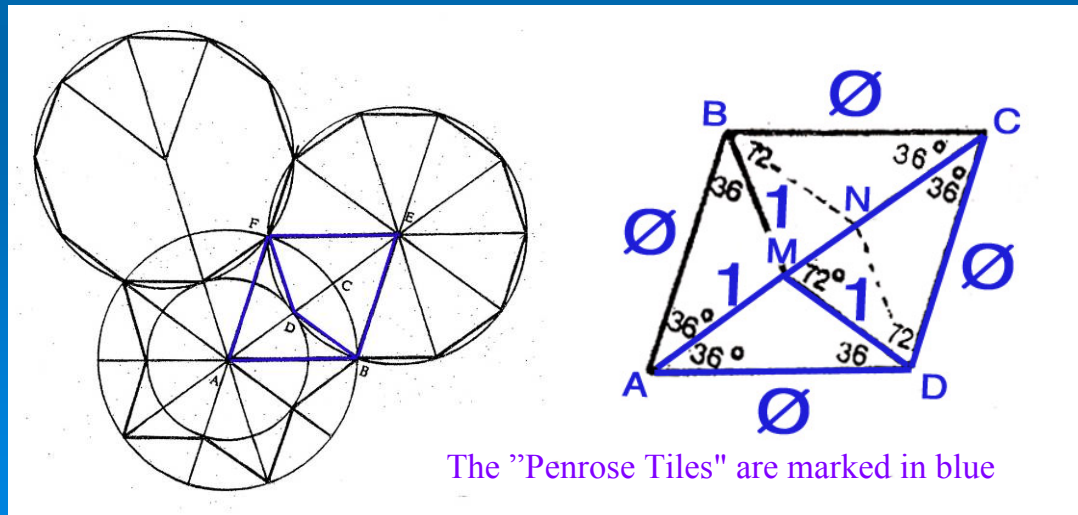
The manuscript describes a series of problems in the design of geometric patterns and their constructions. The first problem deals with interlocking convex decagons and pentagonal stars (Folio 180a/Ms#169).

The geometries illustrated in this early 13th century manuscript describe the 'quasi-periodic' pentagonal tiling of the plane that was re-discovered by Roger Penrose in the 20th century. This tiling is based on the two Pentamodules: the Golden Triangle & Golden Gnomon derived from the Golden Ratio and whose combinations form the 'Penrose Tiles'. [◆SG203.1]

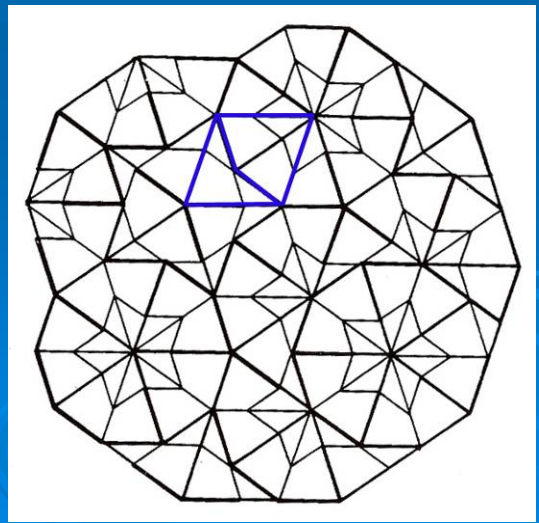


↑ Folio 180a / Ms#169

Data & images. Hargittai. *Fivefold Symmetry*. 1992



The "Penrose Tiles" are marked in blue

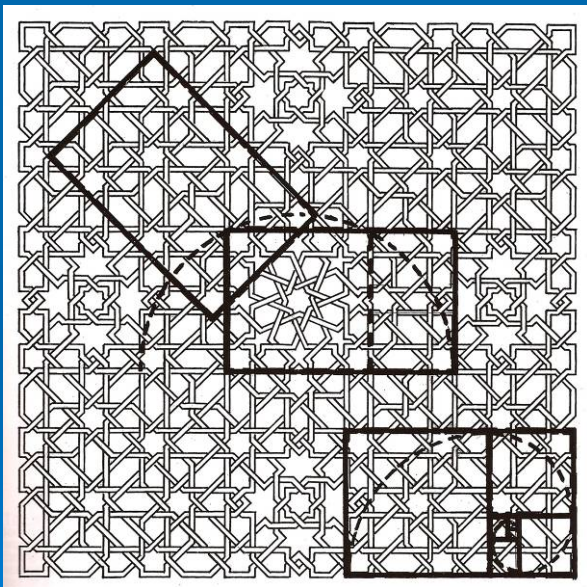
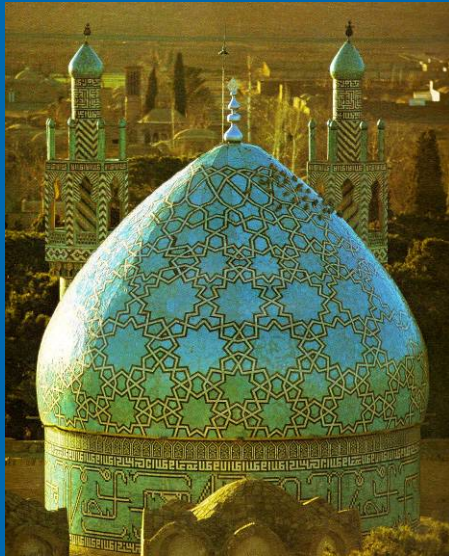


SG102.4.2.3 Islamic Geometric Art & Architecture

In Islamic orthodoxy, images of living creatures are forbidden as external appearances of creation, whereas the quest has been on for the inner, archetypal & perennial essence of the universe.

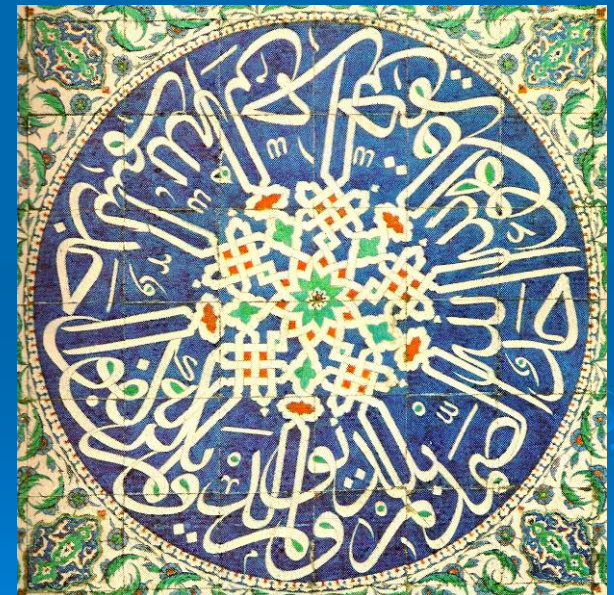
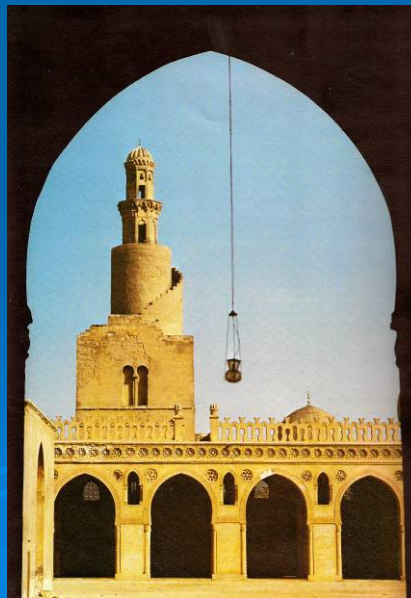
Islamic art therefore focused entirely on *aniconic* (without an image) representations of the Ultimate Reality: numbers, geometries and their multiple harmonies, thus expressing & weaving into arts and crafts the inner web of the universe. Islamic art is a flowering garden of Sacred Geometry.

↓ Mahan, Iran. Sufi Shrine



↑ Alhambra, Spain.
Mosaic based
on the Golden Rectangle.

Ibn Tulun Mosk, Cairo.
Minaret. ↓



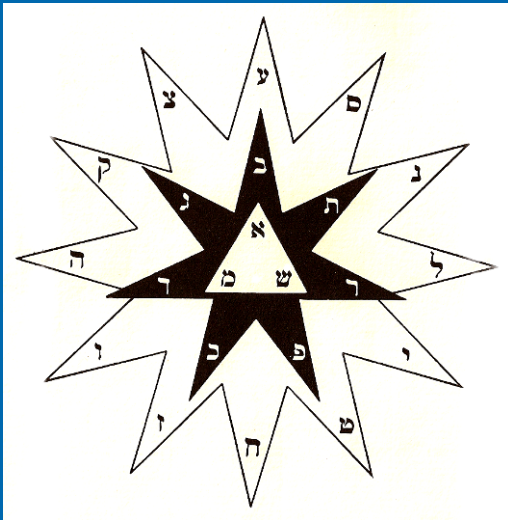
↑ Sokullu Mosk, Istanbul.
Ceramic panel based
on penta-decagonal geometry.

SG102.4.2.4 Mandalic Interlude



Islamic Rose

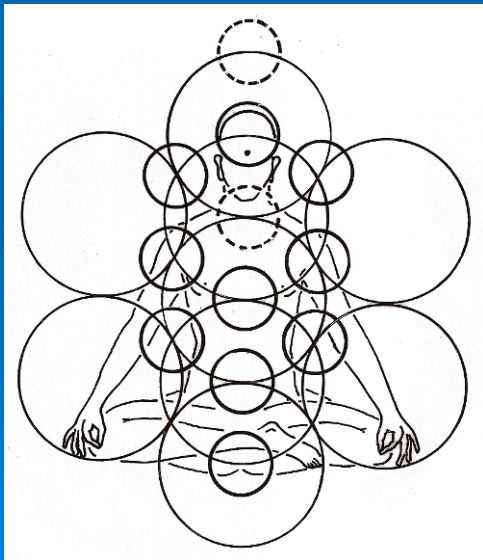
SG102.4.3 Jewish Kabbalah & Gematria



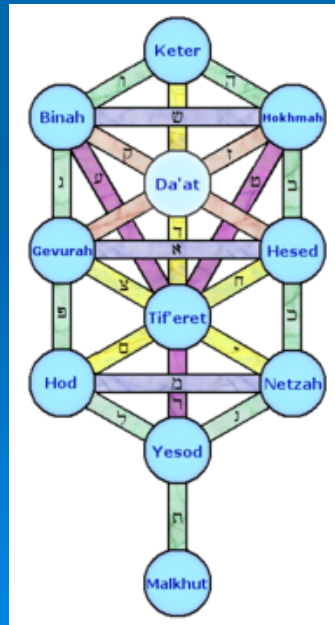
Geometries of the 22 Hebrew letters.

The term “*Kabbalah*”, meaning “*Tradition*” and referring to Jewish esoteric mysticism, appeared around the 11th century. At the time, many gnostic currents were emerging within and around the established religions of Christianity, Islam and Judaism.

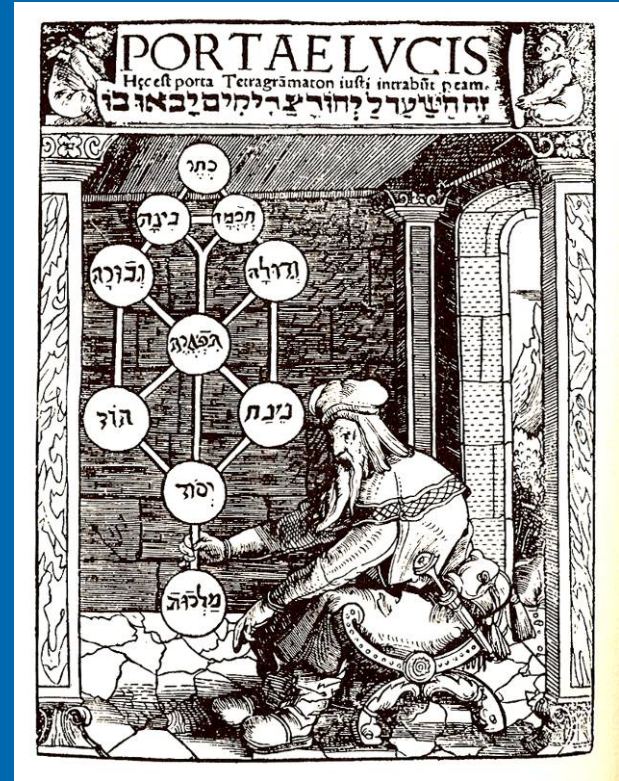
Islamic Spain, in particular, was a hub of cosmopolitan *gnosticism*: Cordoba, Toledo and Seville were busy cultural centers where Jews, Christians, Muslims and Arabic Christians could freely enjoy their traditions while interacting with each other in a rich creative environment.



↑ The *Sefirot* as energy centers



← The 10 *Sefirot* with #11 as Unity



↑ The 10 *Sefirot* as “*Doors of Light*”. Paulus Ricius, Augsburg. 1516.

[◆SG202]

The *Sefer Yetzirah*, a concise (1600 words) and visionary text, is considered to be a foundation of Kabbalism.

The *Sefer Yetzirah* (“*Book of Creation*”) deals with:

- The powers and geometries of the 22 letters of the Hebrew alphabet as channels for the cosmic life force. These letters are coded with numbers and allow for a numerical reading of the Hebrew texts by Gematria (digital sum) or other techniques.
- The 10 *Sefirot* of the Tree of Life diagram as a multi-level mandala map of energy and consciousness.

SG102.4.4.1 Leonardo da Pisa aka Fibonacci (1)

Fibonacci was an Italian mathematician born around 1170 in *Pisa* (Tuscany), a center of European culture and the future home of Galileo and the Leaning Tower. Fibonacci means “*Filius BONACCI*” or “*son of Bonaccus*” (very much like the English “*Smith-son*”).

Fibonacci’s father was a custom and trade official and took his son on extensive travels around the Mediterranean countries. Hence the surname of “*Bigollo*” (“*traveler*”) given to Fibonacci. Fibonacci thus had ample opportunities to study, compare and weigh the advantages & inconveniences of various local and global systems of numeration and calculation.

Fibonacci’s enquiries culminated in selecting the Hindu-Arabic numerals (with their essential place-holder ZERO) as vastly superior to all other methods. Following the lead of Al Khwarizmi (who, in the 8th century, publicized in the Islamic world the then novel Indian numerals), Fibonacci published in 1202 his system of “*New Mathematics*” in his book *Liber Abacus* (or “*Book of the Abacus*” - the *abacus* is the counting board).

The *Liber Abaci* attracted considerable attention and helped to put aside the cumbersome Roman numeral system still in use in medieval times.

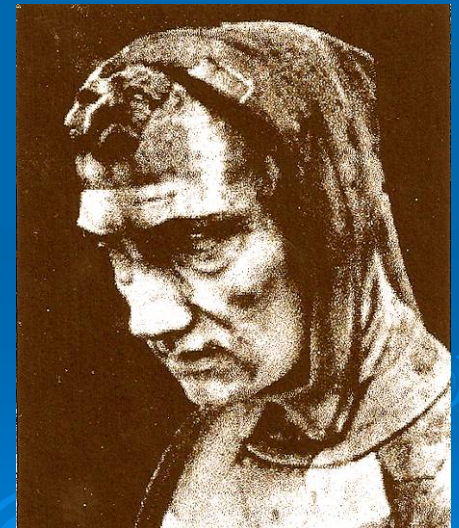
The Roman numerals were, in particular, missing the sign “0” or place-holder concept.

Fibonacci made some direct contributions to Sacred Geometry in his “*Practica Geometricae*” (1223) where he presented new methods of geometric construction linked with the Golden Ratio. In *Practica* he also added to trigonometry. But his real claim to fame was indirect and posthumous: he secured a world-famous place in Mathematics and Sacred Geometry by his study of the breeding patterns in the rabbit population. This “*Rabbit Problem*” led Fibonacci and following mathematicians to the re-discovery of the mathematical progression now called the “*Fibonacci Series*”. This Series is ubiquitous in nature and the quotient of two successive Fibonacci Numbers produces the Golden Number 1.618...

Since 1963, the *Fibonacci Association* has published a professional journal entitled “*The Fibonacci Quarterly*”, which has now grown into a well-respected international publication in Fibonacci-based Number Theory.



The young man



The mature man

SG102.4.4.2 Fibonacci (2) and the Rabbit Problem

In chapter XII of the “*Liber Abaci*” (1202) by Fibonacci, there is the famous “*Rabbit Problem*”:

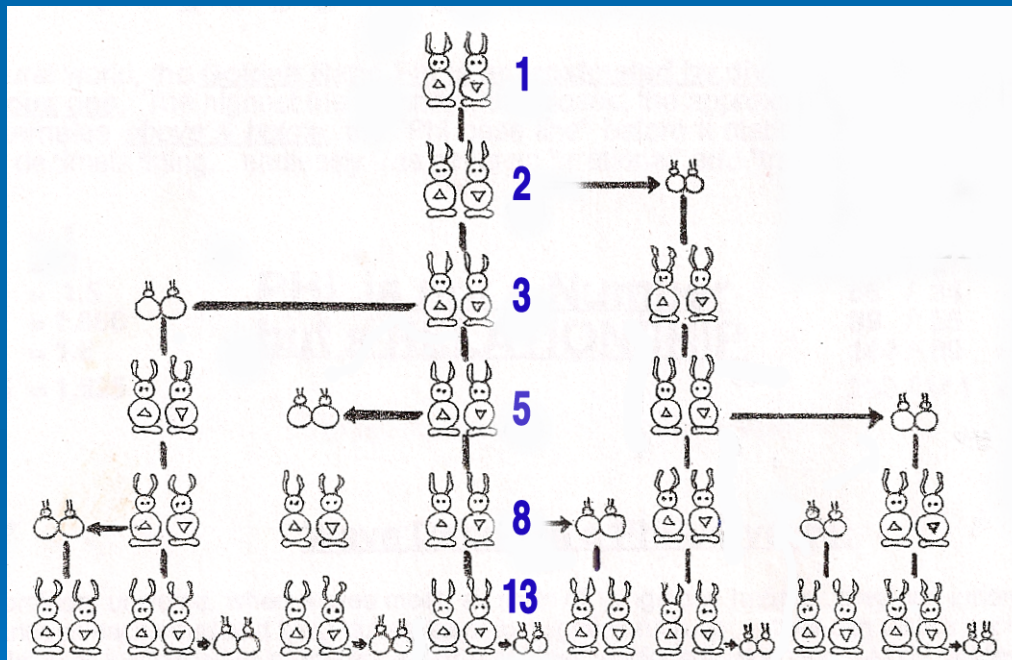
“A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year, if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?”



Fibonacci's teachers

In this (ideal) situation, the solution is that the number of rabbits follows the “Fibonacci Series”:

1 - 1 - 2 - 3 - 5 - 8 - 13 - 21 - 34 - 55 - 89 - 144 - 233...



Half a year later, there are 13 pairs of rabbits...
and multiplying to the next Fibonacci

A year later, January 1st, there are 233 pairs...
1 1 2 3 5 8 13 21 34 55 89 144 233

In the Fibonacci Series:

- Each term is the sum of the two preceding ones.
- Each term divided by the previous one gives a progressively better approximation of PHI, the Golden Number. Example: $233/144 = 1.618...$

Ever since the “Rabbit Problem”, this sequence has been observed in an astounding variety of phenomena, both natural and cultural. We will document many of these throughout the program.

[◆SG104.1]

Rabbit latest: there is even a digital “*Rabbit Sequence*” called the Golden String.

[◆SG104.5]

SG102.4.5 Algorism & Abacism

773. A diplomatic mission from India, including the astronomer Tanaka, reached Baghdad and shared with the Arabic scholars assembled by Caliph Al Mamoun the knowledge of their Indian numeral system, including the place-holder ZERO.
- C. 810. Al-Khwarizmi published the new system which quietly spread throughout the Arabic diaspora.
1126. Al-Khwarizmi's book is translated in Latin.
1202. Fibonacci publishes *Liber Abaci*.
1588. Montaigne, one of the most influential scholars of his days, wrote in his Essays: "I cannot yet cast account either with penne (numerals) or counters (abacus)".

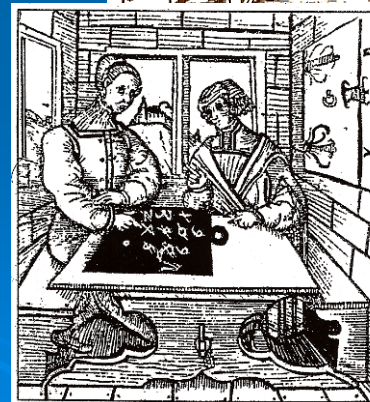
It took many centuries for western Europe to accept the Hindu-Arabic numerals and the knowledge of ZERO, the "circle".

It looks like Fibonacci used the word "*Abacus*" in the title of his book to protect himself against the wrath of the "Abacists" - the established accountants and clerics, a powerful lobby backed by the Church.

The *Abacists* saw the use of the new Hindu-Arabic numerals as the work of the Devil - and also a threat to their control of the people by keeping them uneducated. The Church-Abacus lobby issued a veto against the new numerals. *Algorists* plied their trade in hiding, as if using a secret conspiracy code. The knowledge of Zero (the place-holder "circle") was dangerous: the Inquisition sent quite a few "*algorist heretics*" to the stake.

Hindu-Arabic numerals & calculation made arithmetic and mathematics available to the masses. Fibonacci, in the West, was very influential in bringing about this knowledge.

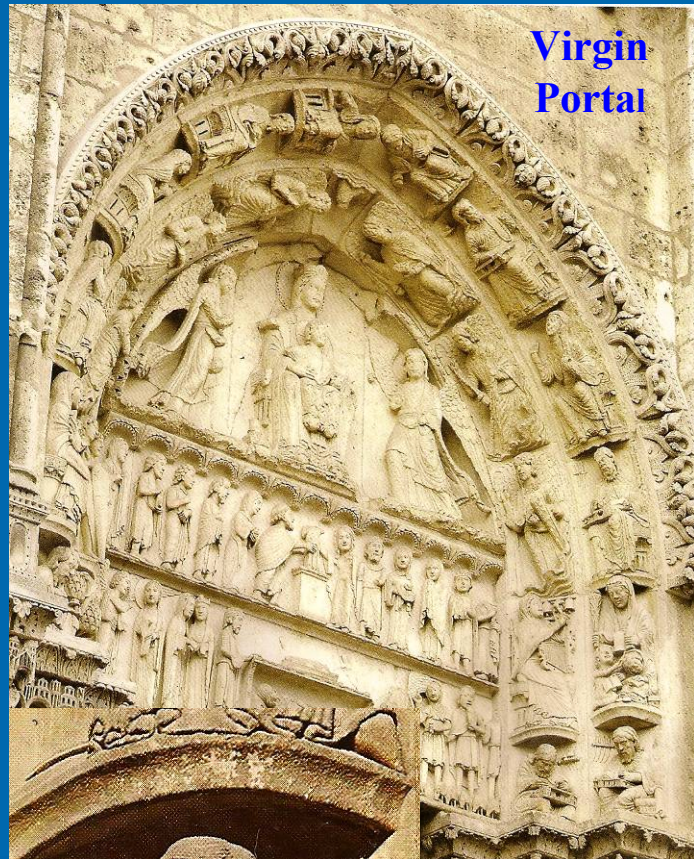
1503: Lady Arithmetic SMILES to the Algorist.
Hindu-Arabic numerals and Zero HAVE WON.



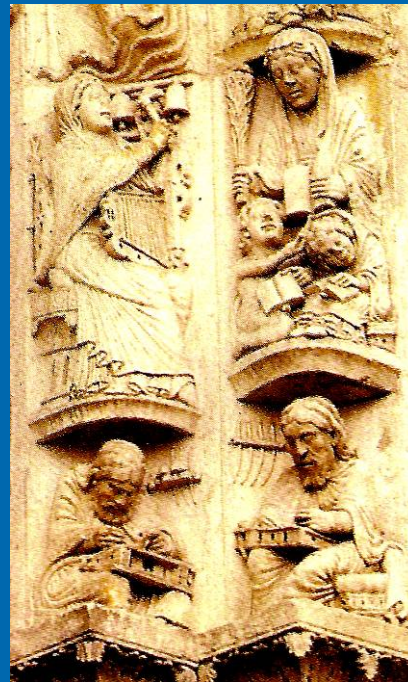
Algorists



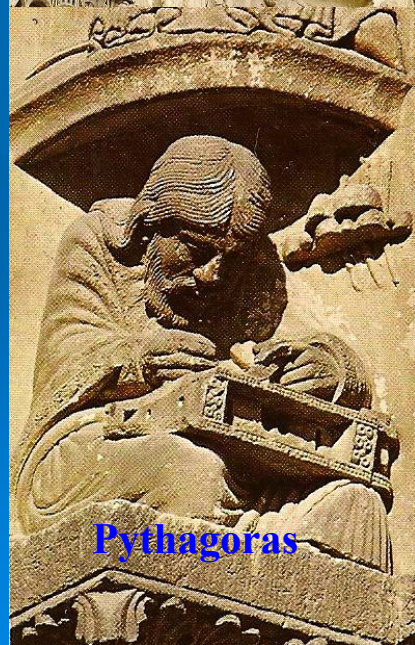
Abacists



Virgin Portal

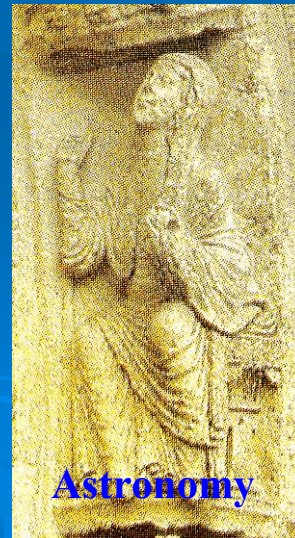


↑ Left: Music & Pythagoras
Right: Grammar & Donatus



Pythagoras

Grammar	Donatus
Rhetoric	Cicero
Dialectic	Aristotle
Arithmetic	Boethius
Music	Pythagoras
Geometry	Euclid
Astronomy	Ptolemy



Astronomy

SG102.4.6 The Seven Liberal Arts

In antiquity and the Middle Ages, the *Seven Liberal Arts* of the classical curriculum were divided into two study groups:

- *Trivium* (“three roads”): Grammar (Moon), Rhetoric (Venus) and Dialectic (Mercury).
- *Quadrivium* (“four roads”): Arithmetic (Sun), Music (Mars), Geometry (Jupiter) and Astronomy (Saturn).
Respectively: pure **Numbers**, numbers in time (**Harmony**), numbers in space (**Proportion**), and numbers in space/time (**Rhythm**).

These arts are represented on the southern tympanum (outer vousoir) of the Western portal in Chartres, sculpted around Virgin Mary as *Sedes Sapientiae* (“Source of Wisdom”). They are associated, right below them, with the corresponding famous teachers from antiquity.

SG102.4.7 The School of Chartres

From the 10th to the 13th century, the Chartres Cathedral School rose to preeminence under the early leadership of Fulbert, a brilliant scholar, who was called the “*Venerable Socrates of the Chartres Academy*”. The real golden age of Chartres was the second half of the 12th century, under the guidance of remarkable chancellors: Bernard of Chartres, Gilbert de la Porée, Guillaume de Conches, Thierry de Chartres.

Bernard de Chartres, renowned philosopher and “*the most perfect platonician of his times*” (John of Salisbury), is known to us by his famous comparison: “*We (the Moderns) are like dwarves seating on the shoulders of giants (the Ancients)*”.



The *Quadrivium*: Arithmetic, Music, Geometry and Astronomy.
Church of Blenod-les-Toul, France



Chartres: South view.
Insert: Labyrinth.

At the School of Chartres, the curriculum was funded on the classical 7 Liberal Arts: *Trivium* (preparatory cycle) & *Quadrivium* (advanced cycle). To these were added physics and ethics as well as some crafts, including medicine. A variety of translations from antiquity and from original Islamic works were becoming available. Thierry de Chartres assembled, in his *Heptateuque*, the main classical texts.

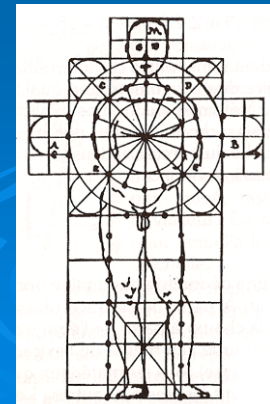
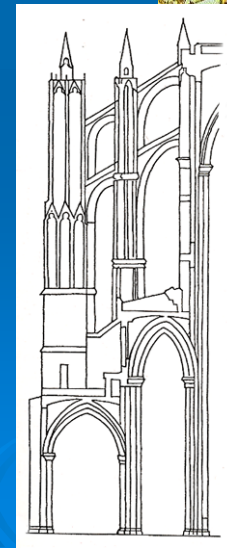
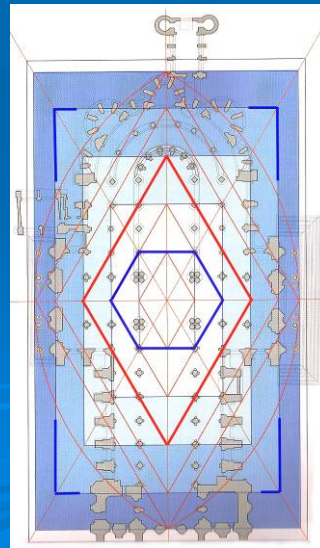
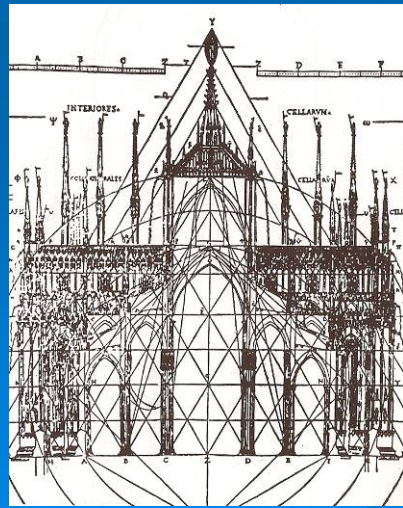
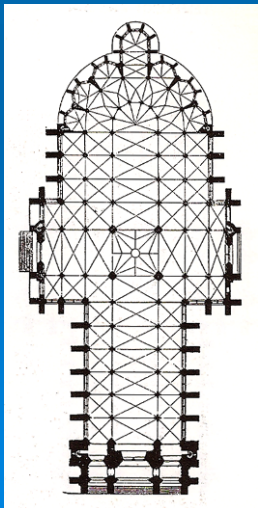
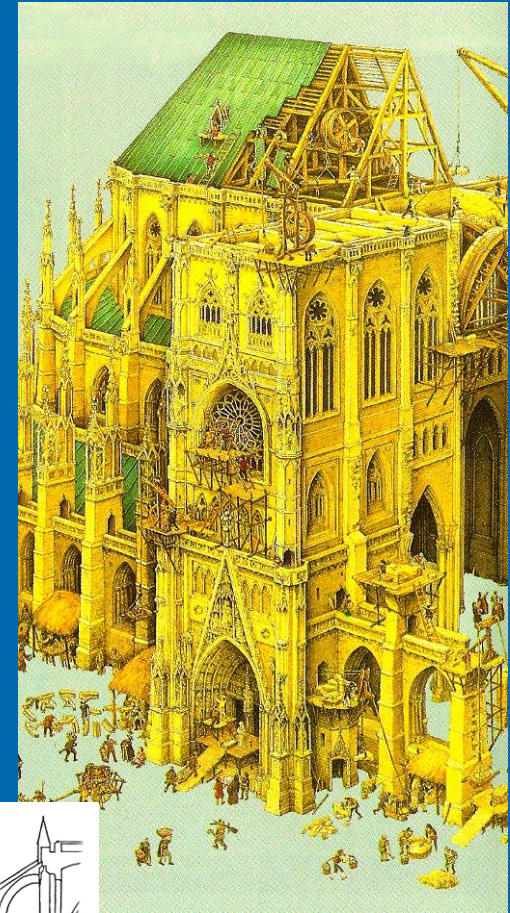
After the 13th century, the School of Chartres progressively lost its popularity to the new school founded in Paris by Pierre de Sorbon. (later known as “*La Sorbonne*”).

SG102.4.8.1 The Age of the Cathedrals

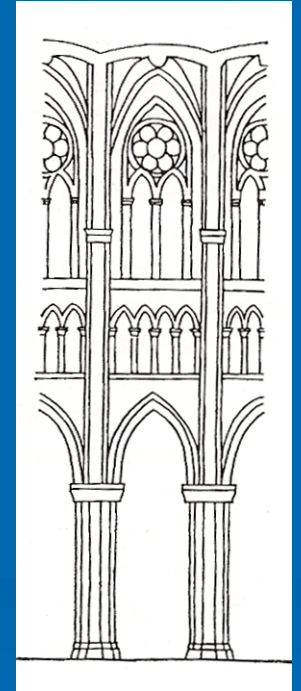
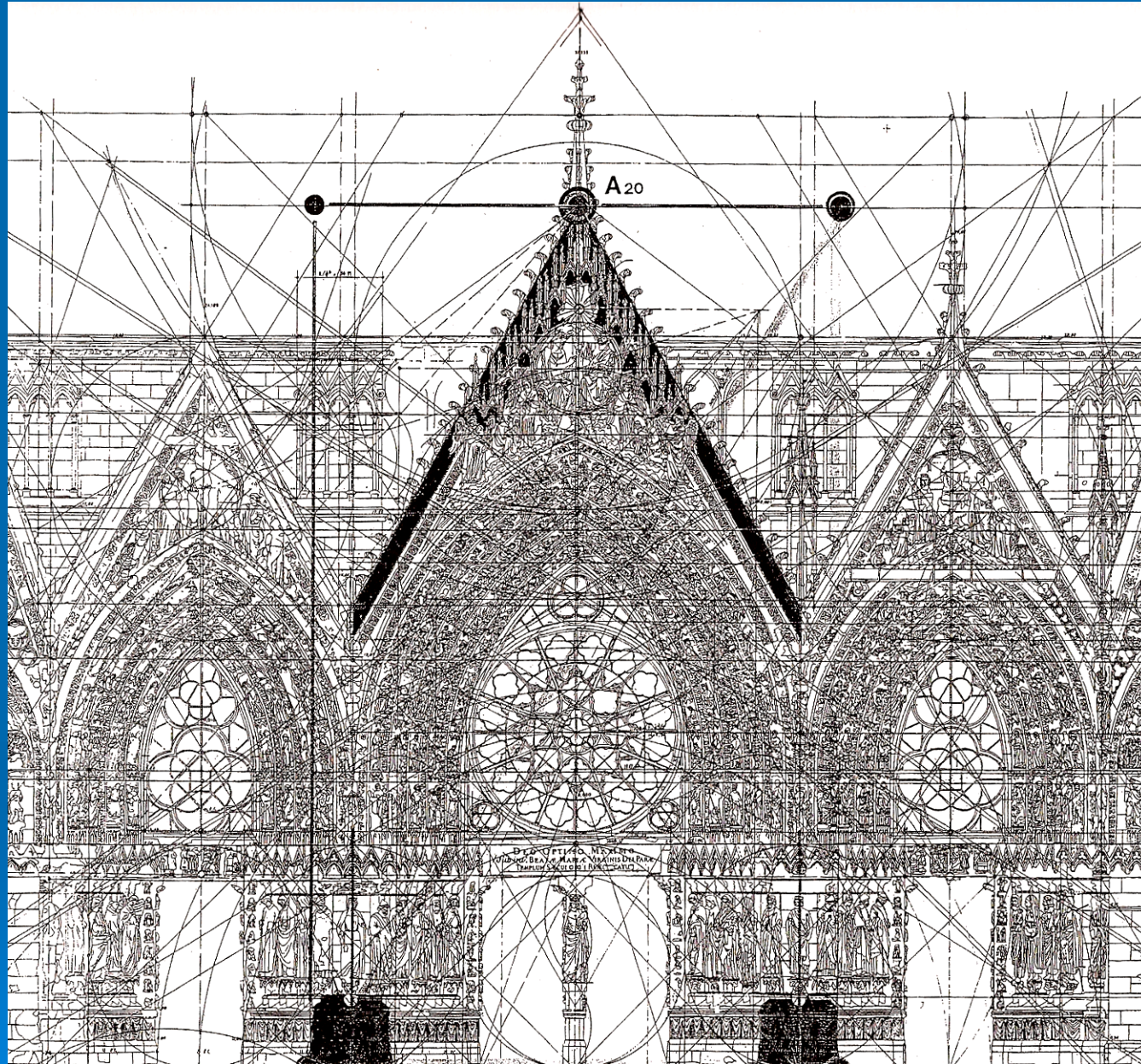
The 11th, 12th and 13th centuries were the times of a global intellectual, cultural and spiritual Renaissance in western Europe:

- The wisdom of antiquity was pouring out of the Islamic universities in Baghdad and around the Mediterranean world.
- The crusades were mixing “enemy” cultures and bringing to western Europe Islamic art, mathematics and architecture.
- The Knight Templars were consolidating a new social order.
- Saint Bernard and his Cistercian order were sprinkling France with dynamic monasteries (530 monasteries built during the 12th century).
- The cult of Mary, as the Great Goddess and the Black Virgin, was encouraged and eagerly embraced by the people.
- The Cathar and other spiritual wisdoms were rising in popularity.
- Abbot Suger was launching the Gothic style of “*lux continua*”.

... A fervent Gnostic Renaissance was blossoming and culminated in the “Age of the Cathedrals” (80 cathedrals) with astounding Sacred Geometry harmonics.

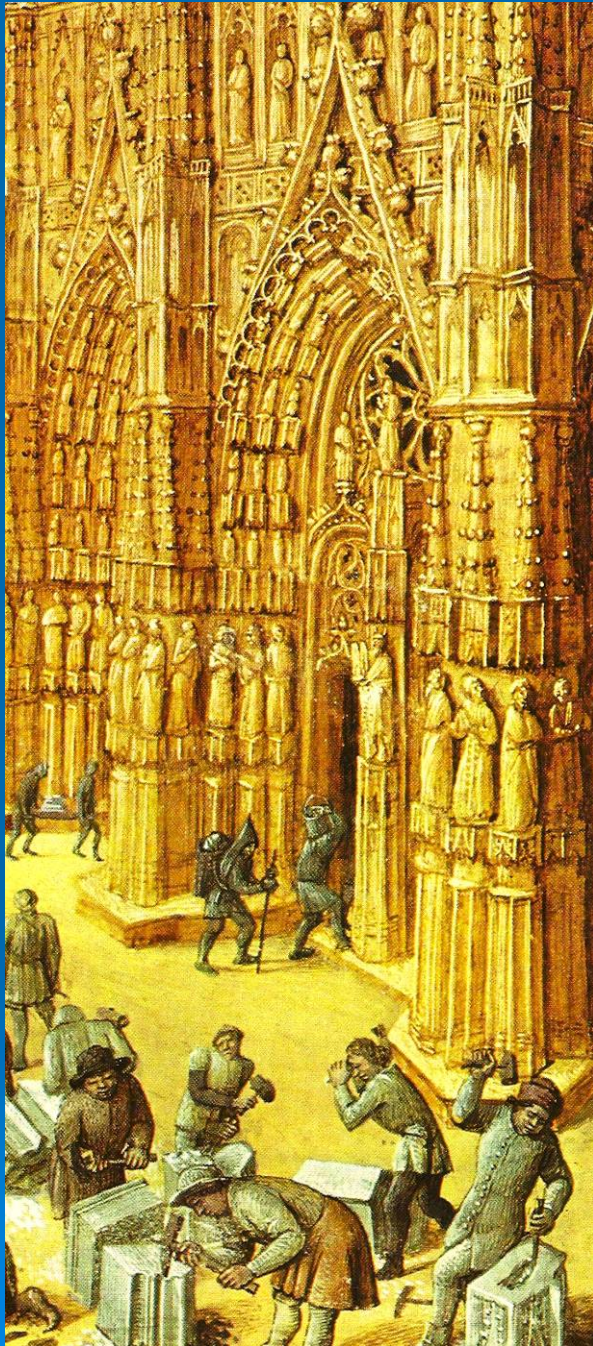


SG102.4.8.2 Notre Dame of Reims

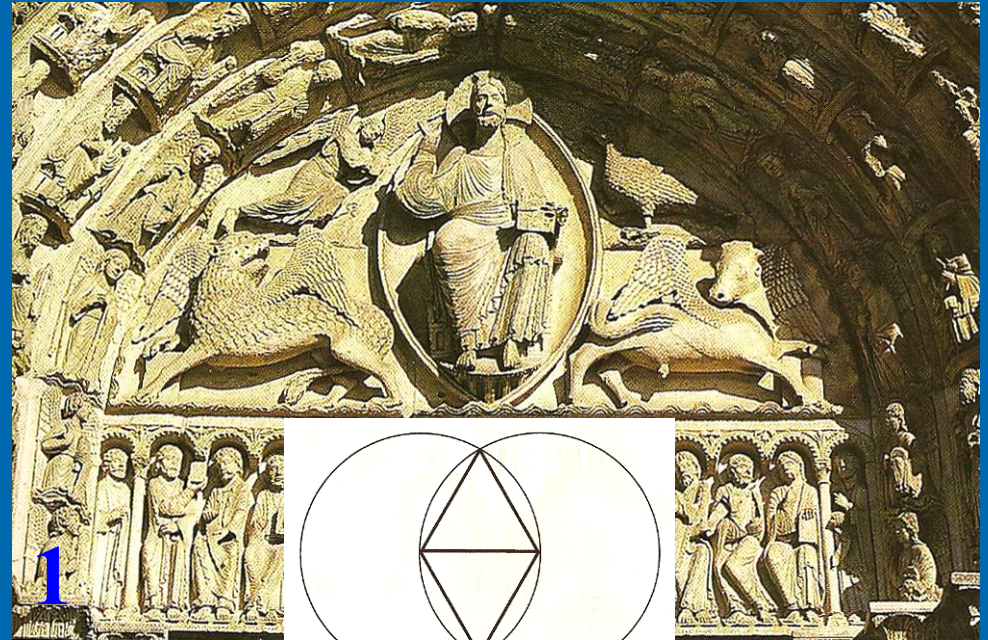


Reims.
Elevation.

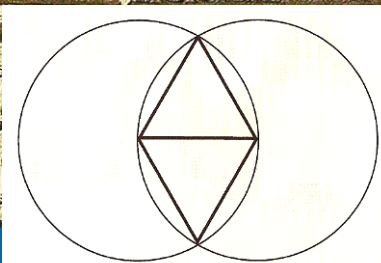
Geometries in Reims Cathedral.
Thierry de Champris. *Cathédrales, Le Verbe Géométrique*. France, 2004.



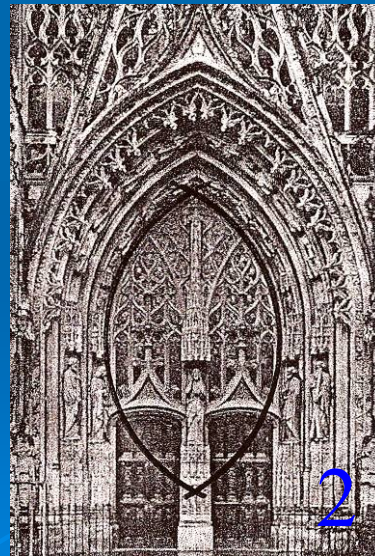
SG102.4.8.3 The Medieval Vesica



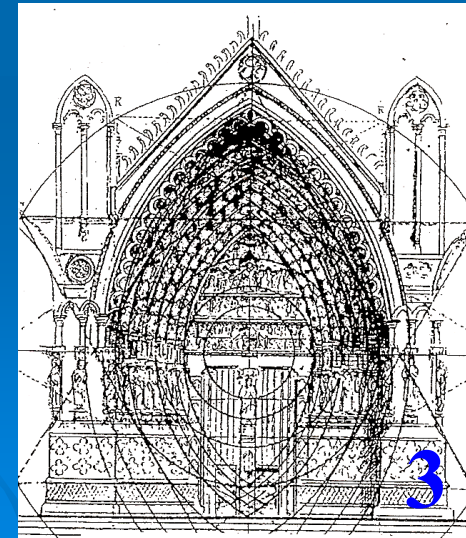
1



Vesica Piscis with inscribed Rhombus



2



3

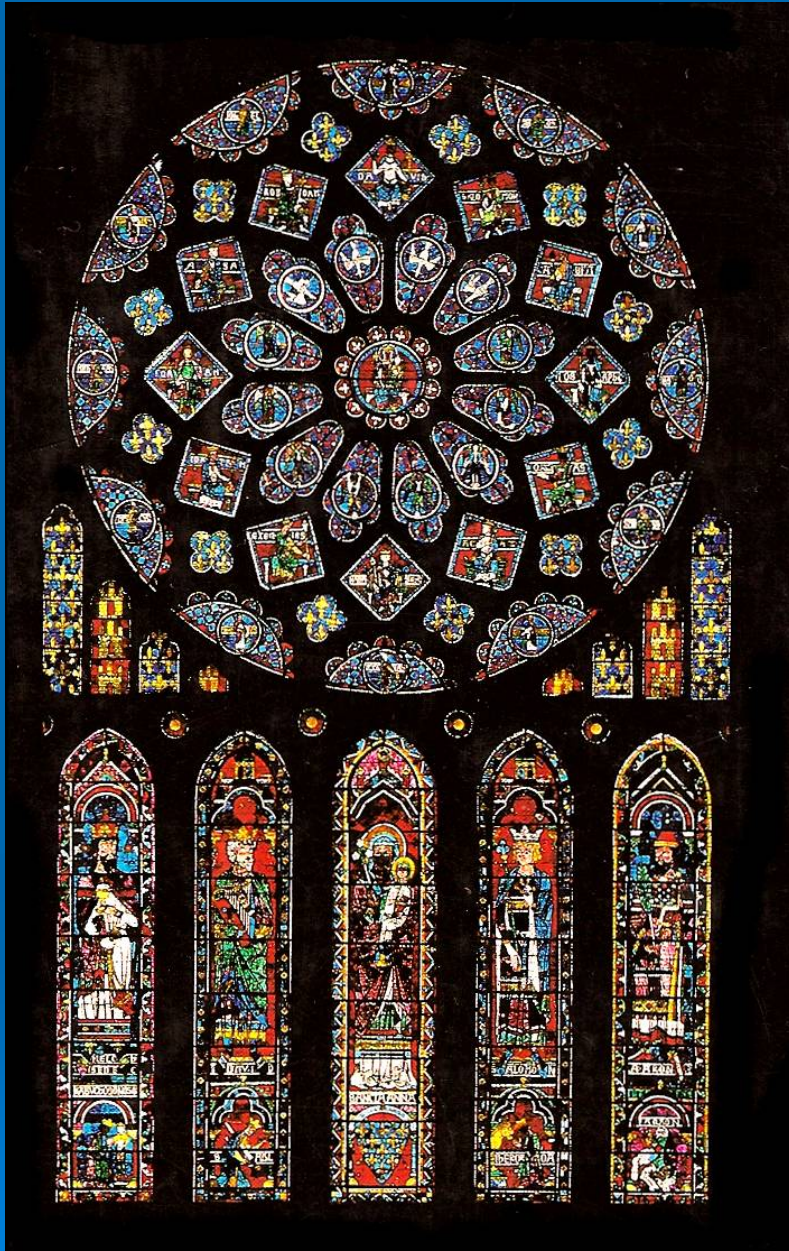
← Miniature by Fouquet, 15th century.

1. Chartres
2. Vendome
3. Amiens

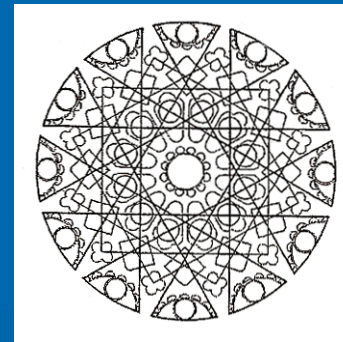
SG102.4.8.4 Labyrinths & Rose Windows

In the medieval cathedrals, Rose Windows and Labyrinths are expressions of the *Universal Mandala*, *Wheel of Life*, *Cosmic Vortex* or *Inner StarGate*.

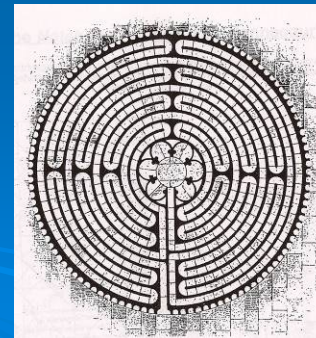
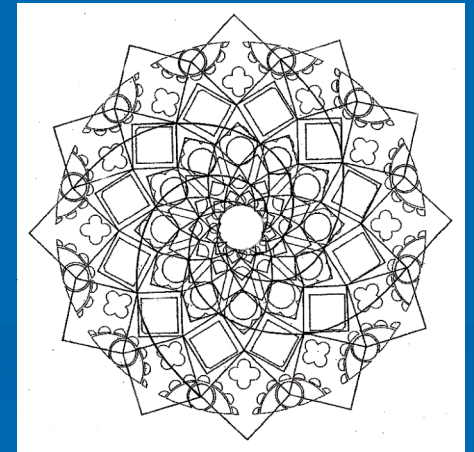
In the stained glass Rose Windows, intricate systems of rhythmic proportions create a hi-frequency lens to animate the flows of colored light pouring from the alchemically-treated glass. The Labyrinth experience offers a pathway of dancing geometry akin to a mini-pilgrimage into the Self, an initiation into Source in order to bring back spirit power into daily life.



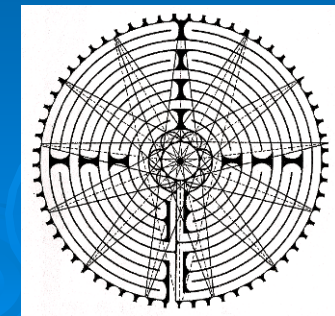
↑ Chartres Cathedral. North Rose Window



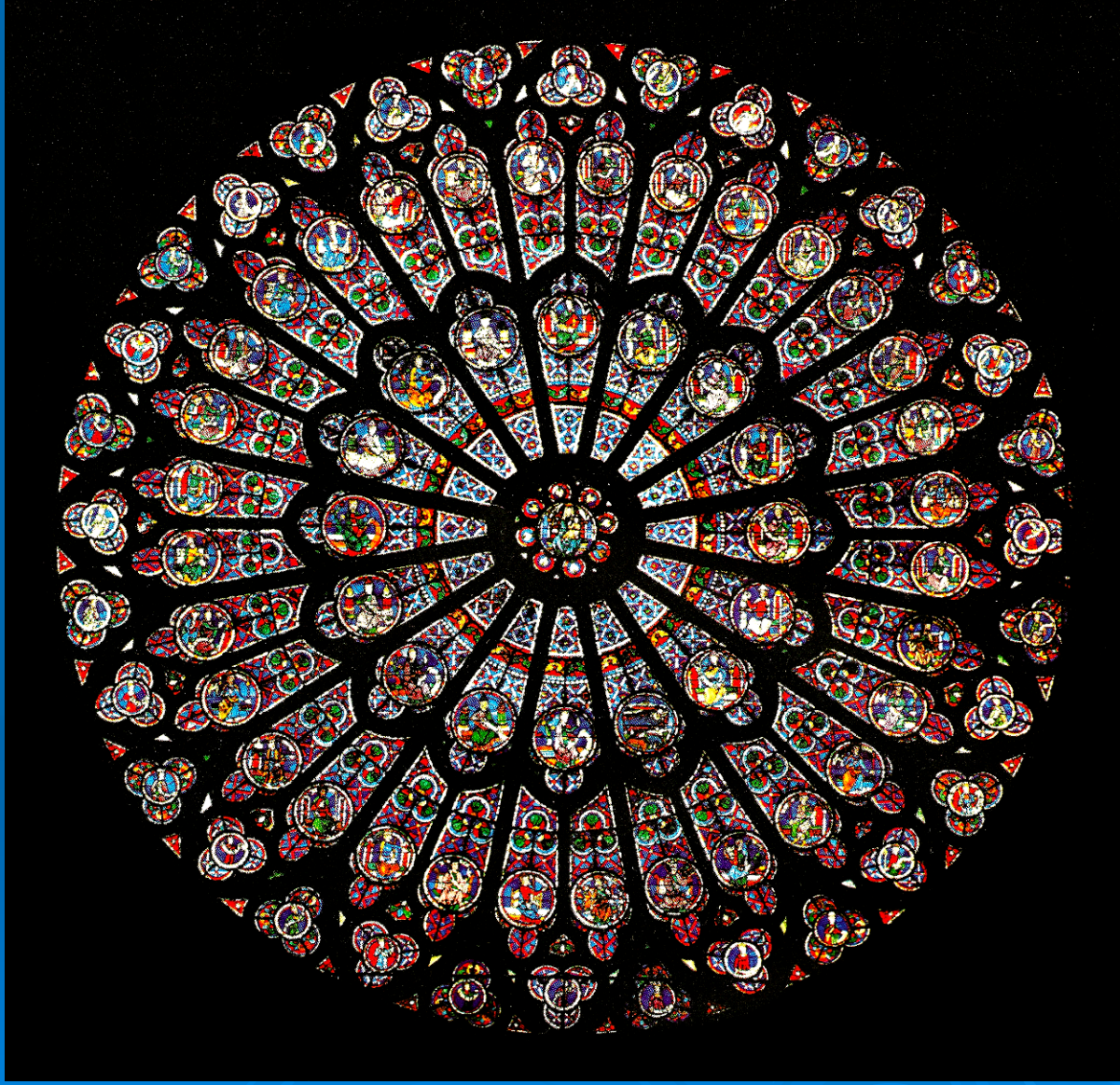
Chartres North Rose
↓ Geometries ↑



↑ Chartres Labyrinth →



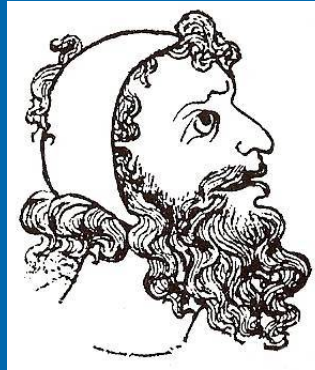
SG 102.4.8.5 Mandalic Interlude



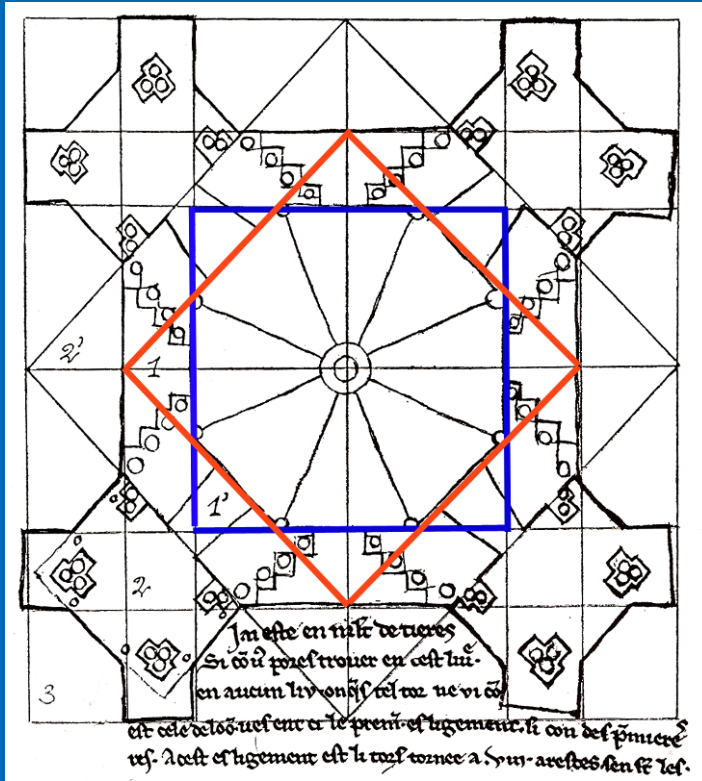
Rose Window

Paris North. c. 1268

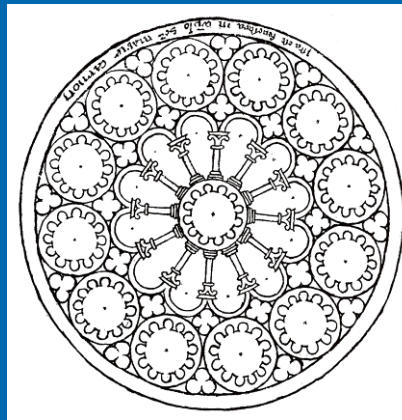
SG102.4.9 The Notebooks of Villard de Honnecourt



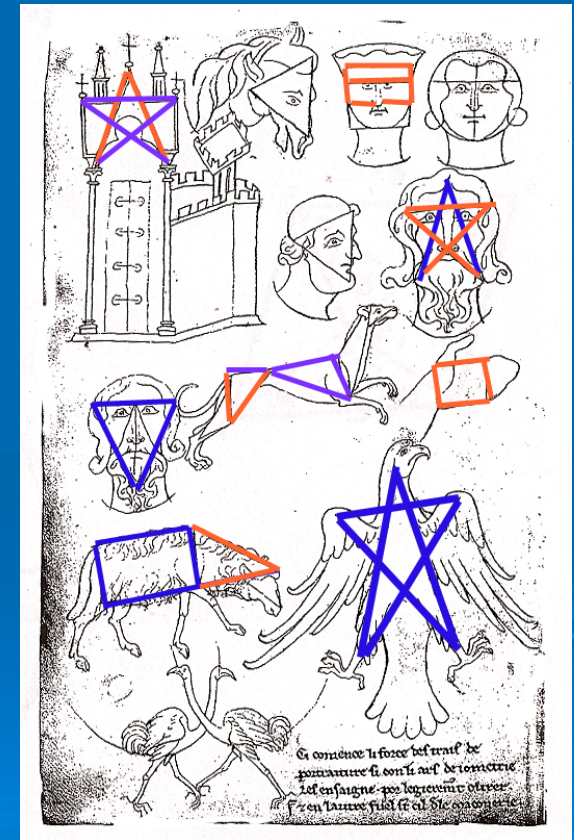
Villard de Honnecourt was a mid-13th century architect whose “*Carnets*” (notebooks) illustrate the well established “*Ars de Geometria*” - the Art of Geometry, grounded in Sacred Geometry.



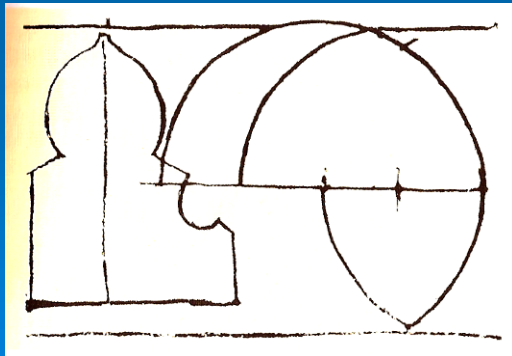
↑ Groundplan of a tower.
Laon Cathedral.



“Wheel Window”



↑ “Figures from nature”

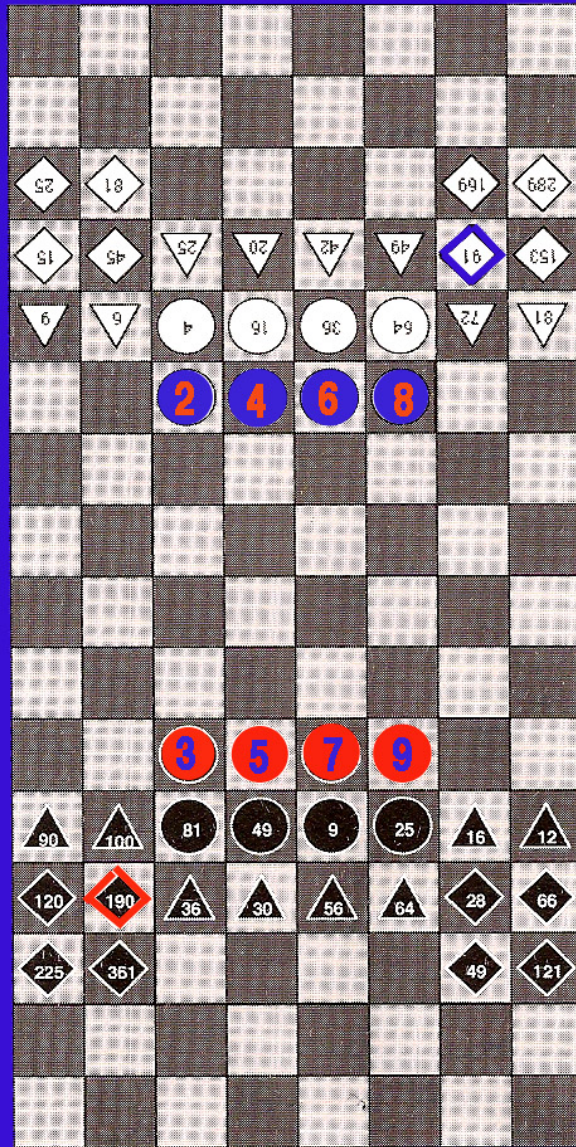


“How to make three kind of arches
with a single opening of the compass”



Tracing of ogival windows

SG102.4.10 Ludus Philosophorum



The opening positions of *Rithmomachia*.
The pieces in blue are the ‘Kings’:
they are composite pyramids made up
of circle, triangle and square pieces.

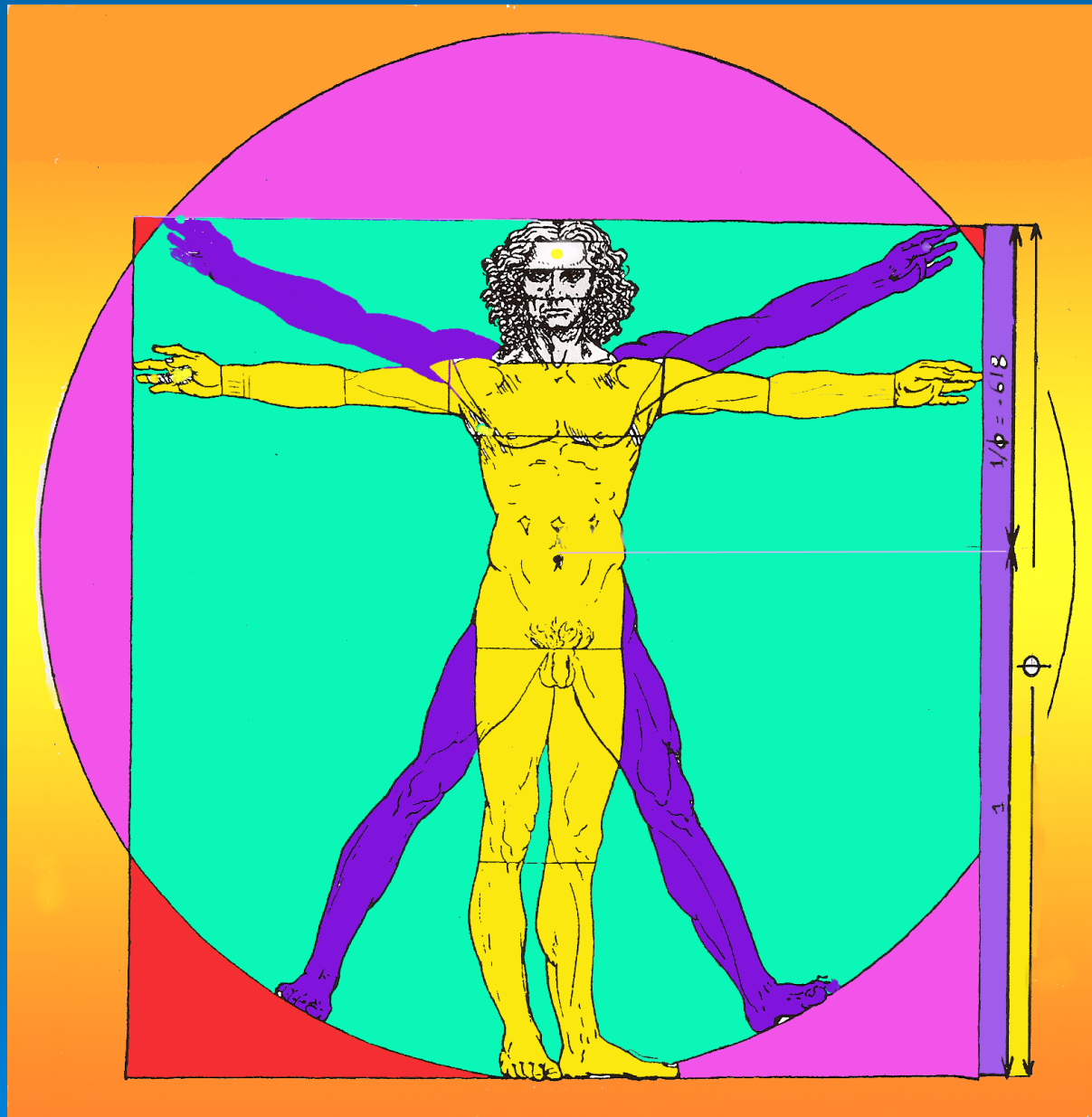
Rithmomachia (Latin = *battle of numbers*) or *Ludus Philosophorum* was a number game played during the middle ages and also the Renaissance. Played on a double chess-like board with 24 pieces on each side, this ‘*nobilissimus et antiquissimus ludus Pythagoreus*’ provided a grand practice in Nichomachean/Pythagorean arithmetic.

This board-game illustrates the arithmetical mindset of the period and an extraordinary agility in mental computation:

- Each of the pieces carries a number but not all numbers are the same on the two sides.
- One side is “even” and the other “odd” (after the leading pieces and by color)
- Each side has a balance of 12 odd and 12 even numbers
- There are 3 geometry for the pieces: circular, triangular and square
- Each side has a modular “king” composed of the 3 shapes.
- When a piece is captured, it can be turned over to show a new value
- As in chess, each piece has a specific move.
- There are several rules/tactics of battle interaction: equality, addition, blockade, multiplication, subtraction, division. Quite a mental gymnastics!
- Various proportionalities are also involved in the battle interactions.
- The goal is to capture the opponent’s King. Then a variety of ‘triumphs’ are displayed, requiring the victor to line up proportionalities in their pieces (arithmetical, geometrical or harmonic).

Among the first adepts of this mental number game were Gerbert (Pope Sylvester II), Francois Rabelais, John Sherwood (Bishop of Durham)... and many more, all the way to the Italian Renaissance. The complexity of this teaching game points to the sophisticated mathematical and sacred geometry mindset of the elite who build the cathedrals and created artworks based on the harmonic proportions of the ideal world of principles.

SG102 - Chapter 5 Renaissance Times



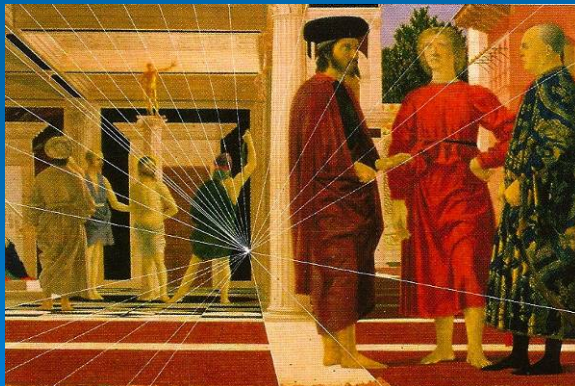
SG102.5.1.1 European Renaissance (1) Context

The fall of Constantinople in 1453 provided Western Europe with a major cultural impetus as many eastern men of knowledge sought refuge in Italy. They brought along their libraries of books & manuscripts containing ancient Greek philosophies and early descriptions of natural sciences. This wave re-emerging from antiquity merged, in Italy, with the dawning spirit of humanism inspired by St. Francis of Assisi, and poets like Dante, Boccaccio and Petrarch.

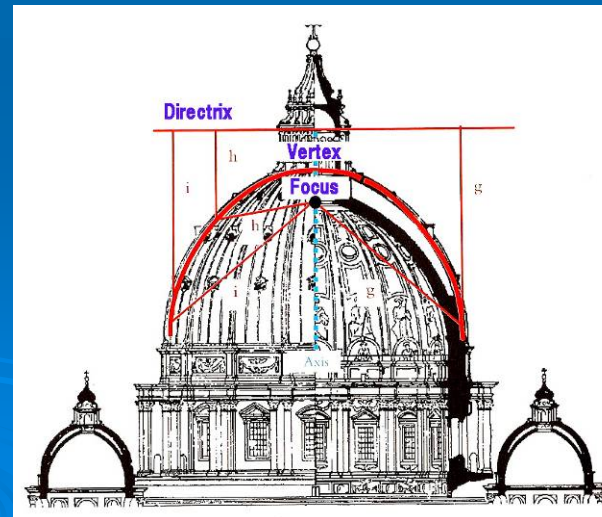
The Renaissance (“*rebirth*”) refers to the period of time in Europe (1450-1600) that saw the decline of the feudal system, the discovery of new continents, the invention of printing (1455) and, in astronomy, the Copernician system superseding the Ptolemaic system. This was a time of reviving the classical wisdom and expanding human knowledge in many new directions.

The spirit of the Renaissance times was mostly revealed in the arts: painting, sculpture and architecture.

While humanist scholars & philosophers (Ficino, Pacioli) were making ancient knowledge available again, painters (Leonardo da Vinci, Masaccio, Fra Angelico, Piero della Francesca, Raphael, Botticelli), sculptors (Pisano, Michelangelo, Donatello, Verrochio) and architects (Alberti, Palladio, Bramante, Brunelleschi) were busy creating a cultural environment expressing for a new world the ancient geometric principles of proportion, harmony and balance.



↑ The *Flagellation of Christ*
by Piero della Francesca.
Perspective lines and balanced geometries.



← Michelangelo
and others.
Partial cross-
section of St.
Peter's dome,
completed 1564.

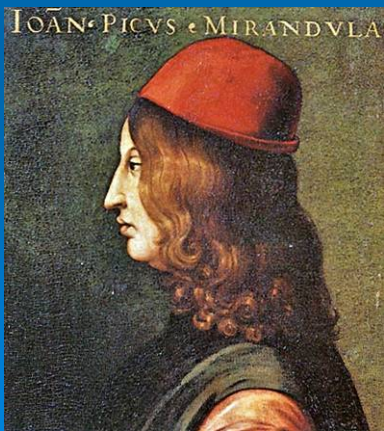
SG102.5.1.2 Renaissance (2) Humanism

Celebrating the awakening from the long medieval night, the Renaissance Humanists brought forward a view of the dignity and creative value of the individual as well as a renewed faith in humankind's goodness, destiny and responsibility.

The most influential of the Renaissance Humanists was Marsilio Ficino (1433-1499), a protégé of Cosimo de Medici. His *Florentine Academy*, a revival of Plato's School, had enormous influence on every thinker and artist of the Renaissance, as it launched a passionate interest in Plato, the "Athenian Moses". Ficino's grand work was to translate from the Greek originals into Latin all of Plato's works, with his own commentaries; also the *Corpus Hermeticum*, a collection of Hellenistic documents about Hermes Trismegistos, and works by Iamblichus, Plotinus and other Neo-Pythagorean writers from antiquity. After Ficino, the full literature of the Pythagorean & Platonic tradition was made accessible to Western Europe, thus providing a foundation for creative achievements in the arts and developments in the new mathematical science of nature.

"Right Proportion" and music (Sacred Geometry) were the fashionable tools of the days for achieving "Heavenly Harmony" on Earth.

In 1492, Ficino wrote: "*This century, like a Golden Age, has restored to light the Liberal Arts which were almost extinct: grammar, poetry, rhetoric, painting, sculpture, architecture, music... astrology.*"



Pico della Mirandola



Marsilio Ficino

In turn, one of Ficino's students, Pico della Mirandola revived Pythagorean numerology, combined with Kabbalistic lore. Pico della Mirandola is famous for his *Oration on the Dignity of Man*, which has been called the "Manifesto of the Renaissance". In his Oration, he refers specifically to the "*method of philosophizing through numbers*" as known to Pythagoras. Meanwhile, in Germany, Johann Reuchlin resurrected for his compatriots the teachings of the Pythagorean School. In Italy, Nicholas of Cusa, a Catholic cardinal, developed a neo-pythagorean theology based on numerical proportion and a system of knowledge seeking to determine the proper ratios through numbers.

Thus, the Renaissance Humanists restored the Pythagorean-Platonic philosophical wisdom of a benevolent universe governed by a Deity of Beauty & Justice, using Right Proportion and Harmony for playing the Music of the Spheres.

SG102.5.2.1 Luca Pacioli (1)



Portrait of Luca Pacioli (1445-1515) by Jacopo de Barbari (1440-1515)
Luca Pacioli is practicing the yoga of Geometric Contemplation

[◆SG101.1]

Born to a poor family in Tuscany, the young *Pacioli*, who loved mathematics, abandoned a business apprenticeship to befriend and work with the artist Piero della Francesca (one of the great Renaissance painters and a pioneer of the new science of ‘perspective’).

In Venice, Della Francesca gave Pacioli access to the large library of Frederico, the Count of Urbino.

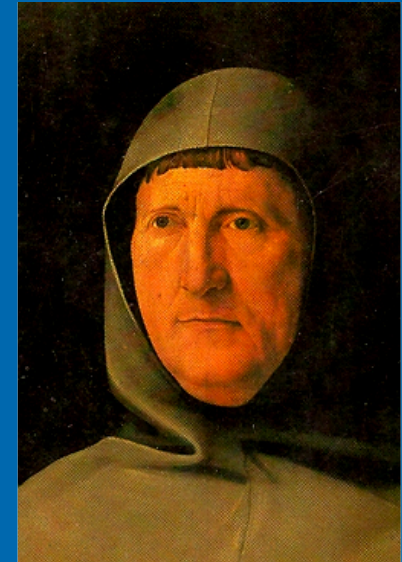
Through assiduous self-teaching, Pacioli furthered his knowledge of his passion: mathematics.

In Rome, he studied with Leone Battista Alberti, another Renaissance man who wrote the first modern manual for Painters. There, Pacioli studied theology and became a friar in the Franciscan Order.

SG102.5.2.2 Luca Pacioli (2)

Now traveling & teaching, Pacioli, around 1489, wrote his first famous book: *Summa de arithmetica, geometria, proportioni et proportionalita*. The Summa provided a foundation for great leaps in European mathematical knowledge.

In 1494, Ludovico Sforza became Duke of Milan. A generous patron to artists & scholars, he wanted his court to be the finest in the whole of Europe. Leonardo da Vinci had already be invited to the Milan court as a painter and engineer. In 1496, Luca Pacioli was invited to teach mathematics. Pacioli and Leonardo became great friends.

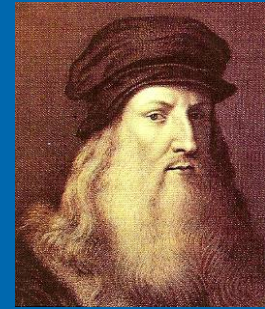


When the French King Louis XII conquered Milan, Pacioli and Leonardo fled to Mantua, to be the guests of Elizabeth d'Este, and then went to Venice to finally return to Florence. There Pacioli was appointed to teach geometry at the university of Florence. He also became the superior of his order and entered the monastery of San Croce.

In 1509, Pacioli published his 3-volume work *De Divina Proportione* as well as a Latin translation of Euclid's Elements. Pacioli died in Florence in 1514 at about age 70.

With the printing in 1509 of his “De Divina Proportione”, Luca Pacioli publicly revealed to artists & scholars the construction & unique properties of the Golden Ratio. Many Italian and European “*Renaissance Men*” (artists, architects, philosophers and natural scientists) came to Pacioli to learn the secrets of the Golden Ratio as well as the rules of harmonic proportion and perspective.

SG102.5.2.3 Luca Pacioli (3) De Divina Proportione



Leonardo Da Vinci
By Giorgio Vasari

Divina

proportione

Opera a tutti gli ingegni perspicaci e curiosi necessaria. **O**ne ciascun studioso di **D**hilosophia: **D**rospectiva **D**ictura **S**culptura: **A**rchitectura: **M**usica: e altre **M**athematiche: sua uissima: sottile: e admirabile doctrina consequira: e delectarassi: cõvarie questione de secretissimi scientia.

In 'De Divina Proportione', Pacioli attributes divine qualities to the Golden Ratio :

UNITY & UNIQUENESS

TRINITY

IMPOSSIBILITY OF BEING DEFINED IN HUMAN TERMS

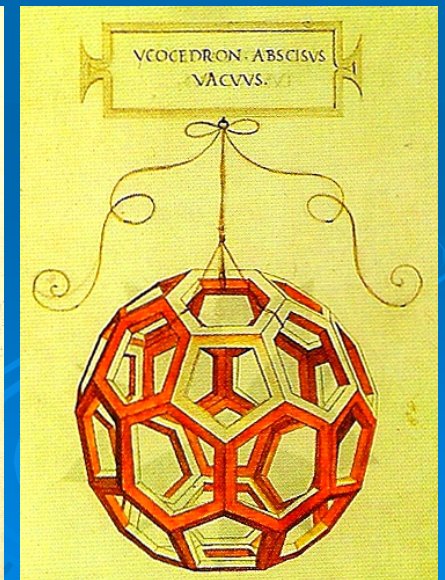
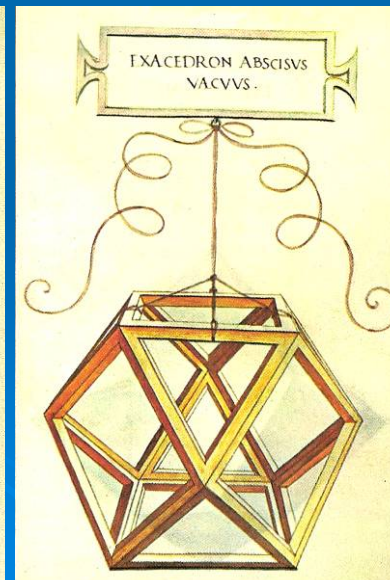
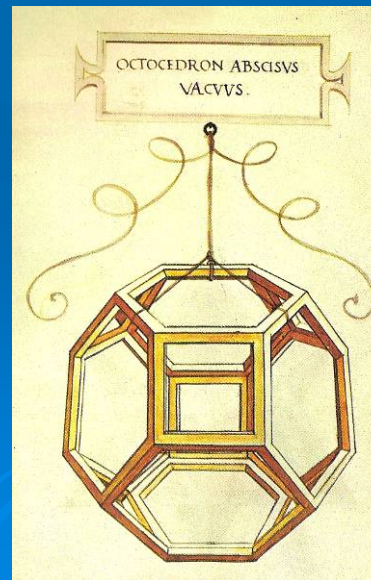
IMMUTABILITY

("Like God, the Divine Proportion is always similar to itself")

"As God breathes life into the cosmos through the Fifth Essence and to the four earthly elements and to everything in nature, so our Divine Proportion breathes life into the Dodecahedron".

↑ Illustrated by Pacioli's famous friend Leonardo Da Vinci, 'De Divina Proportione' comprises three books devoted to the geometry and philosophy of the Golden Ratio.

Da Vinci's drawings →



SG102.5.3.1 Leonardo da Vinci (1)

The archetype of the Renaissance Man, excelling in his art while well versed in engineering, mathematics, science and architecture, Leonardo epitomizes an ideal that is becoming quite popular again: a well-rounded, balanced knowledge of nature & culture, of material questions and spiritual values, of ancient wisdom and future inspirations.

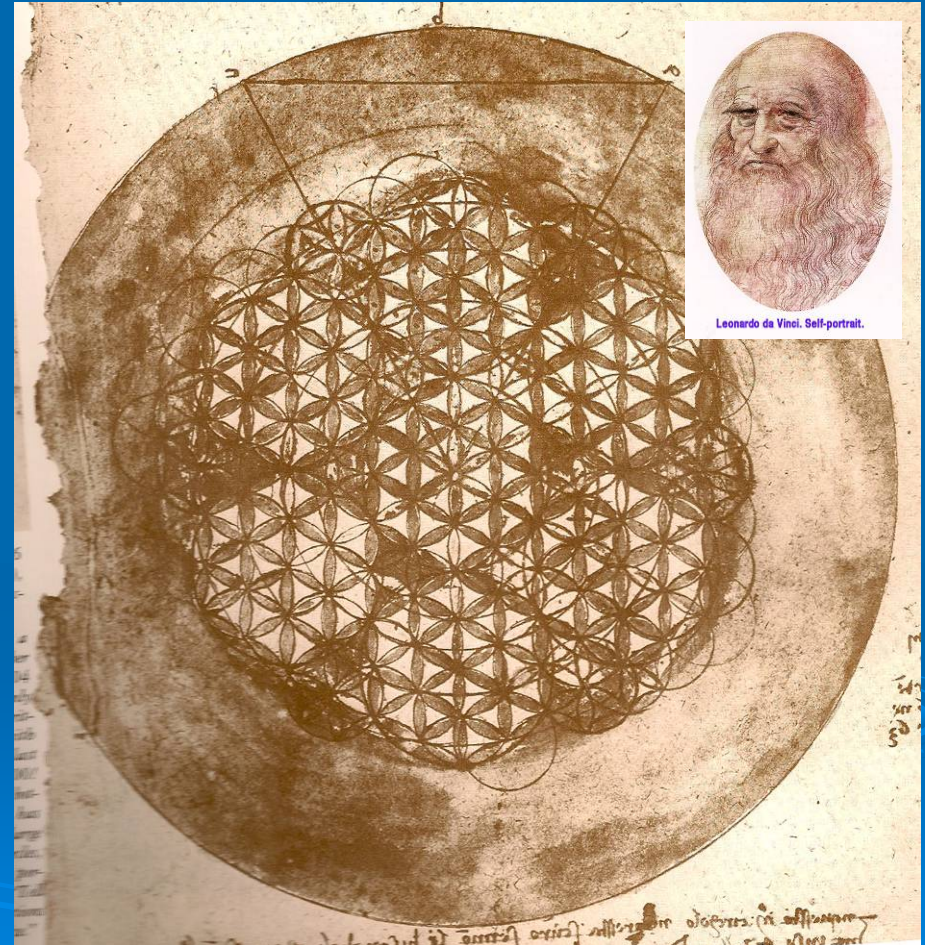
At the time of Leonardo's birth (1452), Florence, a mere twenty miles away from Leonardo's birthplace *Vinci*, was the richest city in Europe. A republic for 30 years when Leonardo was born, Florence was governed by humanist leaders who believed in the dignity of man and encouraged a stable economy based on healthy merchant & craftmen classes.

Leonardo's parents were unmarried but illegitimacy was socially accepted (even from popes) and Leonardo grew up on the estate of his father's family as a legitimate son while receiving a full education.

Apprenticed to artist Andrea del Verrochio at fifteen, Leonardo went on to accept commissions in Florence, and then in Milan, under the patronage of Duke Ludovico. In Milan, Leonardo was multi-tasking as a painter, sculptor, technical adviser in architectural & military affairs, hydraulic & mechanical engineer, and even anatomist.

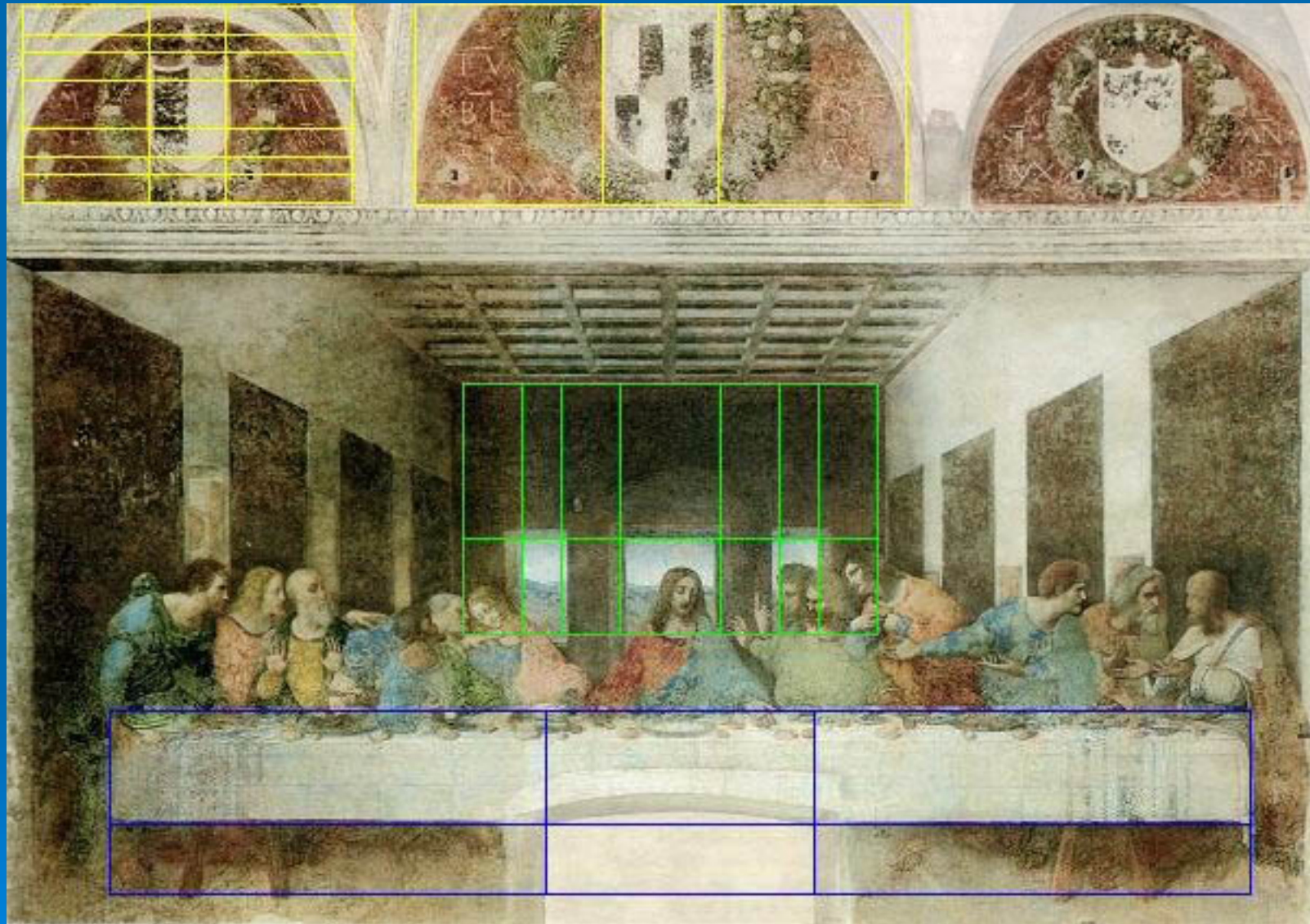
In all his creative projects, Leonardo was driven by the deepening conviction that all of nature was operating according to orderly, harmonious principles and laws that could be replicated by creative minds. His life-long friendship with Luca Pacioli was an ongoing meditation on the Divine Proportion. He said:

"The painter's mind is a copy of the divine mind"



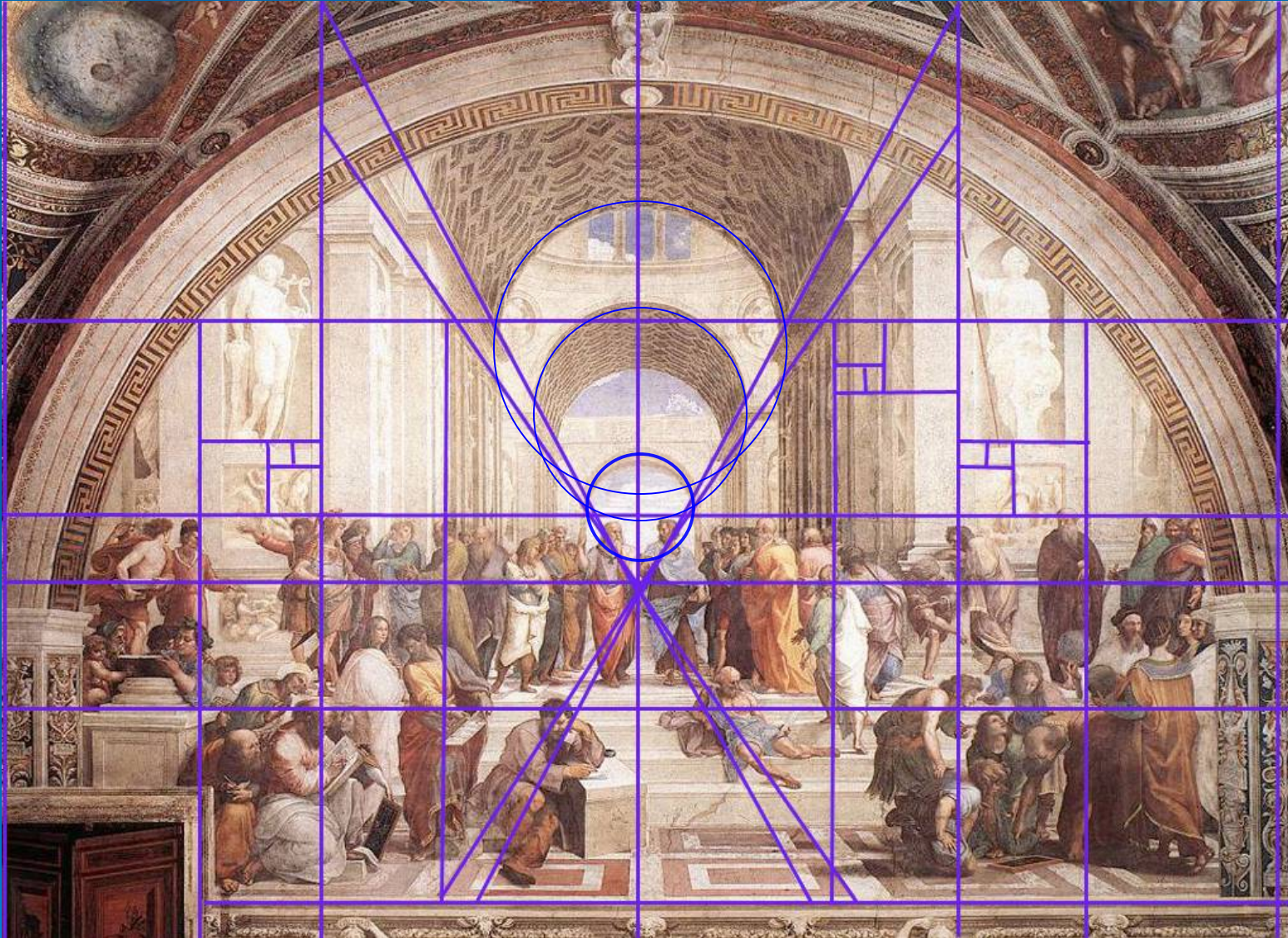
Leonardo's study of the Flower of Life.
Codex Atlanticus, 75/1.

SG102.5.3.2 Leonardo da Vinci (2)



Phi proportions in *The Last Supper*. Leonardo da Vinci, 1495-1498.

SG102.5.4 Phi in the Renaissance Art

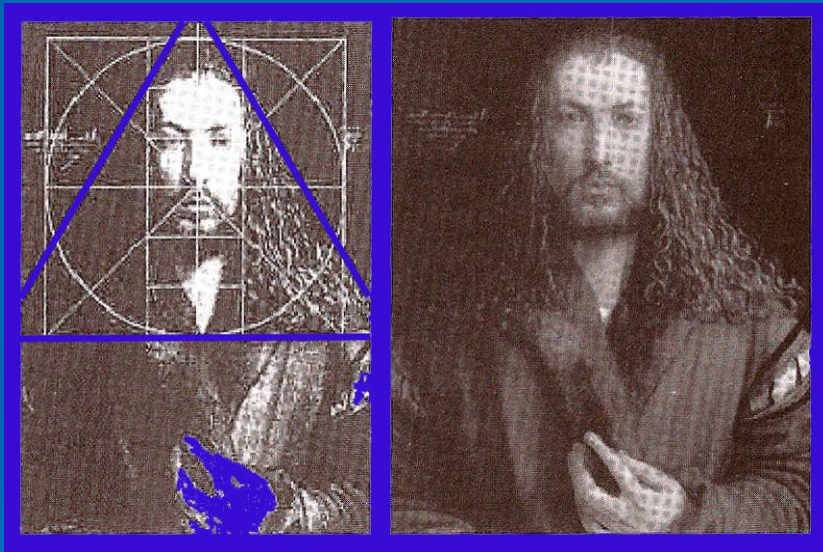


The *School of Athens* by Raphael (1483-1520) is a good example of the use of geometric harmonic proportions (some of them Phi-related) and perspective lines empowering the focus to center.

SG102.5.5.1 Albrecht Dürer (1)

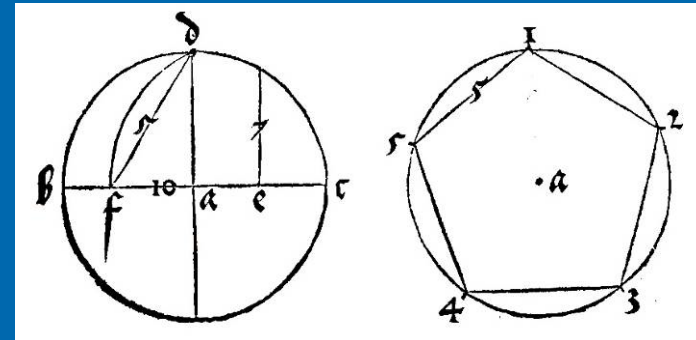
From a letter to his friend in Nuremberg, Willibald Pirckheimer, we know that, in 1506, Albrecht Dürer (1471-1528), the well respected German painter & graphic artist, rode from Venice to Bologna and possibly Florence in order to “*learn more about the secret art of perspective*”. It is likely that Dürer met with Luca Pacioli to be initiated in the secrets of the classical system of proportions and the new science of perspective. Dürer’s enthusiasm about regular figures - polygons and polyhedra - was stimulated by the works of Piero della Francesca and Pacioli. Dürer believed that it was necessary for a painter to know precisely how to measure an object.

In 1523, Dürer set his ideas in print in his *Four Books on Human Proportions*. The Four Books expand on the classical system of proportion using the human body as the unit of measure, with parts expressed as fractions thereof.

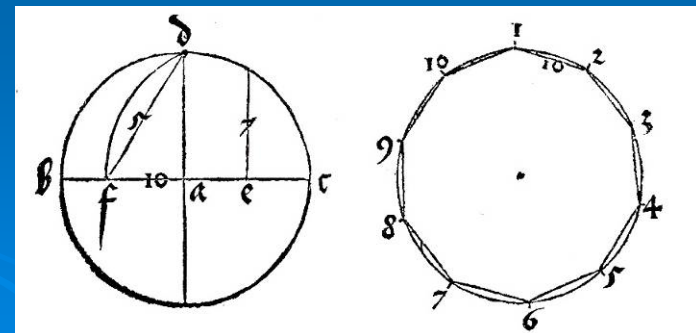


Dürer’s Self-Portrait (1500).

There is a clear equilateral triangle outlining the head of wavy hair. The chin and the base of the triangle divide the height of the picture into a Golden Rectangle.



↑ Dürer. Exact construction of a pentagon.

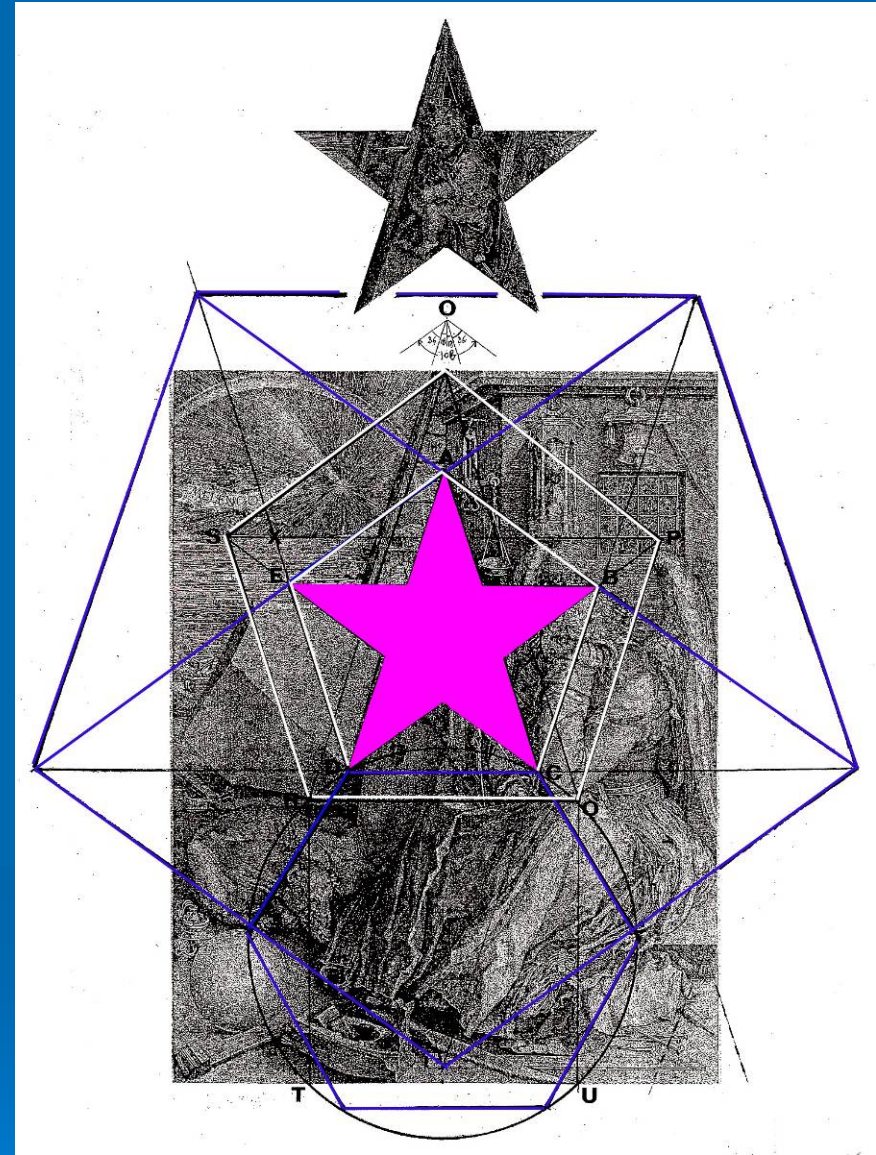


↑ Dürer. Construction of a decagon, after Pappus.

SG102.5.5.2 Albrecht Dürer (2)



Dürer's *Melancholia* is replete with sacred geometry, both in its blueprint layout and in its actual elements. We will study it in detail later [◆SG302]
One hint: the slop angle of the ladder is the exact angle of the Golden Triangle - one of the two Penta-modules necessary to trace a pentagram/pentagon.



“Geometry is the right foundation of all painting”. A. Dürer.

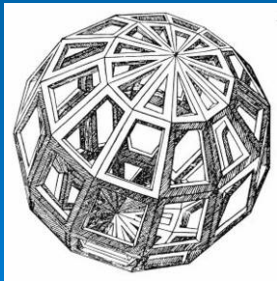
SG102.5.6 Intarsia Art

Intarsia (Italian *intarsio* = mosaic made of pieces of inlaid wood and fitted into a support)

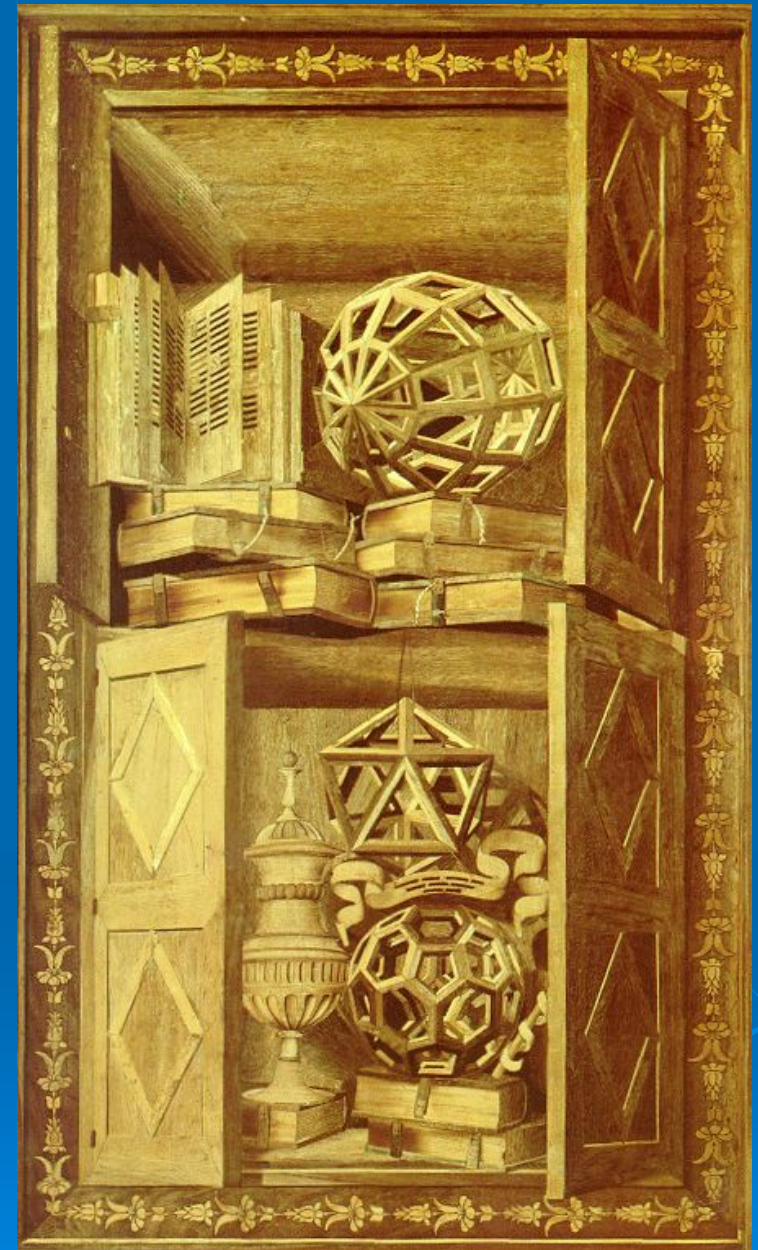
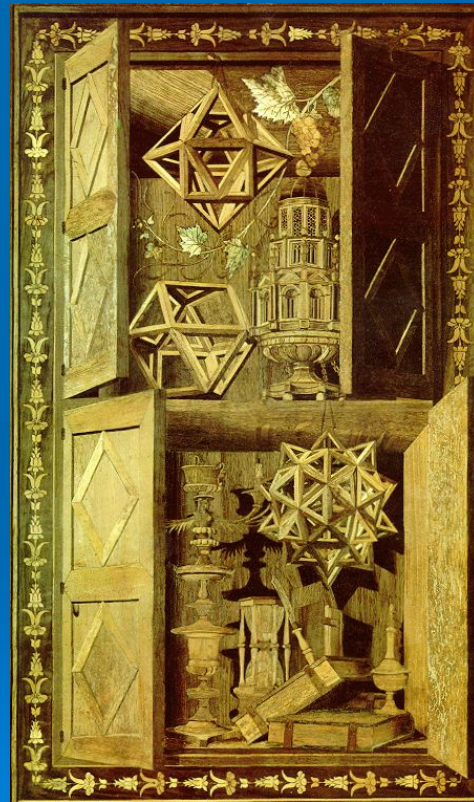
In the 15th - 16th centuries, the art of *Intarsia* reached a peak in Northern Italy. Here are two examples of elaborate polyhedra created by Fra Giovanni da Verona, as elements of his intarsia. They are from the church of Santa Maria in Organo, Verona, and are dated around 1520.

Note: these are flat panels giving a *trompe-l'oeil* impression.

Does it remind you of the drawings by Da Vinci in Pacioli's Book *De Divina Proportione*?



The 72-sided Sphere by da Vinci.



SG102.5.7.1 Johannes Kepler (1)

In standard astronomy books, Johannes Kepler (1571-1630), while he is presented as the discoverer of the three laws of planetary motion, is also disparaged in the rest of his theoretical work as an “astrologer”. This is a misunderstanding of history: Kepler was a brilliant and foresighted theoretician in many areas of knowledge and he lived at a time of unified understanding of the cosmos, before the great split science-spirit.

Kepler is being understood again as we are healing this split. In

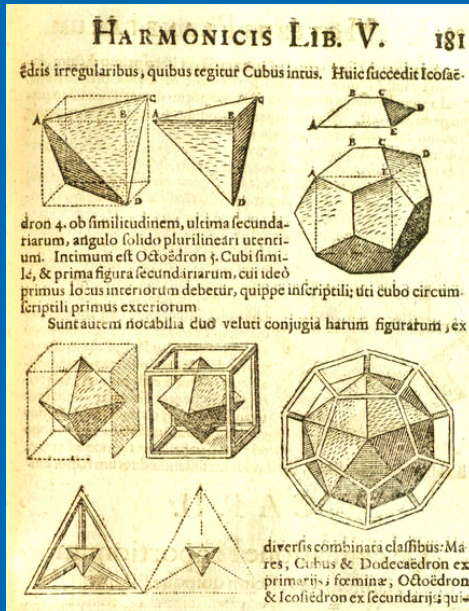
◆SG301, we will encounter the pioneering work of Hartmut Warm, a German engineer who is scientifically validating Kepler's intuitions.

Below are some of the achievements of Johannes Kepler:



- Kepler offered a geometric model of the solar system as a nesting of the five Platonic Solids (see next page). While the model does not exactly fit, Kepler’s overall insight (explaining the number & properties of the planets by the symmetry functions of the Platonic Solids) opened the way to understand the harmonic geometry of the solar system, or, in recent scientific terms, that the rotational energy of the sun is distributed throughout the solar system in a quantized way, according to the Golden Ratio.
- Kepler should also get recognition for his work on the relationship of the musical scale and the planetary distances. Kepler found that a planet’s nearest approach to the sun (perihelion) and its furthest distance from the sun were approximately in the same ratio to each other as the values of the frequencies on the musical scale.
- Based on this sacred geometric framework, Kepler also identified the position of the asteroid belt (which astronomers only discovered in early 19th century) as occurring at the note F# of his musical scale. This is the well-known position of the “register shift” in the human voice. Music and astrophysics may well have a lot in common, as anticipated by Kepler’s study of the “harmonies of the spheres”, and as re-discovered by current research [◆SG201].
- In botany, Kepler was the first to notice the relationship between leaf patterns and the Fibonacci series, writing: “*It is in the likeness of this self-developing series that the faculty of propagation is, in my opinion, formed; and so in a flower the authentic flag of this faculty is shown, the pentagon*”. [◆SG205]. He also discovered the link between the Fibonacci Series and the Golden Ratio.
- When Tycho Brahe died in 1601, Kepler became the Imperial Mathematician in Prague. Using the large body of data left by Brahe, Kepler discovered the first two laws of planetary motion as well as “*Kepler’s Supernova*” (1604). In Linz, Austria, he published *Harmonices Mundi* (1619) containing his third seminal law.
- Much of Kepler’s work is devoted to Sacred Geometry, calling the Golden Ratio “*the creation of likeness out of likeness*”.
- Kepler did fundamental work in optics, inventing an improved version of the refracting telescope.
- Kepler also did pioneering work on tiling (*tesselation*) in preview of the “Penrose Tiles”. [◆SG203.2]

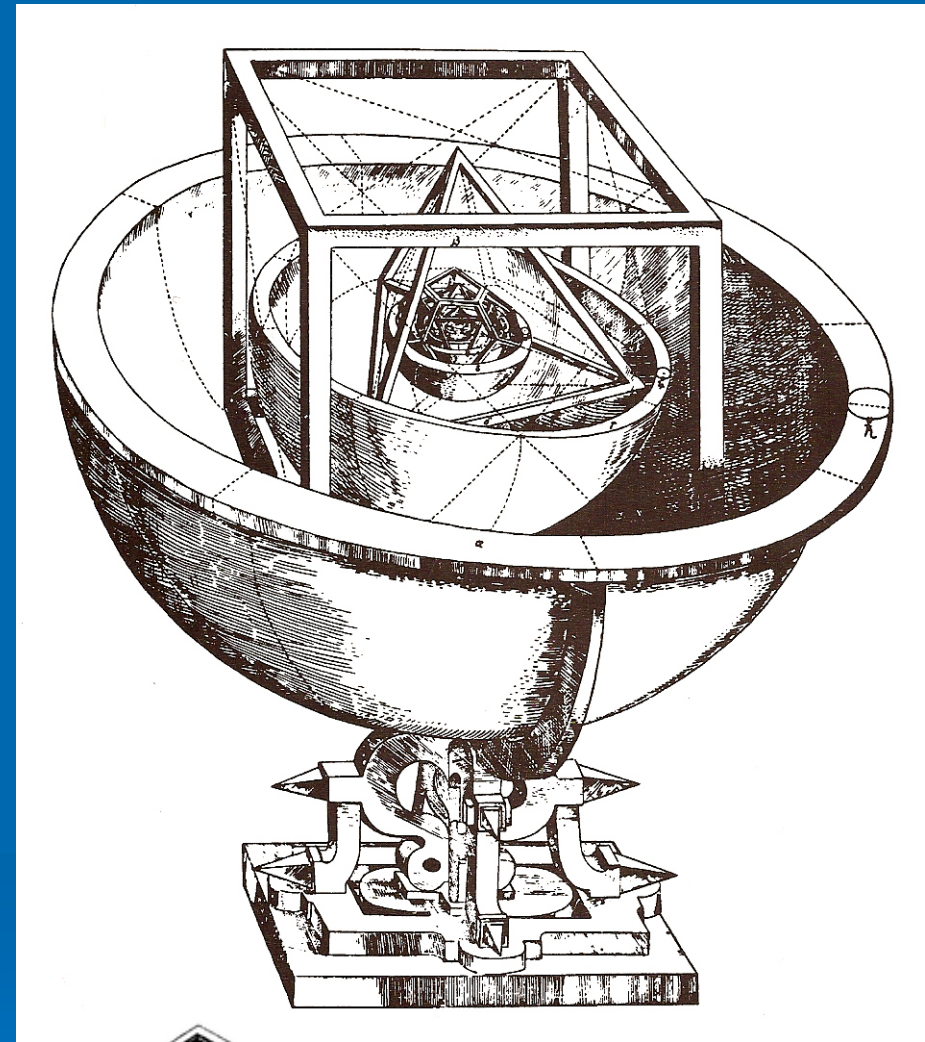
SG102.5.7.2 Johannes Kepler (2)



← Kepler's drawings of some of the five Platonic Solids.

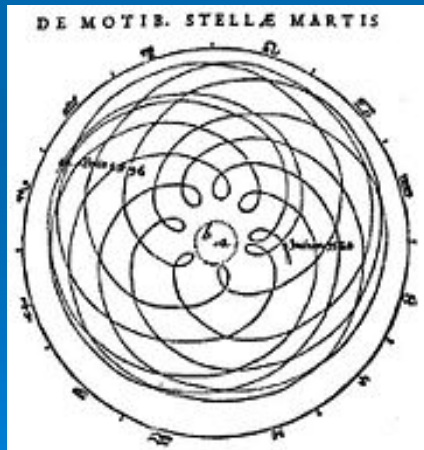
He calls the Tetrahedron an "androgyn".

(Book 5, page 181. *Harmonices Mundi*, 1619.)

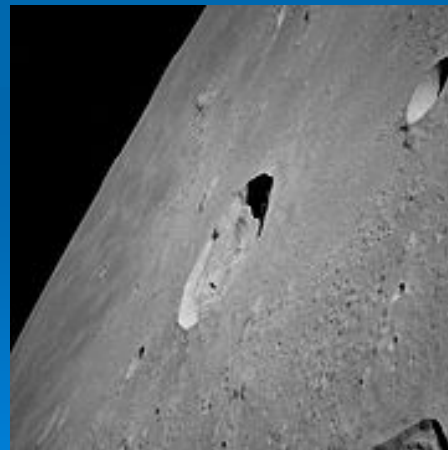


↑ Kepler's planetary model of nested polyhedra (5 Platonic Solids)

← Close-up of the inner planets.



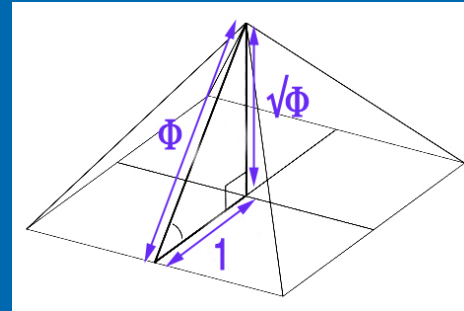
↑ Kepler's diagram of the geocentric trajectory of Mars.



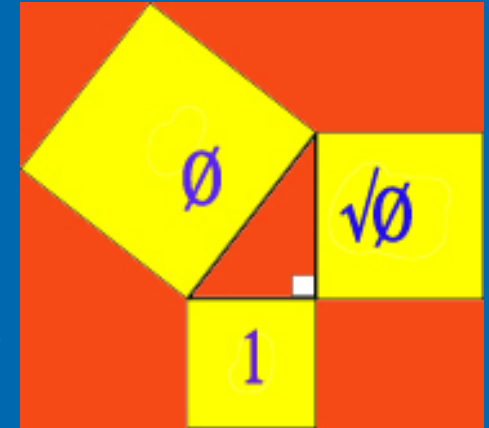
↑ "Kepler's Crater" on the Moon

SG102.5.7.3 Johannes Kepler (3)

*“Before the origin of things,
Geometry was co-eternal
with the Divine Mind”*



↑ Kepler's Triangle in the Great Pyramid.

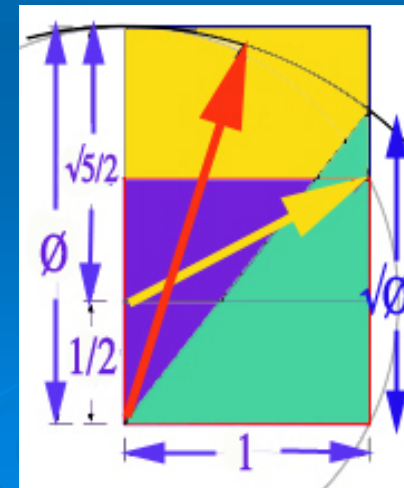


↑ “Kepler’s Triangle” is formed by 3 squares with areas in Phi progression. Two key mathematical principles are combined in Kepler’s Golden Triangle: the Pythagorean Theorem and the Golden Ratio. This fascinated Kepler.

“Geometry has two great treasures: one is the theorem of Pythagoras, the other the division of a line into mean and extreme ratio. The first we may compare to a mass of gold, the second we may call a precious jewel.”
(Kepler)



↑ As a little boy, Kepler witnessed the great comet of 1577

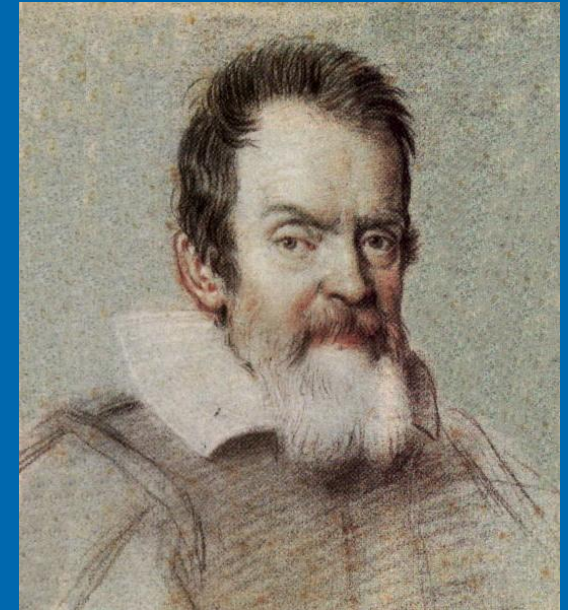


← Construction of Kepler’s Triangle

SG102.5.8 A Quote from Galileo

*“Philosophy is written in this grand book
-I mean the universe-
which stands continually open to our gaze,
but it cannot be understood
Unless one first learn to comprehend the language
and interpret the characters in which it is written.*

*It is written in the language of mathematics,
and its characters are triangles, circles,
and other geometrical figures,
without which it is humanly impossible to understand a single word of it;
without these one is wandering about in a dark labyrinth.”*



Galileo Galilei, Florentino.
By Ottavio Leone

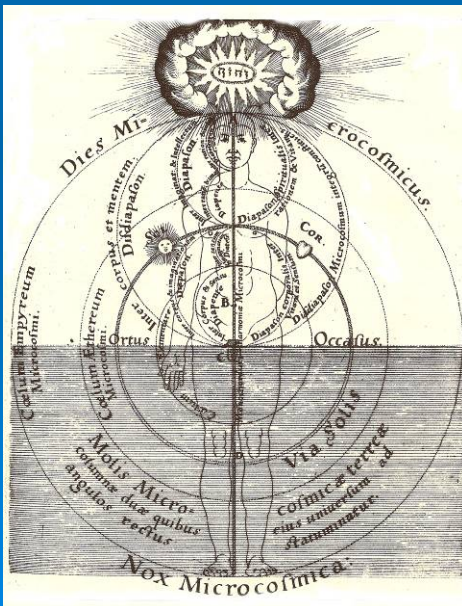
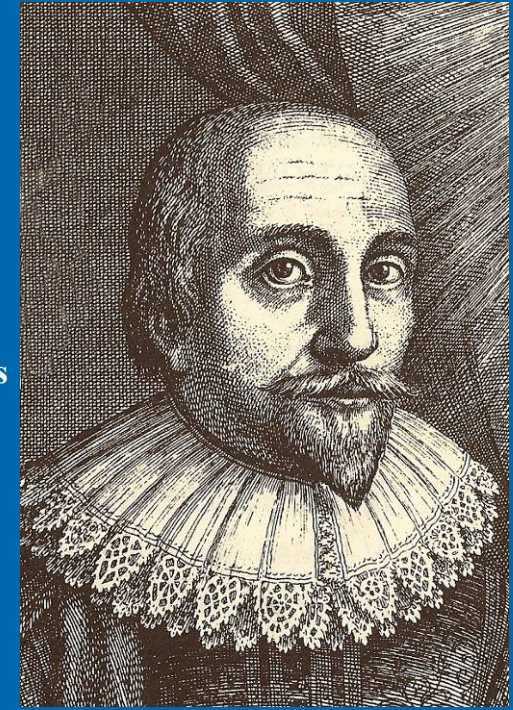
Galileo Galilei (1564-1642)

SG102.5.9.1 Robert Fludd (1)

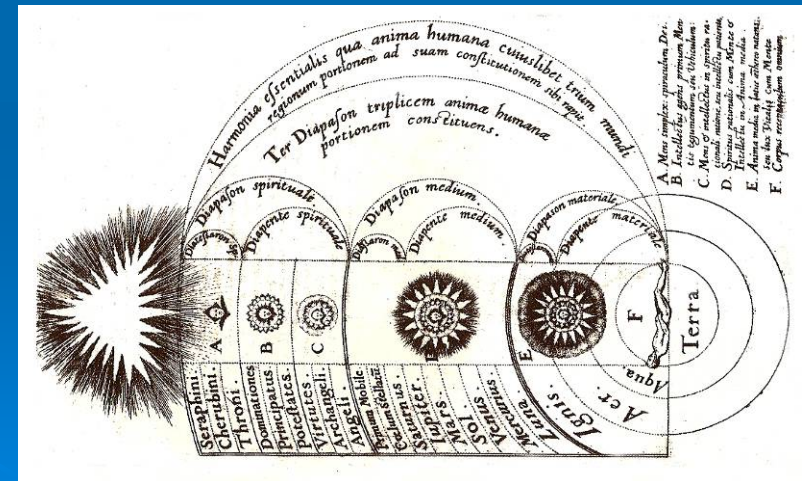
Robert Fludd (1574-1637), a prominent English physician and hermetic philosopher, was possibly the last Renaissance Man who could still aim at integrating the whole of human knowledge with a divine and hierarchically ordered cosmos.

Soon enough, René Descartes would lay the foundation for a separation between matter and spirit, thus launching the Scientific Revolution. Only now is the human consciousness reaching the point where a larger paradigm can incorporate scientific knowledge and metaphysical realities.

Robert Fludd expressed his unified cosmo-vision in visionary diagrams published in his magnum opus: *Utriusque Cosmi, Maioris scilicet Minoris, Metaphysica, Physica atque Technica Historia* - which translates as: *The Metaphysical, Physical and Technical History of the Two Worlds, namely the Greater and the Lesser* (1617-1621).



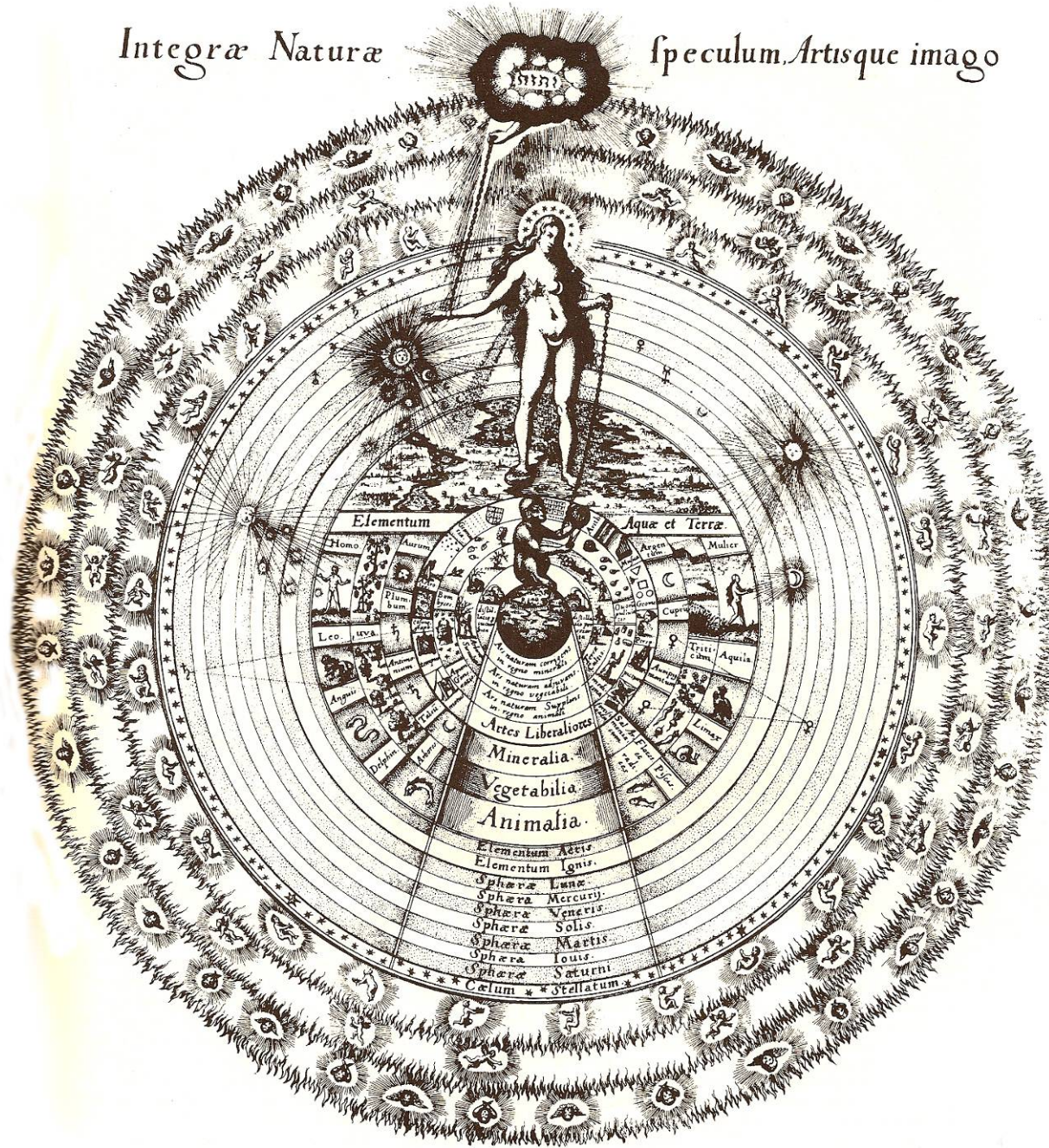
← The *Spiritus Mundi* (Spirit of the World) is represented by Fludd as a cord extending from God to the Earth and participating in both extremes. Along this cord are marked the stages of the soul's descent into the body (*Nox*) and its re-ascend after death (*Dies*).



↑ A Cosmos spanning 3 musical octaves

Integræ Naturæ

Speculum Artis que imago



SG102.5.9.2 Robert Fludd (2)

← “*The Mirror of the Whole of Nature and the Image of Art*” is the most comprehensive of all Fludd’s diagrams.

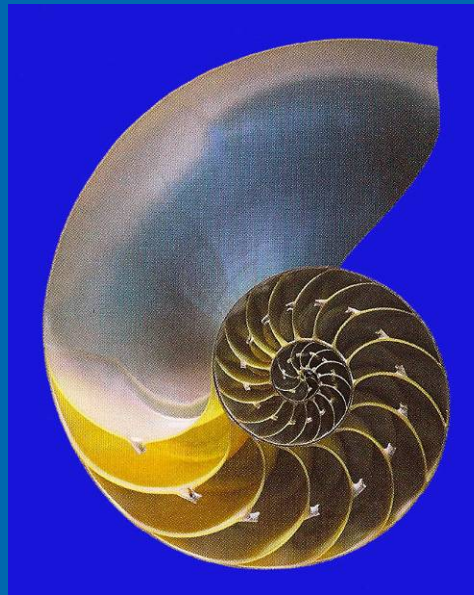
Standing upon the elements of water and earth, the figure of Nature is linked to God through the *catena aurea* (Golden Chain or Thread) but she is the one who turns the sphere of the stars and transmits the influences of the planets to the lower realms. She is the Mother Goddess nurturing all creation from her breast.

Nature, says Fludd, has a helper who imitates her by producing Art. This *Ape of Art* is the terminal in the chain of being, bearing the same relation to nature as She to God.

The Arts (and Sciences), for Fludd, are man’s means to make the Earth a happy and beautiful place - if he uses them rightly.

SG102.5.10 J. Bernoulli & the Spira Mirabilis

Jacob Bernoulli (1654-1705) was the first member of his distinguished family to achieve fame as a mathematician. Following his father's wishes, he first earned degrees in theology and philosophy. But his love was mathematics and he set up traveling throughout Europe to learn his trade. Influenced by Leibniz and his "infinitesimal calculus", Jacob Bernoulli contributed greatly to the foundation of calculus and was the father of the Probability Theory.



Jacob Bernoulli was fascinated by the relationship between mathematics and the cosmos. He studied extensively questions related to spirals and wrote a treatise called *Spira Mirabilis* (Latin = 'Wonderful Spiral') about the logarithmic spiral. He was deeply taken by the self-similarity of the Spira Mirabilis: any section, if properly scaled, is congruent with other portions of the spiral. Jacob wrote:

"(The logarithmic spiral) may be used as a symbol, either of fortitude and constancy in adversity, or of the human body, which after all its changes, even after death, will be restored to its exact and perfect self".

No wonder that Jacob Bernoulli, upon dying, requested the logarithmic spiral to be engraved on his tombstone, along with the motto: EADEM MUTATA RESURGO ('Though changed, I arise the same again'). As fate had it, the spiral engraved by the stone mason was the Archimedean spiral - not the Logarithmic spiral he so loved.



↑ The actual inscription

What is
should
have been →



SG102 Ca A Long Journey



SG102 Cb Conclusion to Module SG 102

Through these few historical windows into Sacred Geometry - over 6,000 years and more - you may have get a sense of a cyclical pulse bringing this knowledge to the forefront of certain cultures and then withdrawing it in the background, only to bring it back again.

This flux & reflux, this dance of appearance & disappearance of the Sacred Geometry wisdom, follows the adventures of human consciousness on planet Earth.

Even though Beauty and Harmony are ever present as the ground of spirit, no matter how the external 3D circumstances appear to be, the development of a culture of Beauty & Harmony requires collective conditions of peace & well-being and the spiritual dedication to co-create the cosmos on Earth.

We have reached the point again, in the evolutionary spiral of humankind, where we have the potential to develop a wise global culture of Peace, Beauty and Harmony.

Just like all of nature - and, closer to us, the human nature of our DNA - is cascading with Phi resonances, so have we the ability, gift and responsibility to consciously weave through the newly born Global-Culture-of-Life golden strands of Beauty and glorious dances of Harmony.

To this goal,
I raise the Golden Cup
of Infinite Beauty!

May you, in turn, be inspired
by a long line
of Sacred Ancestry!

SG102 Cc1 Modern & Contemporary Times (1)

This SG102 module on the History & Traditions of Sacred Geometry stops at the end of the Renaissance. A long hiatus was brought about by the Scientific Revolution and its ways of fragmenting life and separating matter & nature from spirit and cosmic dimensions.

Throughout the rest of this program, we will get acquainted with the forerunning research & discoveries of the 19th century as well as the courageous pioneers in the course of the 20th century who reopened the Great Sacred Book for many more to follow. In later modules & post-graduate seminars we will focus on 20th century luminaries such as Viktor Schauberg, Schwaller de Lubicz, Buckminster Fuller, Hans Kayser, Le Corbusier, Hans Jenny, John Michell and Dan Winter...

Adams, George - Anthroposophical Science
Aya - Sacred Geometry Encyclopedia
Beard, Robert S. - "Patterns in Space"
Bilheust, Henri - Art des Bâisseurs
Brunes, Tons - "The Secrets of Ancient Geometry"
Cathie, Bruce - Harmonic Grids
Church, A. H. - Phyllotaxy
Ernst Chladni (1756 - 1827) - Vibratory Sound Plates
Colman, Samuel - "Nature's Harmonic Unity"
Conner, William - "Math Metasonics"
Cook, Theodore - "The Curves of Life"
Critchlow, Keith - "Order in Space"
Doczi, Gyorgy - "Power of Limits"
Dudon, Jacques - Photosonic Mandalas
Escher, M.C. - Art of Tiling
Fratr Achad - "Anatomy of the Body of God"
Fuller, Buckminster - Geodesics
Ghyka, Matila - "Le Nombre d'Or"
Godwin, Joscelyn - Harmony of the Spheres
Hambidge, Jay - "Dynamic Symmetry"

SG102 Cc2 Modern & Contemporary Times (2)

Hargittai, Istvan - Fivefold Symmetry
Huntley, H. E. - "The Divine Proportion"
Jain - Mathemagics
Jenny, Hans - Cymatics
Heleus, Michael - Astrosonics
Hero, Barbara - Lambdoma Grids
Kayser, Hans (1891 - 1964) - "Lehrbuch Der Harmonik"
Keely, John W. (1837 - 1898) - Sympathetic Physics
Langham, Derald G. - Genesa & "Circle Gardening"
Le Corbusier (1887 - 1965) - The "Modulor" scale
Lawlor, Robert - "Sacred Geometry"
Lonegren, Sig - "Labyrinths"
McCLain, Ernest - Tone Cosmologies
Melchizedek, Drunvalo - "Flower of Life"
Lund, F. M. - Geometry "Ad Quadratum"
Michell, John - "The Dimensions of Paradise"
Penrose, Roger - Phi Tiling
Pennick, Nigel - "Sacred Geometry"
Pietsch, Bernard - Canonic Metrology
Rawles, Bruce - "Sacred Geometry Design Book"
Schwaller de Lubicz, R. A (1887 - 1961) - "The Temple in Man"
Schauberger, Viktor (1885 - 1958) - Implosion Technology
Schneider, Michael - "Constructing the Universe"
Steiner, Rudolf (1861 - 1925) - "The Fourth Dimension"
Taylor, Thomas (1758 - 1898) - "Theoretic Arithmetic"
Tenen, Stan - Language Geometries
Thompson, D'Arcy (1860 - 1948) - "On Growth and Form"
Winter, Dan - Phi Harmonics of the Heart

... and many more teachers & friends
I am honored to acknowledge here...

SG102 Cd1 Sacred Geometry Landmarks in the Western Cultures (1)

Ancient Times

Sumer & Babylon - Pentagon designs
Neolithic polyhedral stones
Egypt - Great Pyramid - Art - "Flower of Life" at Abydos
Hermes Trismegistus / Thoth - Numbers
Bible tradition - Gematria & measures - Temple of Solomon
Zoroastrian tradition
Orpheus & Orphic traditions
Greece - Art & architecture - Parthenon - Phidias
Thales of Milet (~ 625-547 BCE) - Gnomonic Geometry
Pythagoras (6th century BCE)
Hippocrates of Chios (5th century BCE) - Study of Proportions
Plato (428-348 BCE) - "Timaeus" - Number Cosmology - Platonic Solids
Euclid (3rd century BCE) - "The Elements"
Archimedes (c. 287-212 BCE) - Approximations of Pi
Roman-era dodecahedra
Vitruvius (1st century BCE) - "Ten Books on Architecture"
Gnostic traditions - Numerical mysticism
Ptolemy (100-179 CE) - "The Almagest" - Angular Trigonometry
Neo-Pythagoreans (1st - 2nd c. CE): Theon of Smyrna - Nichomachus of Gerasa
Greek Arithmology: Philo of Alexandria - Clement of Alexandria
Neo-Platonists (2nd - 3rd c.): Plotinus ("Enneads") - Porphyry
Iamblichus (c. 250-330) - "Life of Pythagoras" - "Theology of numbers"
Diophantes (325-400) - Polygonal Numbers

SG102 Cd2 Sacred Geometry Landmarks in the Western Cultures (2)

Middle Ages & Renaissance

Medieval Magic Squares

Boethius (480-525) - "On Arithmetic" - "On Music"

Islamic mathematics & art - Al-Khwarizmi - Al Biruni - Islamic Mosques

Jewish mysticism: Kabbalah & Gematria

Templar tradition (11th - 13th century) - Occult Geometry

Saint Bernard - Cistercian abbeys

"Ecole de Chartres" - Cathedrals - Labyrinths

Leonardo of Pisa, aka Fibonacci (1170-1240) - "Fibonacci" series

Islamic pentagonal geometries (10th - 13th c.)

Villard de Honnecourt (13th c.) - "Les Carnets d'Esquisse"

Marsilio Ficino - Platonic Academy of Cosimo de Medici (1389 - 1464)

Piero della Francesca (1415-1492) - "De Quinque Corporibus"

Luca Pacioli (1445-1515) - "De Divina Proportione"

Leonardo da Vinci (1452-1519) - Illustr. "De Divina Proportione"

Albrecht Dürer (1471-1528) - "Melancholia"

Fra Giovanni's Intarsia Polyhedra (c. 1520)

Johannes Kepler (1571-1630) - "Harmonices Mundi"

Robert Fludd (1574-1637) - "Utriusque..."

Jacob Bernoulli (1654-1705) - Spira Mirabilis

Cornelius Agrippa (16th c.) - Numbers & Letters divination

SG102 Ce1. Credits & References (1)

Friendly Note: Some images in our picture files were not referenced.
Kindly contact us if you find an image or design that you would like us to reference.
Thank you.

- 102.1.1.2 *Great Pyramid within hemisphere.* Peter Tompkins. *Secrets of the Great Pyramid.* Harper, 1971.
- 102.1.1.3 *Aprons.* Schwaller de Lubicz. *Le Temple de l'Homme.* Dervy, 1977.
Temple in Man. Schwaller de Lubicz. *The Egyptian Miracle. Inner traditions,* 1985.
- 102.1.1.4 *Temple of Khafra.* Moustafa Gadalla. *Egyptian Harmony,* Tehuti, 2000.
- 102.1.1.5 *Karnak.* Ibid.
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- 102.1.3.3 *Stonehenge.* John Michell. *City of Revelation.* Ballantine, 1976.
- 102.1.5.1 *Megalithic Polyhedra.* Keith Critchlow. *Time Stands Still.* Fraser, 1979.
- 102.2.1.1 *Pythagoras.* K. S. Guthrie. *Pythagorean Sourcebook.* Phanes, 1987.
- 102.2.1.2 *Pythagoras.* M.P. Hall. *Secret Teachings of All Ages.* PRS, 1987.
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St Peter's Dome. George L. Hersey. Architecture & Geometry in the Age of Humanism. U of Chicago, 2000

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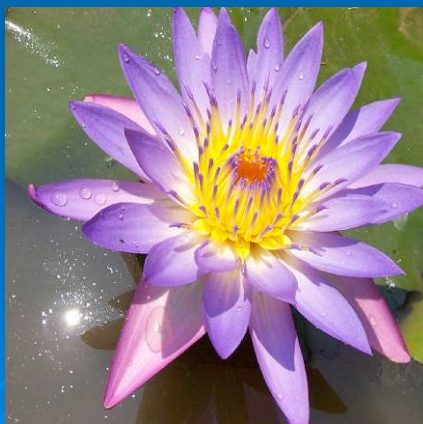
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SG102 Cf1 Online SG School Curriculum: Intro & Intermediate

Sacred Geometry Introductory Level: 8 Modules

SG 101	Intro I	Sacred Geometry: Universal Order & Beauty
SG 102	Intro II	History & Traditions of Sacred Geometry
SG 103	Intro III	Sacred Geometry: A Grand Tour
SG 104	Intro IV	PHI: the Golden Ratio & the Fibonacci Series
SG 105	Intro V	Pentagons, Pentagrams & the Penta-Modules
SG 106	Intro VI	The Golden Rectangle & Golden Spiral
SG 107	Intro VII	The Five Platonic & 13 Archimedean Solids
SG 108	Intro VIII	The Vesica Piscis: Womb of Creation

Sacred Geometry Intermediate Level: 8 modules

SG 201	Interm I	The Monochord, Music & Cymatics
SG 202	Interm II	The Power of Archetypal Numbers
SG 203A	Interm IIIA	Sacred Geometry Resurgence in Science - Part 1
SG 203B	Interm IIIB	Sacred Geometry Resurgence in Science - Part 2
SG 204	Interm IV	PHI in the Human Body, Biology & DNA
SG 205A	Interm VA	The SG of Nature - Part 1: Plants & Phyllotaxis
SG 205B	Interm VB	The SG of Nature - Part 2: Animals & Minerals
SG 207	Interm VII	SG in Architecture, Sacred Sites & Green Design

SG102 Cf2 Online SG School Curriculum: Advanced

Sacred Geometry Advanced Level: 8 modules

SG 301	Adv I	Golden Cosmos: Planets, Stars & Cosmology
SG 302	Adv II	SG in Art, Culture & Creativity
SG 303	Adv III	Universal Symbols: Primordial Knowledge
SG 304	Adv IV	Labyrinths: a Mini-Pilgrimage to Self
SG 305	Adv V	Mandalas & Yantras: Sacred Vortices
SG 306	Adv VI	Languages & Gematrias: Sacred Communication
SG 307	Adv VII	Sacred Geometry in the Healing Arts
SG 308	Adv VIII	Harmony on Earth: Science & Consciousness of Harmony

Upon completion of each level (Introductory, Intermediate & Advanced), a Certificate of Graduation from the Sedona School of Sacred Geometry will be presented to Certification Students.

Postgraduate seminars on current Sacred Geometry research, discoveries & updates will be organized in the harmonic future.

Questions: phi@schoolofsacredgeometry.org

SG102.Cg Reading Suggestions

For this SG102 module, you are invited to enjoy some home reading about the History & Traditions of Sacred Geometry. Here are some suggestions:

Skinner, Stephen. *Sacred Geometry, Deciphering the Code*. Sterling, New York, 2006.

A well formatted and visually pleasant book with full color images. Many sidebars on historical aspects of Sacred Geometry. Perfect to thumb through and to pictorially absorb the gist of the subject.

Lawlor, Robert. *Sacred Geometry, Philosophy & Practice*. Thames & Hudson, London, 1982.

A now classic introduction from a philosophical & metaphysical perspective with hands-on practices and many unique insights on Sacred Geometry as a perennial unifying knowledge. A serious but deep reading.

Livio, Mario. *The Golden Ratio: the Story of Phi*. Broadway Books, New York, 2002. Written by an astrophysicist from the scientific perspective. Contains new science developments about Sacred Geometry as well as many historical descriptions & anecdotes. An enjoyable reading experience.

Hemenway, Priya. *Divine Proportion, Phi in Art, Nature & Science*. Sterling, New York, 2005.

Another full color pictures panoramic view of Sacred Geometry and its expression: Beauty. Many historical highlights. An insightful text pointing back to the wisdom of Who We Are.

For website links, go to our website:

www.schoolofsacredgeometry.org

SG102Ch StarWheel Blessing



SW#22. "Octo-Gate" . www.starwheels.com



SG102.Ci Contact Info

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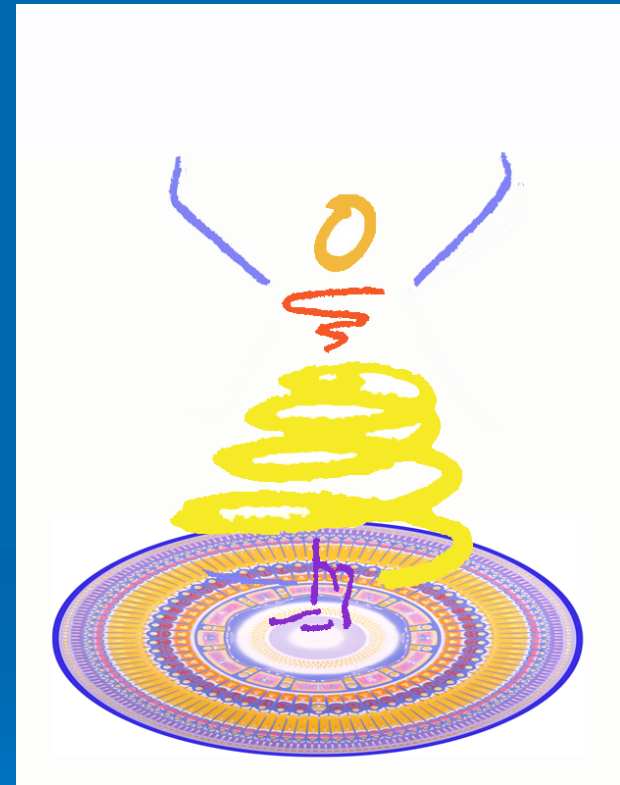
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On Facebook: Aya Sheevaya
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Φ celebration



SG102.Cj About Aya

A native of France, Aya is a visionary artist and celebration yogi who has dedicated his life to serve humanity and to develop sacred arts education. In his late 20's, Aya realized that his professional life in the French diplomatic service was not fulfilling his heart's desires; he quit everything to go on an extended vision quest. His path took him around the world to visit a variety of sacred sites & cultures and to receive inspiration from many teachers.

In 1985, in Santa Monica, CA, Aya was gifted with a spiritual vision prompting him to create a series of 108 airbrushed neo-mandala paintings: the "StarWheels". The StarWheels, a happy family of vibratory flowers for the Earth, are looking for sacred spaces to be graced with their presence... (www.starwheels.com / www.starwheelmandalas.com)

Moving to Sedona, Arizona, in 1997, Aya has been involved with sacred arts classes & events, mandala creation, Sedona guided tours, labyrinth making and Sacred Geometry teaching. Aya has presented several StarWheel art exhibits, has sponsored community awareness events at the Sedona Library, has developed, in collaboration with Gardens for Humanity, the Peace Garden arboretum at the Sedona Creative Life Center, was a speaker at the Sacred Geometry Conference (Sedona, 2004), co-designed several labyrinth sites (The Lodge at Sedona, Magos' Ranch...), and was on the management team of the Raw Spirit Festival in 2006 - 2008.

Realizing that Sedona was progressively becoming a global spiritual university for many seekers from around the world, Aya founded in 2005 the Sedona School of Sacred Geometry. The school is offering online access to Sacred Geometry PDF modules, with 17 modules completed so far. In the school's website, Aya states: *"We are living at the extraordinary and exciting times of a global transformation to a higher order of human consciousness... Sacred Geometry is the expression and resurrection of our deep innate wisdom, now awakening from a long sleep: seeing again the all-encompassing, fractal-holographic unity of nature, life and spirit... The keyword is HARMONY."* (www.schoolofsacredgeometry.org)

Aya's visionary dream, supported by his non-profit educational organization, The StarWheel Foundation, is the co-creation of an international eco-village "The School of Celebratory Arts" - a green, tropical environment encouraging young people of all nations to develop their creative consciousness and thus contribute to a new, spirited, life-respecting global civilization on Earth. (www.starwheelfoundation.org).

Since 2012, Aya is dancing the body divine, after his re-discovery of Yoga, Partner Yoga and AcroYoga. Aya is currently the AcroYoga.org Jam coordinator for Sedona and a teacher of yoga swing asanas.

Blessings in Anjali!